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New Results On Intuitionistic Fuzzy Soft Topological Space

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Abstract : In this paper , We introduce the notion of intuitionistic fuzzy soft topological space in intuitionistic fuzzy soft set theory . Also , We introduce a quasi-coincident relation and Q-intuitionistic fuzzy soft neighborhood . system . Also the concepts of intuitionistic fuzzy soft base , intuitionistic fuzzy soft sub base are introduced her and established some important theorems . and introduced the base for generated intuitionistic fuzzy soft topological space .

Key words : Intuitionistic fuzzy soft set , IFSTS , quasi-coincident , base of IFSTS , IFS closure ,IFS interior

1-Introduction

After having initiated the notion of fuzzy set first by zadeh [19]in1965 much research has been carried out in the areas of general theories as well as application in 1968 chang [9] introduced the theory of fuzzy topological spaces. Then after a long time the concept of a soft set was introduced by Molodtsov in 1999 [1] . Initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties .He has shown several applications of this theory in solving many practical problems in economics , engineering , social science , medical science etc. In the present times researchers have combined these two above concepts to generalize the spaces and to solve more complicated problems . In 2001 , Maji and etal . first combined these two sets and called it fuzzy soft set . also studied fuzzy soft topological spaces in another suitable form [7] .The concept of intuitionistic fuzzy set introduced by K.T. Atanassov [2,18] . Coker in 1995 [20] defined the notion intuitionistic fuzzy topology and studied the basic concept of intuitionistic fuzzy point .And introduced the concept intuitionistic fuzzy soft topological space in [4,5,6] . In this paper , We introduce the notion of intuitionistic fuzzy soft topological space in intuitionistic fuzzy soft set theory . Also , We introduce a quasi-coincident relation and Q-intuitionistic fuzzy soft neighborhood . system . Also the concepts of intuitionistic fuzzy soft base , intuitionistic fuzzy soft sub base are introduced her and established some important theorems . and introduced the base for generated intuitionistic fuzzy soft topological space.

2- Preliminaries

Definition 2.1. [1] Let X be and initial universe set and E be a set of parameters. A pair (F,E) is called a soft set over X if and only if F is a mapping from E into the set of all subsets of the set X , i.e.,$F:E\longrightarrow P(X)$, where P(X) is the power set of X.

Definition 2.2. [10,11,12,13,14] Let $A \subseteq E$. A pair (f , A) , denoted by fA , is called fuzzy soft set over X , where f is a mapping given by f : A $\rightarrow Ix$ defined by fA (e)= $\mu fA\ e$; where $\mu fAe=\overline{0}$ if e $\notin A$, and $\mu fAe \neq \overline{0}$ if e $\in A$, where $\overline{0}(x) = 0\ \forall x \in X$. The family of all these fuzzy soft sets over X denoted by FSS (x)E .

Definition 2.3. [2,18,] An intuitionistic fuzzy set A over the universe set X can be defined as follows

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$A=\{(x,\mu A(x),\nu A(x)):x\in X\}$

Where the function $\mu A:X\longrightarrow[0,1]$ and $\nu A:X\longrightarrow[0,1]$ with the property $0\leq\mu A(x)+\nu A(x)\leq1$ for each $x\in X$. The values $\mu A(x)$ and $\nu A(x)$ represent the degree of membership and non-membership of x to A respectively .

Definition 2.4. [2] Let X be asset and A ,B are an intuitionistic fuzzy set in the form :

A ={ < x, μA(x) , υA(x) > , x $\in$X }

B ={ < x, μA(x) , υA(x) > , x $\in$X } ,then :

A$\subseteq$ B if and only if μA(x) $\leq$ $\mu B(x)$ and $\upsilon A(x) \geq \upsilon B\ (x)$, x $\in X$ (1)

A=B if and only if A $\subseteq$ B and B $\subseteq$ A (2)

Ac = { <x , νA (x) , μA (x) > , x $\in$ X } (3)

