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Solution of the second and fourth order differential equations using irbfn method

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Abstract:

In this paper we introduce presents a numerical approach, based on radial basis function networks (RBFNS), for the approximation of a function and its derivatives (scattered data interpolation), The proposed approach here is called the indirect radial basis function network (IRBFN) to solve second and fourth order differential equations the procedure can start with the second derivative. First, the second order derivative is approximated by a RBFN, then the first order derivative is obtained by integration. Finally the original function is similarly obtained, i.e. by integrating the first derivative function. This second method is here referred to as the second indirect method or IRBFN2 likewise fourth -grade derivative IRBFN4

1. INTRODUCTION:

It can be seen that the differentiation process is very sensitive to even a small level of noise. In contrast it is expected that on average the integration process is much less sensitive to noise. Based on this observation, it is proposed here that the approxi- mation procedure starts with the derivative function using RBFNS original function is obtained by integration. [1]

Numerical methods for differentiation are of significant interest and importance in the study of numerical solutions of many problems in engineering and science. For example, the approximation of derivatives is needed either to convert the relevant governing equa tions into a discrete form or to numerically estimate various terms from a set of discrete or scattered data. This is commonly achieved by discretizing the domain of analysis into a number of elements which are denied by a small number of nodes. The interpolation of a function and its derivatives over such an element from the nodal values can
then be achieved analytically via the chosen shape functions. an alternative approach based on integration to construct the RBF expressions
for the interpolation of functions and the solution of differential equations was proposed
(Mai-Duy and Tran-Cong, 2001,2003). It was found that the indirect/integration-based
RBFN approach (IRBFN) outperforms the direct/differentiation-based RBFN approach
(DRBFN) regarding accuracy and convergence rate over a wide range of the RBF width.
The improvement is attributable to the fact that integration is a smoothing operation
and is more numerically stable[2].. Thus there are great interests in element-free numerical methods in both engineering and scientific communities. In particular, neural networks have been developed and become one of the main fields of research in numerical analysis.

Radial Basis Function Networks (RBFNs) [3] can be used for a wide range of applications primarily because it can approximate any regular function [4] and its training is faster than that of a multilayer perceptron when the RBFN combines self-organised and supervised learning [3]. The design of an RBFN is considered as a curve-fitting (approximation) problem in a high-dimensional space. Correspondingly, the generalization of the approach is equivalent to the use of a multidimensional surface to interpolate the test data [5]. The networks just need an unstructured distribution of collocation points throughout a volume for the approximation and hence the need for discretisation of the volume of the analysis domain is eliminated. In this paper new approximation methods based on RBFNs are reported. The primary aim of the presented methods is the achievement of a more accurate approximation of a target function's derivatives. From the results obtained here it is suggested that the present IRBFN approach could be, in addition to its ability to approximate scattered data, a potential candidate for future development of element-free methods for engineering modelling and analyses. Divo and Kassab (2007) developed a localized radial basis function meshless method (LCMM) for a solution of coupled viscous uid ow and conjugate heat transfer problem [6] . Mai-Duy (2007) presented a one dimensional integrated ra- dial basis function network (1D-IRBFN) collocation method for the solution of second-and fourth-order PDES [7] . Ngo-Cong, Mai-Duy, Karunasena, and Tran-Cong (2012) proposed a local moving least square-one dimensional integrated radial basis function network method (LMLS-1D-IRBFN) for simulating 2-D incompressible viscous ows in terms of stream function and vorticity [8].

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2. ID-IRBFN METHOD:

The ID-IRBFN methods [Mai-Duy and Tanner (2007)] including ID-IRBFN-2 and 1D-IRBFN-4 schemes are briefly described here [7]

2.1. Second-order 1D-IRBFN (1D-IRBFN-2 scheme):

The second-order derivative of u is decomposed into RBFs. The RBF networks are then integrated twice to obtain the first-order derivatives and the function itself.

$$\frac{\partial^2 \mathrm{u(x)}}{\partial \mathrm{x}^2} = \sum_{\mathrm{i=1}}^{\mathrm{N_x}} \mathrm{W^{(i)} G^{(i)}(x)} = \sum_{\mathrm{i=1}}^{\mathrm{N_x}} \mathrm{W^{(i)} H_{(i)}^{[2]}(x)}, \tag{1}$$

$$\frac{\partial u(x)}{\partial x} = \sum_{i=1}^{N_x} W^{(i)} H_{(i)}^{[1]}(x) + c_1 \tag{2}$$

$$u(x) = \sum_{i=1}^{N_z} W^{(i)} H_{[0]}^{(i)}(x) + c_1 x + c_2 \tag{3}$$

Where $N_x^{|j|}$is the number of nodes on the grid linc$|j|$; $\{\mathrm{W^{(i)}}\}_{\mathrm{n-1}}^{N_x^{|j|}}$ RBF weights to be determined; $\{\mathrm{G^i(x)}\}_{\mathrm{i=1}}^{N_x^{|j|}} = \left\{\mathrm{H_{[2]}^{(i)}(x)}\right\}_{\mathrm{i=1}}^{N_x^{|j|}}$ known RBFs; $\mathrm{H_{[1]}^{(i)}(x)} = \int \mathrm{H_{[1]}^{(i)}(x)dx}$; $\mathrm{H_{[0]}^{(i)}(x)} = \int \mathrm{H_{[1]}^{(i)}(x)dx}$; and c_1 *and* c_2 integration constants which are also unknown.

An example of RBF, used in this work , is the multiquadrics $G^{(i)}(x) = \sqrt{(x - x^{(i)})^2 + a^{(i)^2}}$, $a^{(i)}$ is the RBF width determined as $a^{(k)} = \beta d^{(k)}, \beta$ a positive factorand $d^{(k)}$ the distance from the k^{th} centre to the neareste neighbor.

Example 2.1.1:

Solve 2^{th} order partial differential equation.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

- We solve this equation using IRBFN and devlope matlab code for it.

We used intial condition which is $u_0 = 5\exp(M)$ with time priod [0,5pi] , $M = -(-x-2)(x-2)$ and $t = 10$ and we get the following resultis.