A $\cap$ B = {< x , (μA (x) $\wedge$ μB (x)) , (νA (x) $\vee$ υ (x)) , x $\in$ X } (4)

A $\cup$ B = { < x , (μA (x) $\vee$ μB (x)) , ($\upsilon A\ (x) \wedge \upsilon B\ (x)$) ,$x \in X$ } (5)

Definition 2.5. [3] Let U be an initial universal set and Let E be set of parameters . Let p (U) denoted the set of all intuitionistic fuzzy sets of U . A pair (F , A) is called an intuitionistic fuzzy soft set over U if F is a mapping

given by F : A $\longrightarrow P\ (U\ )$.

We write an Intuitionistic fuzzy soft set shortly as IF soft set .

Example 2.6. We give an example of an IF soft set. Suppose that there are five people in the universe given by , U ={ $p1$,$p2$,$p3$,$p4$,$p5$ } and E= { $e1$,$e2$,$e3$ } where $e1$stands for young , $e2$ stands for smart , $e3$ stands for middle – aged . suppose that

F ($e1$)={ $p1$(0.5 ,0.2) ,$p2$0.9 ,0.1) ,$p3$0.4 ,0.3) ,$p4$(0 ,0.56) ,$p5$(0.2 ,0.5) }

F ($e2$) = { $p1$(0.3 ,0.2 ,$p2$(0.9 ,0.1) ,$p3$(1.1 ,0.77) ,$p4$(0.8 ,0.13) ,$p5$(0.5 ,0.5) }

F ($e3$) = { $p1$(0.5 ,0.1) ,$p2$(0.3 ,0.75) ,$p3$(0.7 ,0.27) ,$p4$(0.7 ,0.13) ,$p5$(0.7 ,0.31) }

Thus IF soft set is a parameterized family of all IFS of U and gives us approximate description of the object .

Definition 2.7. [3] For two intuitionistic fuzzy soft sets (F , A) and (G ,B) over the common universe U , we say that (F , A) is an intuitionistic fuzzy soft subset of (G , B) if

(1) A$\subset B$

(2) F (e) is an intuitionistic fuzzy subset of G (e)

We write (F , A) $\tilde{\subseteq}$ (G ,B) . (F , A) is said to be an intuitionistic fuzzy soft supper set of (G , B) if (G , B) is intuitionistic fuzzy soft supper of(G , B) if (G ,B) is intuitionistic fussy soft subset of (F,A) .We denote this as

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(F, A) $\tilde{\supseteq}$ (G ,B)

Definition 2.8. [3] Two intuitionistic fuzzy soft sets (F , A)and (G , B) over the common universe U are said to be intuitionistic fuzzy soft equal if (F , A) is an intuitionistic fuzzy soft subset of (G , B) and (G , B) is intuitionistic fuzzy soft sub set of (F , A) .

Definition 2.9[3] The compliment of an intuitionistic fuzzy soft set (F , A) , denoted by (F , A)C is defined by (F , A)c=(Fc ,A) , where Fc:A→$P(X)$ is mapping given by $Fc(e)$=$(F(e))c$ for all e∈A . Thus if $F(e)$={$(x$,$\mu F(e)(x)$,$\nu F(e)(x))$:x∈X} , then for all e∈A , $Fc(e)$=$(F(e))c$={$(x$,$\nu F(e)(x)$,$\mu F(e)(x)$):x∈X }.

Definition 2.10. [2,3] A soft set (F , A) over U is said to be null intuitionistic fuzzy soft set denoted by Φ if ∀e ∈A F(e) = intuitionistic fuzzy set 0 of U where : 0 ={(x , 0 , 1) : $x \in U$ } .

Definition 2.11. [3] A soft set (F , A) over U is said to be absolute intuitionistic fuzzy soft set denoted by $A\sim$, if ∀e ∈A

F(e) = intuitionistic fuzzy set 1 of U where 1= {(x , 1 , 0) : x ∈U } .

Definition 2.12. [3] Intersection of two intuitionistic fuzzy soft sets (F , A) and (G , B) over a common universe U is the intuitionistic fuzzy soft set (H , C) where C = A$\cap B$ and ∀e ∈c ,$H(e)$=$F(e)\cap G(e)$. We

write $(F ,A)\tilde{\cap}(G ,B)$=(H ,C) .

Definition 2.13. [3] Union of two intuitionistic fuzzy soft sets (F , A) and (G , B) over common universe U is the intuitionistic fuzzy soft set (H , C) where C = $A \cup B$ and for all e $\in C$

We write (F , A) Ũ (G ,B) = (H , C) $H(e)=\{F(e)\ G(e)\ F(e) \cup G(e)$ if e∈A−Bif e∈B−Aif e∈A∩B

Definition 2.14. [4] Let (F,A) and (G,B) be two intuitionistic fuzzy soft sets over X .We define the difference of (F,A) and (G,B) as the intuitionistic fuzzy soft (H,C) written as$(F,A)-(G,B)=(H,C)$ where $C=A\cap B$

and for all $e\in C$,$x\in X$,$\mu H(e)(x)=min(\mu F(e)(x),\nu G(e)(x))$ and $\nu H(e)(x)=max(\nu F(e)(x),\mu G(e)(x))$.

Definition 2.16.[5,8] Let τ an IFSS (X , E) be the collection of intuionistic fuzzy soft set over X , then τ is said to be an intuitionistic fuzzy soft topology on X if :

(1) $\tilde{\Phi}$,$\tilde{I}$ belong to τ

(2) the union of any number of intuitionistic fuzzy soft sets in τ belong to τ .

(3) the intersection of any two intuitionistic fuzzy soft sets in τ belong to τ .

The triple (X, τ, E) is called an intuitionistic fuzzy soft topological space over X . If (F,E) $\in \tau$, then the intuitionistic fuzzy soft set (F , E)is said to be intuionistic fuzzy soft open set .

Definition 2.17. [8] Let X be initial universe set , E be the parameters and

(1) $\tau=\{\tilde{\Phi}\ ,\tilde{I}\}$ is called the intuitionistic fuzzy soft indiscrete topology on X .

(2) τ be the collection of all intuitionistic fuzzy soft sets which can be defined over X . then τ is called the IFS

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discrete topology on X .

Definition 2.18. [6] Let $(X\ ,\tau,E)$ be an intuitionistic fuzzy soft topological space over X and let (F,A) be an intuitionistic fuzzy soft set (F ,E) is said to be intuitionistic fuzzy soft closed set in X if its complement (F ,E)c be long to τ

Definition2.19 [6,8] Let (X,τ,E) be an intuitionistic fuzzy soft topological space over X and (F,E) be an intuitionistic fuzzy soft set over (X,E) .The intuitionistic fuzzy soft closure of (F,A) denoted by $\overline{(F,A)}$ is the intersection of all intuitionistic fuzzy soft closed super sets of (F,A) . Clearly$\overline{(F,A)}$ is the smallest intuitionistic fuzzy soft closed set over X which contains (F,A) $\overline{(F,E)}=\tilde{\cap}\{(H,C):(H,C)\in\tau c$ $and\ (F,E)\tilde{\subseteq}(H,C)\}$

Theorem 2.20. [8] Let (X,τ,E) be an intuitionistic fuzzy soft topological space over X .

$(F1,A)$,$(F2,B)$are intuitionistic fuzzy soft set over X . Then :

(1) $\overline{\tilde{\Phi}}=\tilde{\Phi}\ and\ \overline{\tilde{I}}=\tilde{I}$

(2) $(F1,A)\subset\overline{(F1,A)}$

(3) $(F1,A)$ is an intuitionistic fuzzy soft closed set if and only if $(F1,A)=\overline{(F1,A)}$

(4) $\overline{\overline{(F1,A)}}=\overline{(F1,A)}$

(5) $(F1,A)\subset\overline{(F2,B)}$ implies $\overline{(F1,A)}\subset\overline{(F2,B)}$