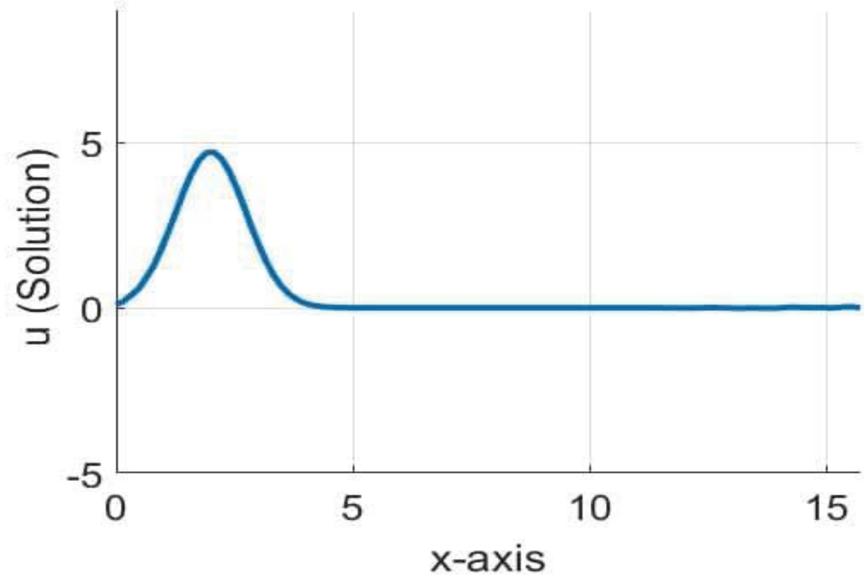


Figure 1: The solution at to total time=10

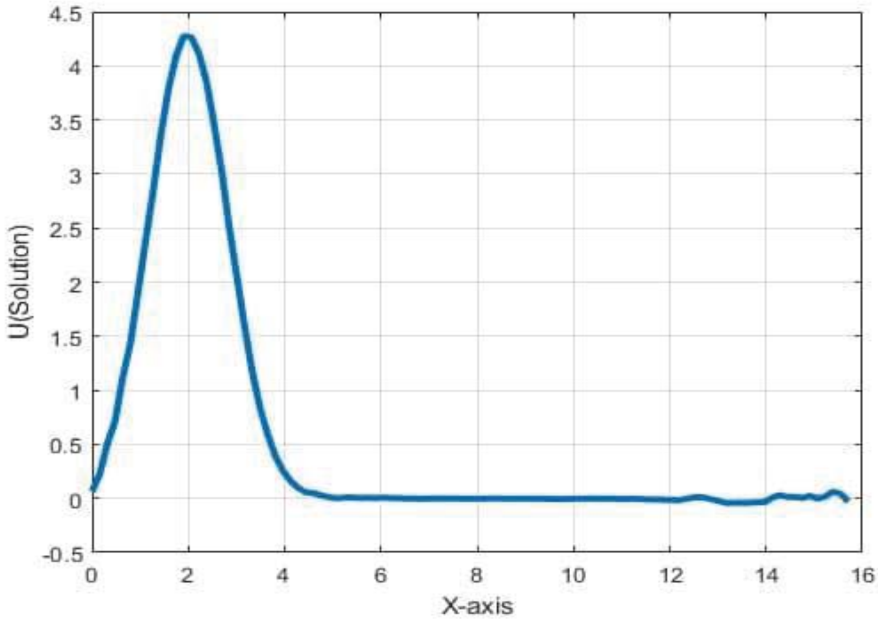


Figure 2: The solution at time step 90

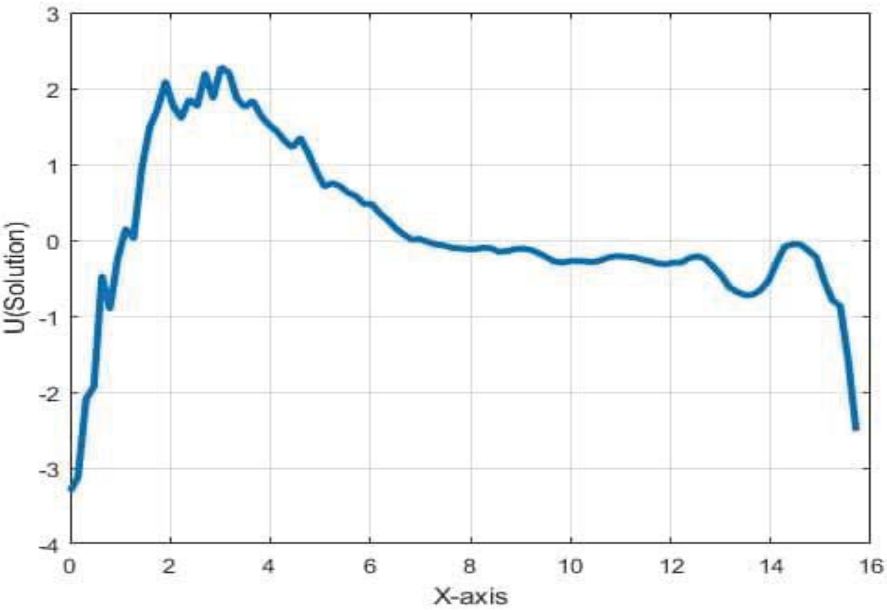


Figure 3: The solution at time step 900

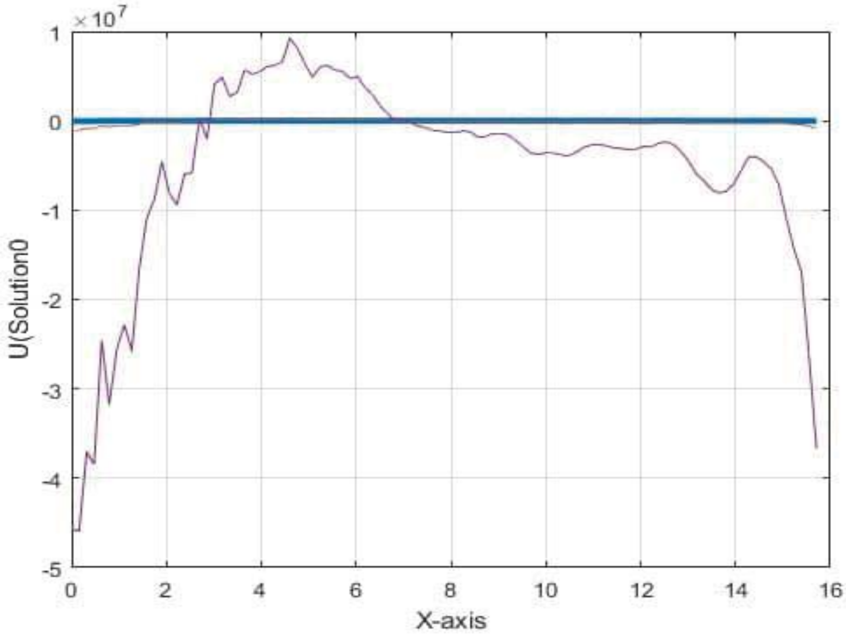


Figure 4: The solution at time step 10001

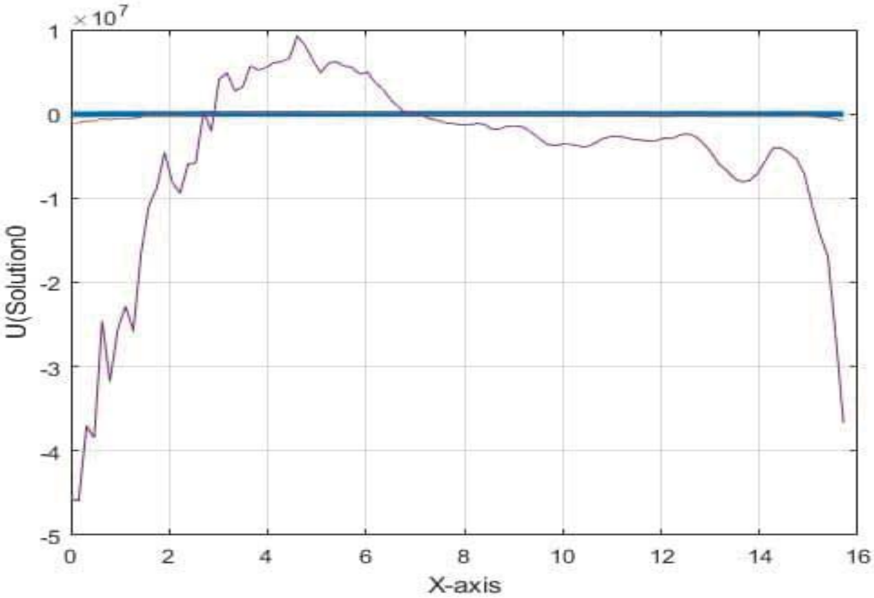


Figure 5: The solution at different moment time (6000, 8000, 10000, 10001)

- The same equation in example (2.1.1) will solve it usinq different initial condition which is $u_0 = \cos(x)$ with time priod [0,5pi] and $t = 10$ and we get the following resultis.

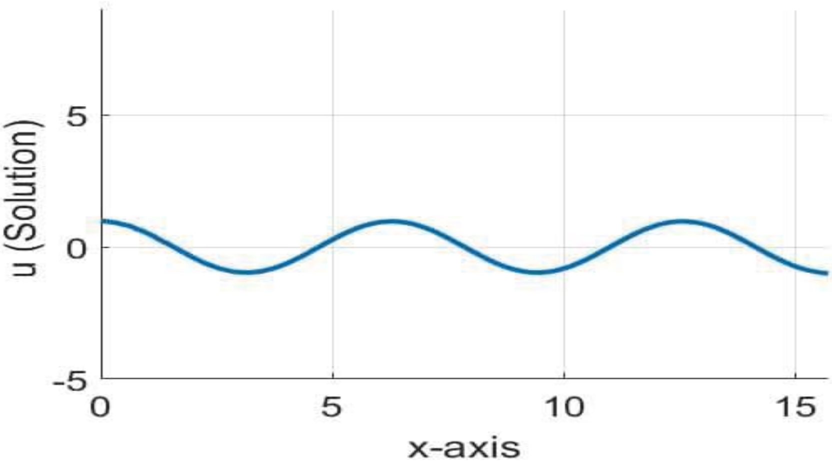


Figure 6: The solution at total time =10

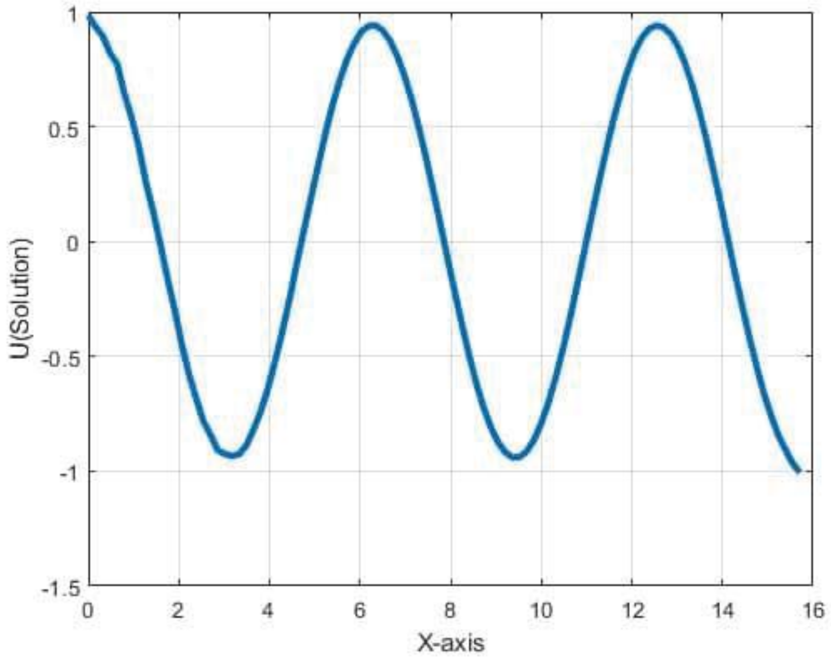


Figure 7: The solution at time step 60

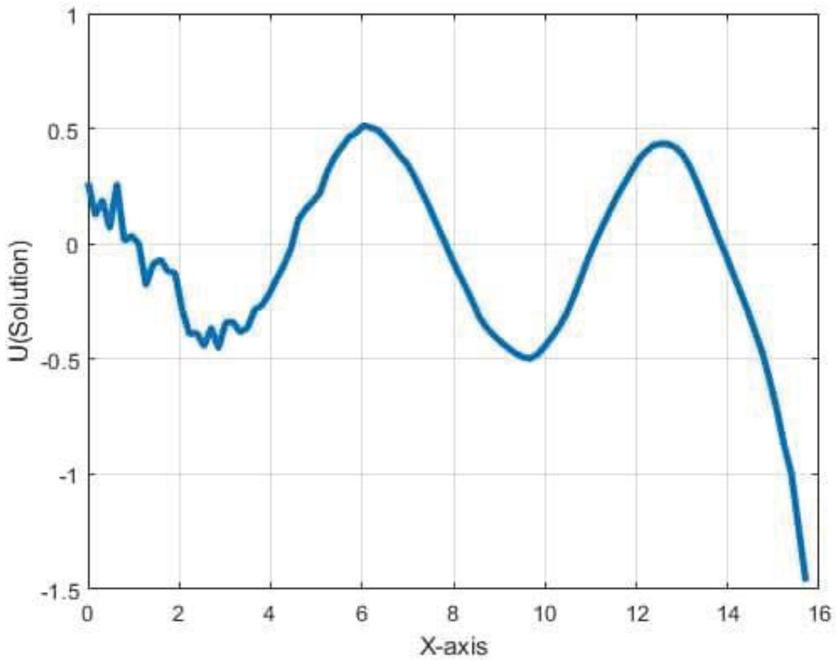


Figure 8: The solution at time step 800

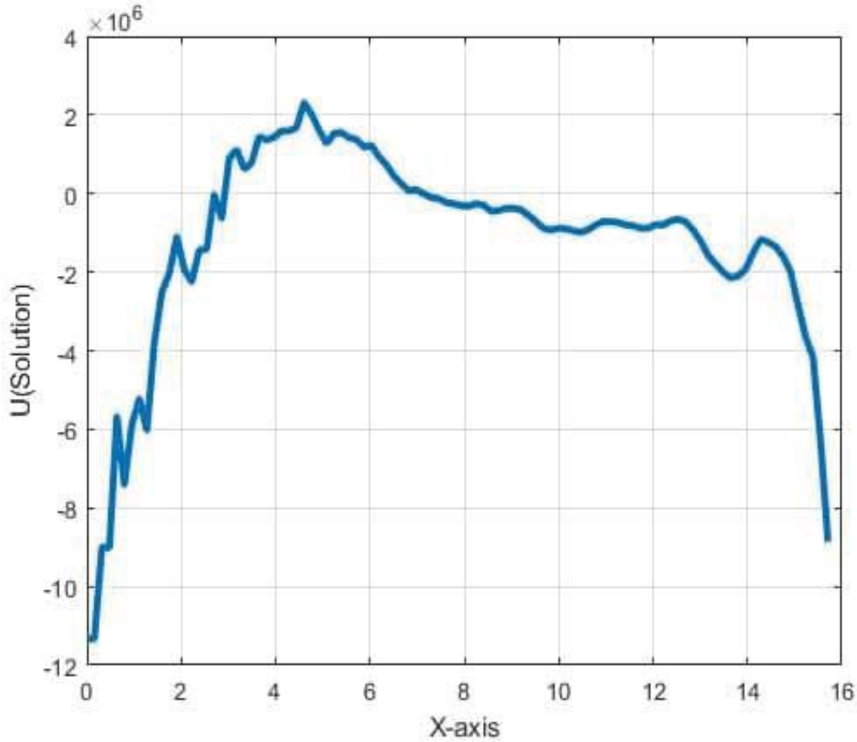


Figure 9: The solution at time step 10001

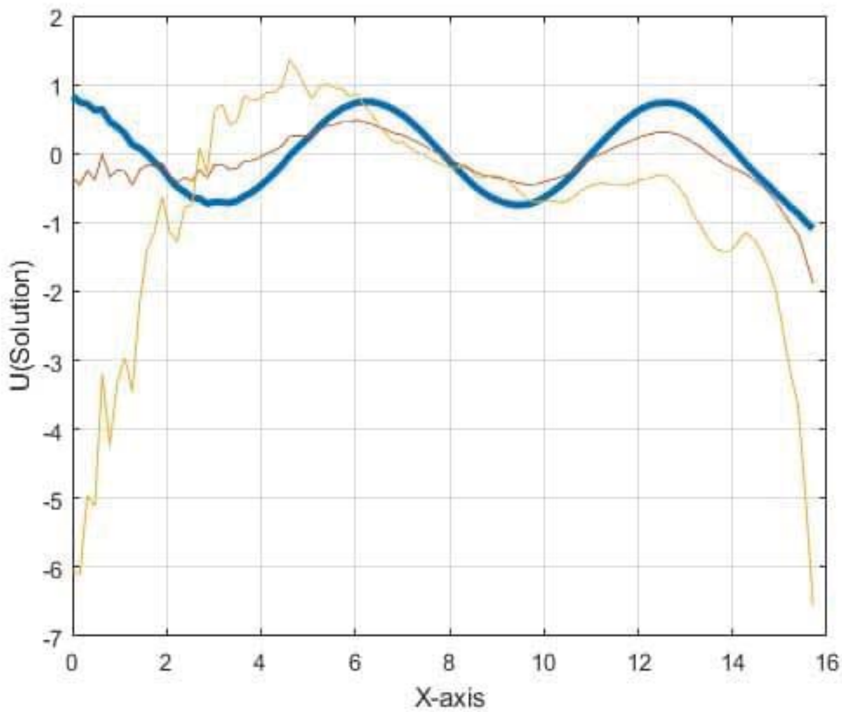


Figure 10: The solution at different moment time (300, 1050, 2000)

- With the same example (2.1.1) will solve it using different initial condition which is $u_0 = \sin(x)$ with time priod [0,5pi], and $t = 10$ and we get the following resultis