(6) $\overline{(F1,A)\vee(F2,B)}=\overline{(F1,A)}\vee\overline{(F2,B)}$

3-Intuitionistic Fuzzy Soft Topological Spaces

Definition 3.1. [6] Let (X,τ,E) be an intuitionistic fuzzy soft topological space and let (F,A) be an intuitionistic fuzzy soft set over X . The intuitionistic fuzzy soft interior of (F,A) is defined as the union of all intuitionistic fuzzy soft open sets content in (F,A) and is denoted by $int(F,A)$. Thus int(F,A) is the largest IF soft open set contained in (F,A) . We write $int(F,A)=\tilde{\cup}\{(G,B): (G,B)\tilde{\in}\tau$ $IFSopen\ set\ and\ (G,B)\tilde{\subseteq}(F,A)\}$

Proposition 3.2. $int((F,A)\tilde{\cap}(G,B))=int(F,A)\tilde{\cap}int(G,B)$

Proof :Since $(F,A)\tilde{\cap}(G,B)\tilde{\subseteq}(F,A)$

$(H,C)\tilde{\subseteq}(F,A)$ $C=A\cap B\subseteq A$,$\forall e\in C$ $H(e)=F(e)\tilde{\cap}G(e)$ $\Rightarrow int((F,A)\tilde{\cap}(G,B))\tilde{\subseteq}$ $int(F,A)$

Similarly $int((F,A)\tilde{\cap}(G,B))\tilde{\subseteq}int(G,B)$

Therefore $int((F,A)\tilde{\cap}(G,B))\tilde{\subseteq}$ $int(F,A)\tilde{\cap}int(G,B)$

Let $(W,D)\tilde{\in}\tau$ such that $(W,D)\tilde{\subseteq}$ $int(F,A)\tilde{\cap}(G,B)$

Then $(W,D)\tilde{\subseteq}$ $int(F,A)$ and $(W,D)\tilde{\subseteq}$ $int(G,B)$

$W(e)\tilde{\subseteq}$ $F(e)$ and W(e) $\tilde{\subseteq}$ $G(e)\forall e\in E$

So, $W(e)\tilde{\subseteq}$ $F(e)\tilde{\cap}G(e)=(F\tilde{\cap}G)(e)\forall e\in E$

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$\Rightarrow(W,D)\tilde{\subseteq}(F,A)\tilde{\cap}(G,B)$

So, $(W,D)=int$ (W,D)$\tilde{\subseteq}$ int $(F,A)\tilde{\cap}(G,B)$.

This implies that int$(F,A)\tilde{\cap}int$ $(G,B)\tilde{\subseteq}$ int $((F,A)\tilde{\cap}(G,B))$. This complete proof .

Definition 3.3 [6] A If soft set (F,A) is said to be a If soft point , denoted by eF , if for the element $e\in A,F(e)\neq\tilde{0}$ and $F(\acute{e})=\tilde{0}\ \forall\acute{e}\in A-\{e\}$.

Definition 3.4 [6] A If soft point eF is said to be in a IF soft set (G,A) , denoted by $eF\in(G,A)$ if for the element $e\in A$,$F(e)\leq G(e)$.

Remark : this intuitionistic fuzzy soft point is denoted by $xe(\alpha,\beta)$ or eF ,(f,e) ,(fe,A) .The class of all intuitionistic fuzzy soft points of x denoted by IFSP$(x)E$

Definition 3.5. [6] Let (X,τ,E) be an intuitionistic fuzzy soft topological space . An intuitionistic fuzzy soft set (F,A) is a neighborhood of an intuitionistic fuzzy soft set (G,B) if and only if there exists an intuitionistic fuzzy soft open set $(Q,C)\tilde{\in}\tau$ such that $(F,A)\tilde{\subseteq}(Q,C)\tilde{\subseteq}(G,B)$.

Definition 3.6. [6] Let (X,τ,E) be an intuitionistic fuzzy soft topological space . An intuitionistic fuzzy soft set (F,A) is a neighborhood of an intuitionistic fuzzy soft point $eF\tilde{\in}(F,A)$ if and only if there exists an intuitionistic fuzzy soft open set $(Q,C)\tilde{\in}\tau$ such that $eF\subseteq$

$\tilde{}(Q,C)\tilde{\subseteq}(F,A)$

Definition3.7 . An intuitionistic fuzzy soft point $xe(\alpha,\beta)$ is said to be aquasi-coincident with an intuitionistic fuzzy soft set (G,B) denoted by $xe(\alpha,\beta)q(G,B)$ if for the element $e\in A$,$F(e)q(G,B)$ for some x$\in X$ negation of this statement is written as $xe(\alpha,\beta)\tilde{q}(F,A)$.

Definition3.8. An intuitionistic fuzzy soft set(F,A) is said quasi-coincident with (G,B), denoted by (F,A) q(G,B) if and only if there exist element $x\in X$ such that

1-AqB

2- $F(e)qG(e)$,$\forall e\in E\ i.e.\ \mu F(e)(x)>\nu G(e)(x)$ or $\nu F(e)(x)<\mu G(e)(x)$

If this is true we can say that (F,A) and (G,B) are quasi-coincident at x . otherwise (F,A) not quasi-coincident with (G,B) and denoted by (F,A)$\tilde{q}$ (G,B) .

Definition 3.9. Let (X,τ,E) be an IFS topological space and $xe(\alpha,\beta)$ be an IF soft point in X . An IFS set (F,A) is called Q-IFS neighborhood of $xe(\alpha,\beta)$ (Q-IFSS nbd . forshort) if there exists $(G,B)\tilde{\in}\tau$ such that $xe(\alpha,\beta)q(G,B)$ and $(G,B)\tilde{\subseteq}(F,A)$. Let $N(xe\alpha,\beta)$ be the family of all Q-IFS nbd. Of $xe(\alpha,\beta)$ in an IF soft topological space (X,τ,E) . Q-IFS nbd. Of $xe(\alpha,\beta)$ in an IF soft topological space (X,τ,E) .

Proposition 3.10. Let (F,A) and (G,B) be two IFS sets

Then : (1) $(F,A)\tilde{\subseteq}(G,B)$ if and only if $(F,A)\tilde{q}\ (G,B)c$

(2) $xe(\alpha,\beta)\ \tilde{\in}(\tilde{F},A)$ if and only if $xe(\alpha,\beta))\ \tilde{q}\ (\tilde{G},A)c$

(3) if (F,A)q(G,B) and (G,B) $\tilde{\subseteq}$ (H,C) then (F,A) q (H,C)

Proof : (1) ($\Rightarrow$) suppose that $(\tilde{F},A)\subseteq(\tilde{G},B)$

To show that $(\tilde{F},A)\tilde{q}\ (\tilde{G},B)c$

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$\because(\tilde{F},A)\subseteq(\tilde{G},B)$

$\Rightarrow(i)A\subseteq B\ i.e\ \mu(x)A<\mu(x)B$ and $\nu(x)A>\nu(x)B$

$\Rightarrow(ii)\forall e\in A\subseteq E\ F(e)\subseteq G(e)$

$(x\ ,\mu F(e)(x),\nu F(e)(x))\tilde{\subseteq}(x,\mu G(e)(x),\nu G(e)(x))$

$\mu F(e)(x)\leq\mu G(e)(x)$ and $\nu F(e)(x)\geq\nu G(e)(x)$

$\Rightarrow F(e)\tilde{q}G(e)c\ \forall e\in E\ \Rightarrow A\tilde{q}Bc$

Then $(F,A)\tilde{q}(G,B)c$ by definition 3.8

(2) Obvious

(3) suppose that (F,A)q (G,B)

(i) AqB

(ii) $\forall e \in E$ and $x \in X$ F(e)q G(e)