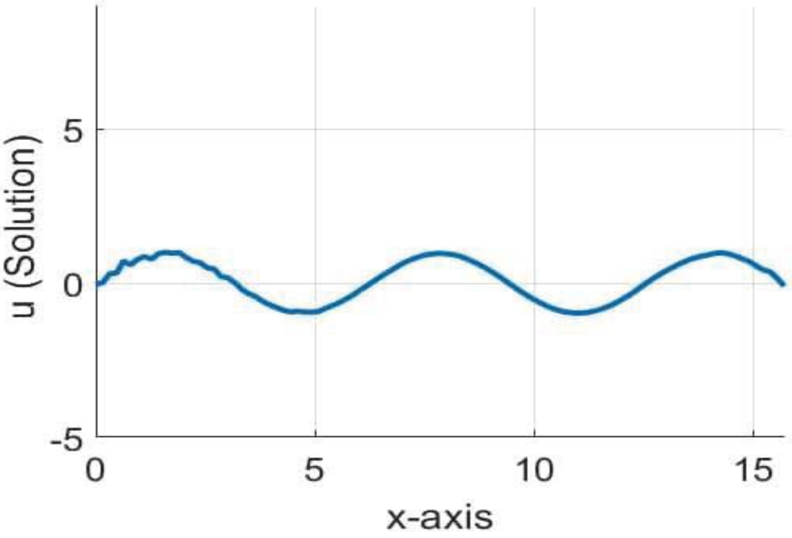


Figure11: The solution at total time =10

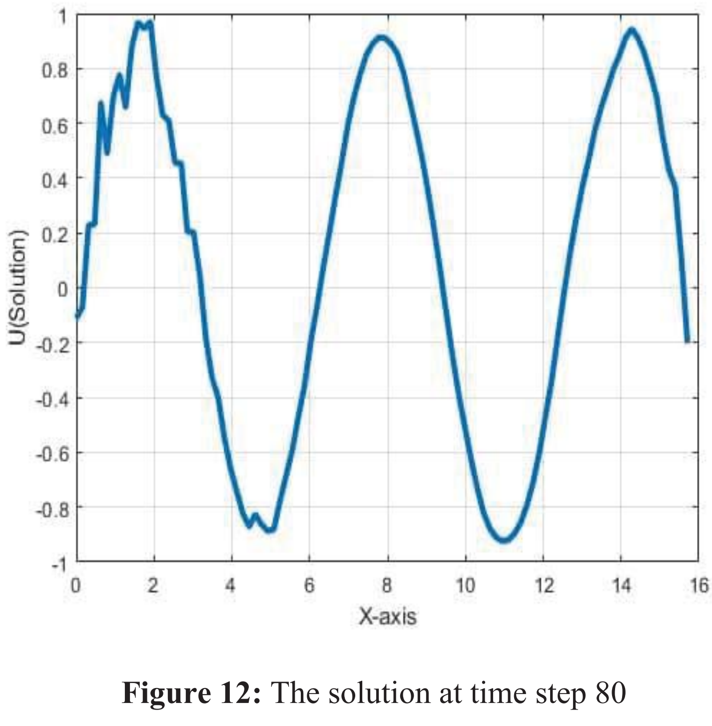


Figure 12: The solution at time step 80

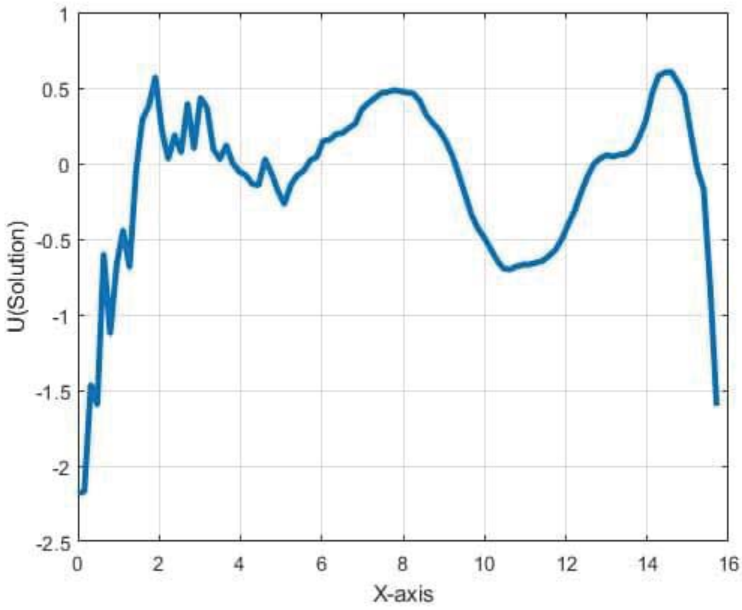


Figure 13: The solution at time step 600

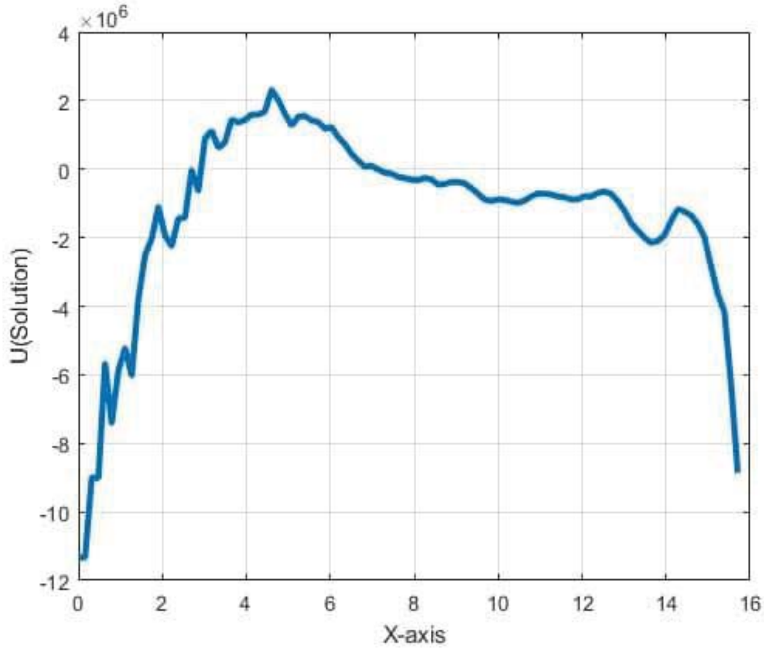
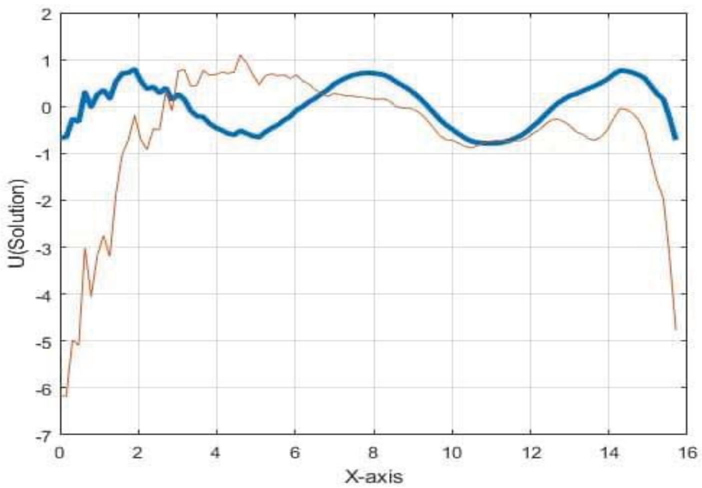


Figure 14: The solution at time steps =10001



Figer 15: The solution at different moment time (300, 1050)

2.2. Fourth-order 1D-IRBFN (1D-IRBFN-4 scheme):

In the 1D -IRBFN-4 scheme, the fourth-order derivative is decomposed into RBFs. The RBF networks are then integrated to obtain the lower-order derivatives and the function itself.

$$\frac{\partial^4 u(x)}{\partial x^4} = \sum_{i=1}^{N_x} W^{(i)} G^{(i)}(x) = \sum_{i=1}^{N_x} W^{(i)} H_{[4]}^{(i)}(x), \tag{4}$$

$$\frac{\partial^3 u(x)}{\partial x^3} = \sum_{i=1}^{N_x} W^{(i)} H_{[3]}^{(i)}(x) + c_1, \tag{5}$$

$$\frac{\partial^2 u(x)}{\partial x^2} = \sum_{i=1}^{N_x} W^{(i)} H_{[2]}^{(i)}(x) + c_1 x + c_2, \tag{6}$$

$$\frac{\partial u(x)}{\partial x} = \sum_{i=1}^{N_x} W^{(i)} H_{[1]}^{(i)}(x) + \frac{c_1}{2} x^2 + c_2 x + c_3, \tag{7}$$

$$u(x) = \sum_{i=1}^{N_x} W^{(i)} H_{[0]}^{(i)}(x) + \frac{c_1}{6} x^3 + \frac{c_2}{2} x^2 + c_3 x + c_4, \tag{8}$$

where $\left\{G^{(i)}(x)\right\}_{i=1}^{N_x} = \left\{H_{[4]}^{(i)}\right\}_{i=1}^{N_x}$ are known RBFs; $H_{[3]}^{(i)}(x) = \int H_{[4]}^{(i)}(x)dx$; $H_{[2]}^{(i)}(x) = \int H_{[3]}^{(i)}(x)dx H_{[1]}^{(i)}(x) = \int H_{[2]}^{(i)}(x)dx$; $H_{[0]}^{(i)}(x) = \int_C H_{[1]}^{(i)}(x)dx$; and $\{c_i\}_{i=1}^{4}$ integration constants which are also unknown.

Example 2.2.1:

Solve 4th order partial differential equation.

$$\frac{\partial u}{\partial t} = 2\left(\frac{\partial u}{\partial x}\right)^2 \frac{\partial^2 u}{\partial x^2}$$

- We solve this equation using IRBFN and devlope matlab code for it.

We used intial condition which is $u_0 = 5\exp(M)$ with time priod [0,5pi] , $M = -(-x-2)(x-2)$ and $t = 10$ and we get the following resultis.

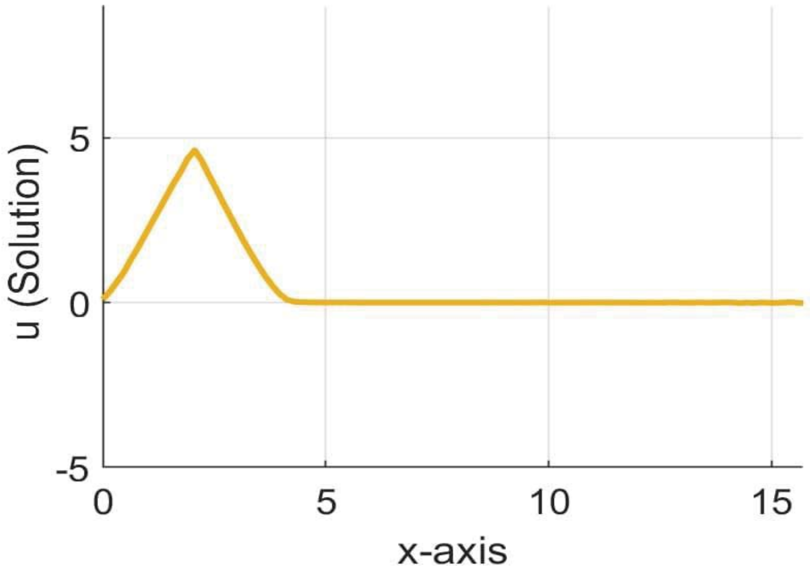


Figure 16: The solution at total time =10

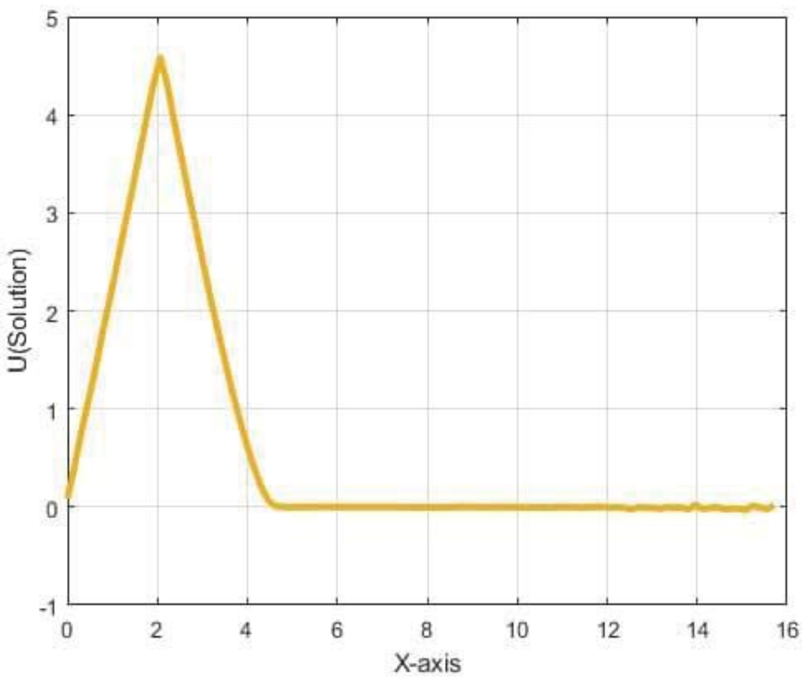


Figure 17: The solution at time step 90

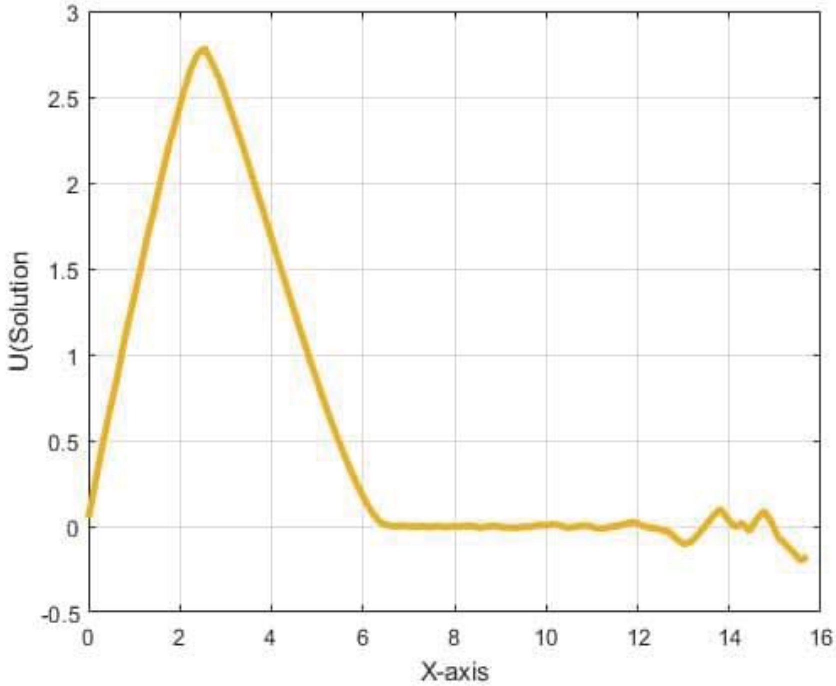


Figure 18: The solution at time step 1500

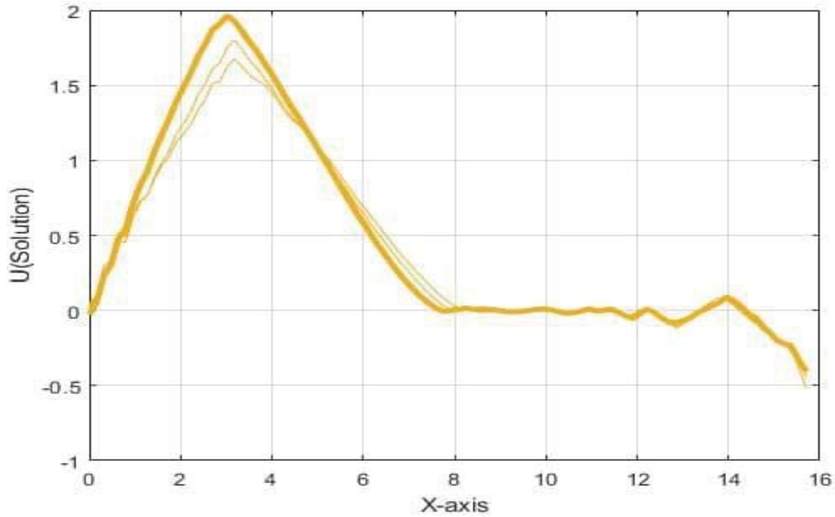


Figure 19: The solution at different moment time (6000, 8000, 10000)

- The same equation in example (2.2.1) will solve it usinq different initial condition which is.

$u_0 = \cos(x)$ with time priod [0,5pi] and $t = 10$ and we get the following resultis.

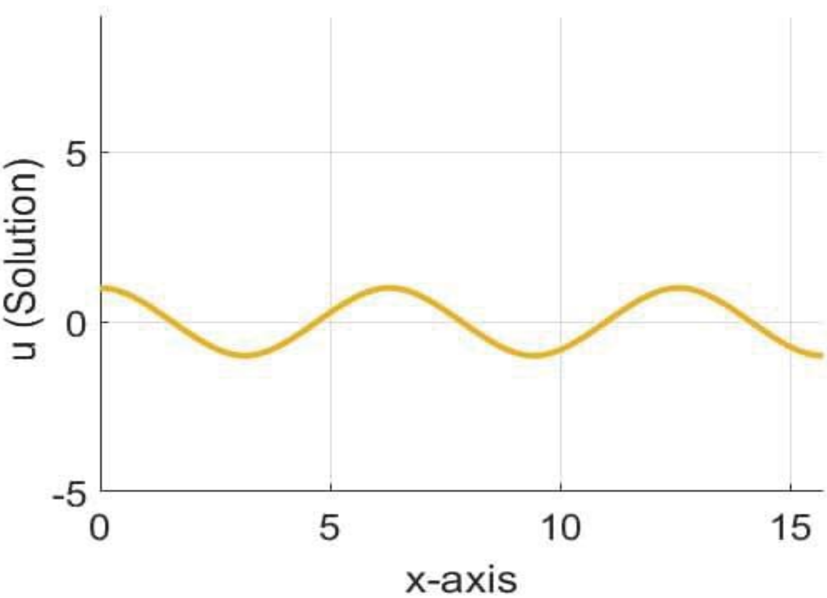


Figure 20: The solution at total time =10

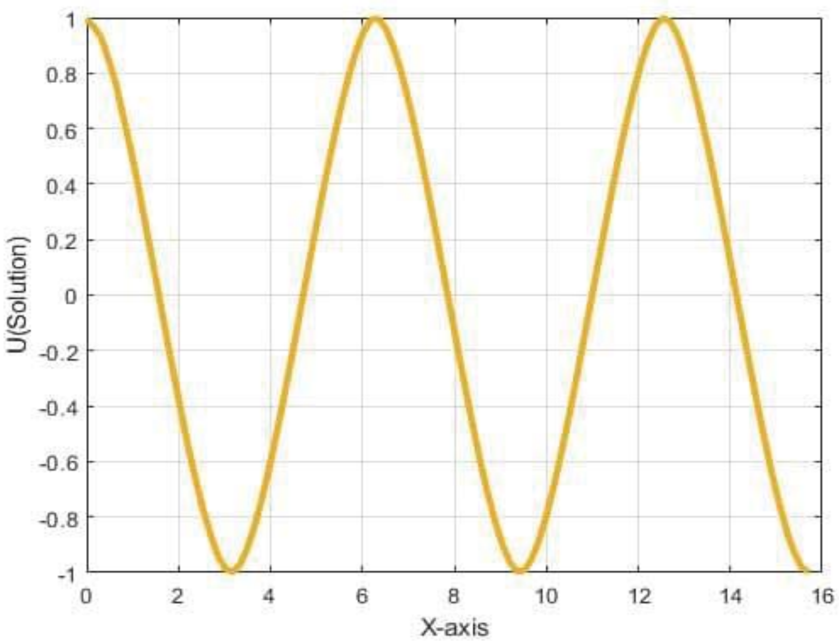


Figure 21: The solution at time step 90

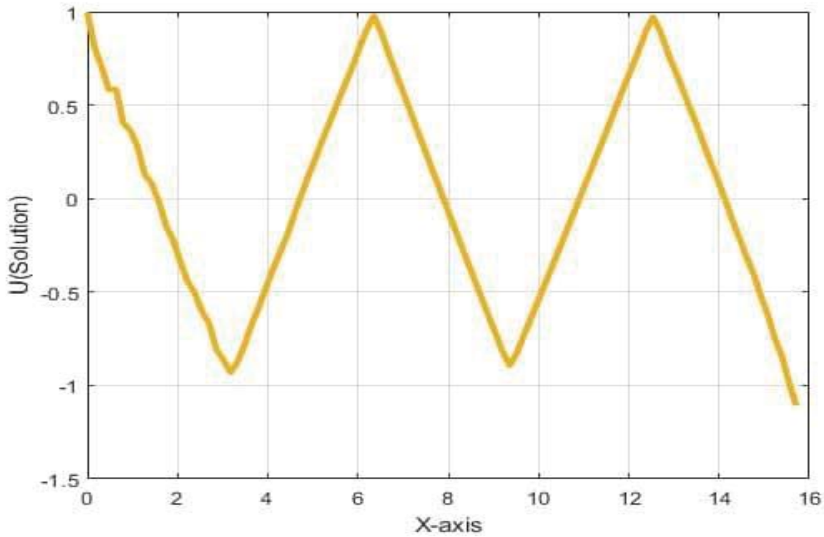


Figure 22: The solution at time step 3000

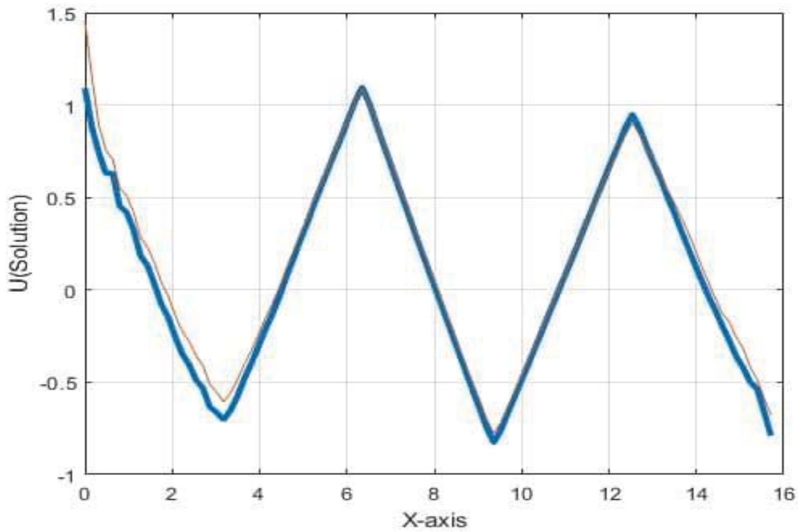
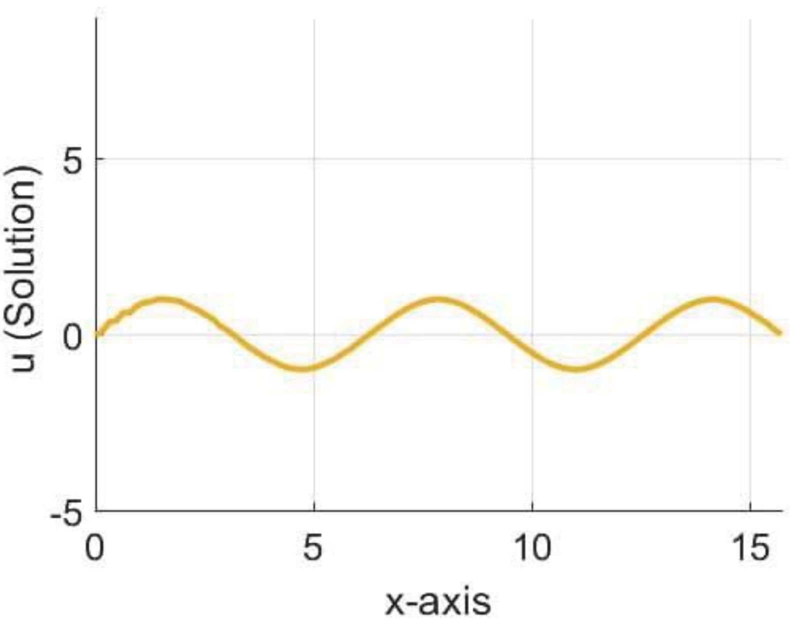
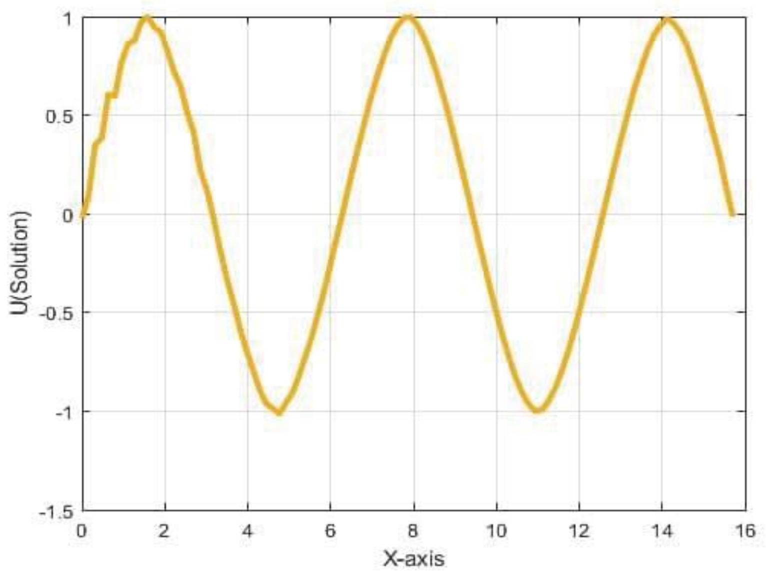


Figure 23: The solution at different moment time (8000, 10000)

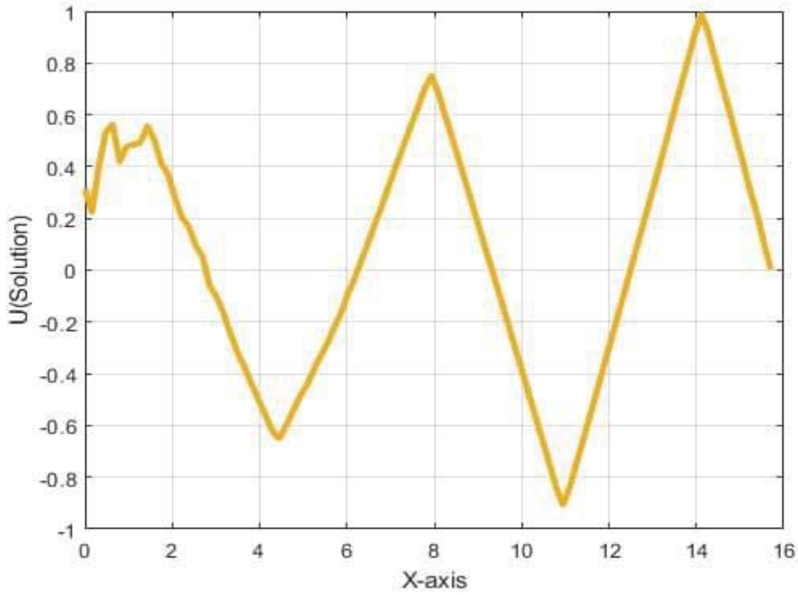
- With the same example (2.2.1) will solve it using differert initial condition which is $u_0 = \sin(x)$ with time priod [0,5pi], and $t = 10$ and we get the following resultis



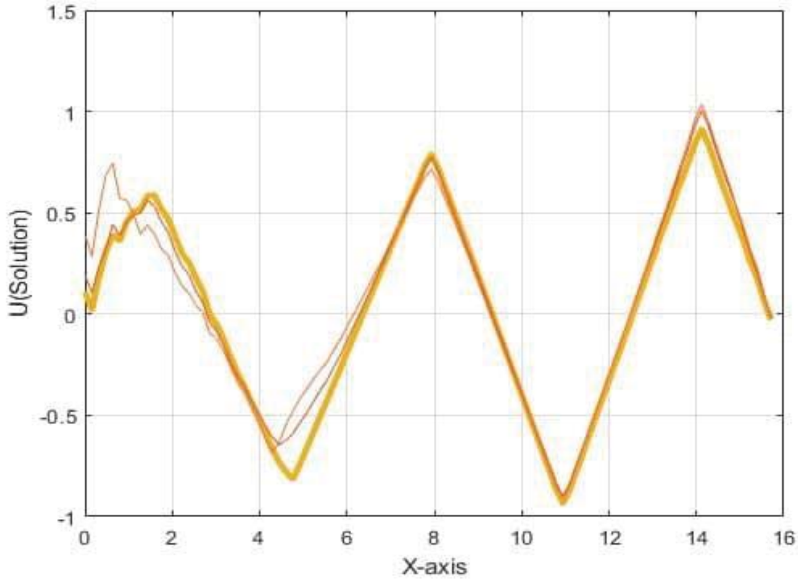
Figer 24: The solution at total time =10



Figer 25: The solution at time step 80



Figer 26: The solution at time step 9000



Figuer27: The solution at different moment time (6000, 8000, 100000)

3. CONCLUSION:

This paper presents the solution of differential equations of the second and the fourth order the procedure can start with the second derivative. First, the second order derivative is by a RBFN, then the first order derivative is obtained by integration. Finally, this original function is obtained, i.e. by integrating the first derivative function. This second method is here referred to as the second indirect method or IRBFN2. likewise, fourth -grade derivative IRBFN4. Where the fourth derivative is analyzed by integration then the third derivative and then the second here it

Finally, the original function is obtained by combining derivatives. is referred to as a fourth-order 1D-IRBF scheme, denoted by IRBF.

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