$\Rightarrow \mu F(e)(x) > \nu G(e)(x)$ or $\nu F(e)(x) < \mu G(e)(x)$

Since $(G,\beta) \tilde{\subseteq} (H,C)$

$\Rightarrow (i) \beta \subseteq c$

$\Rightarrow \forall e \in E, x \in X\ G(e) \tilde{\subseteq} H(e)$

$\mu G(e)(x) \leq \mu H(e)(x)$ and $\nu G(e)(x) \geq \nu H(e)(x)$

Then $\mu F(e)(x) > \nu G(e)(x) \geq \nu H(e)(x)$ or $\nu F(e)(x) < \mu G(e)(x) \leq \mu H(e)(x)$

$\Rightarrow \mu F(e)(x) > \nu H(e)(x)$ or $\nu F(e)(x) < \mu H(e)(x)$

(ii) F(e) q H(e)

$\Rightarrow (1) A\ q\ C$

$\Rightarrow (2)\ (F,A) q\ (H,C)$

Theorem 3.11. Let (X,τ,E) be an IFS topological space and $xe(\alpha,\beta)$ be an IFS point in X . Then $xe(\alpha,\beta) \tilde{\in} \overline{(F,A)}$ if and only if for each $(G,B) \tilde{\in} N(xe(\alpha,\beta))$,$(G,B) q (F,A)$

Proof (⇒) suppose that $xe(\alpha,\beta) \tilde{\epsilon}\ (F,A)$

To show that $\forall (G,B) \tilde{\in} N(xe(\alpha,\beta))$,$(G,B) q\ (F,A)$

$\exists (G,B) \tilde{\in} N(xe(\alpha,\beta))\ s.t\ (G,B)\ \tilde{q}\ (F,A)$

$\because ((F,A)c)c = (F,A)$

$\Rightarrow (G,B) \tilde{q} ((F,A)c)c$

By proposition 3.10 $\Rightarrow (G,B) \tilde{\subseteq} (F,A)c$

$(G,B)c \subseteq$

$\tilde{}((F,A)c)c = (F,A)$…….(1)

$\because xe(\alpha,\beta) \tilde{\in} \overline{(F,A)}$

$\Rightarrow \exists (G,B) \tilde{\in} \tau c\ s.t\ (F,A) \tilde{\subseteq} (G,B)$……….(2)

$\Rightarrow (G,B)c \subseteq$

$\tilde{}(G,B)$ cont #

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$\Rightarrow \forall (G,B) \tilde{\in} N(xe(\alpha,\beta))\ s.t\ (G,B) q (F,A)$

(⇐) suppose that $\forall (G,B) \tilde{\in} N(xe(\alpha,\beta))\ s.t\ (G,B) q\ (F,A)$ to show that $xe(\alpha,\beta) \tilde{\in} \overline{F,A}$

$\because (G,B) \tilde{\in} N(xe(\alpha,\beta)) \Rightarrow \exists (F,A) \tilde{\in} \tau\ s.t$

$xe(\alpha,\beta)$ q (F,A) and (F,A) $\subseteq$

$\tilde{}(G,B)$

$\Rightarrow xe(\alpha,\beta)qA$ and $H(e)qF(e)$, $H(e)=(x,\alpha,\beta)$ $\alpha\in I0$,$\beta\in I1$

$\Rightarrow \alpha>\nu F(e)(x)$ or $\beta<\mu F(e)(x)$

$\Rightarrow xe(\alpha,\beta)\notin$

$\tilde{}(F,A)c$

$\Rightarrow xe(\alpha,\beta)\tilde{\in}(F,A)$

$\because (F,A)\tilde{\subseteq}(F,A)$ By Theorem 2.20

$\Rightarrow xe(\alpha,\beta)\tilde{\in}(F,A)$

Definition 3.12. An IFSP eF is called an adherence point of an intuitionistic fuzzy soft set (F,A) if every Q- IFSS nbd. of eF is a aquasi-coindent with (F,A) .

Proposition 3.13. Every intuitionistic fuzzy soft point of (F,A) is an adherence point of (F,A)

Proof : Let $eF\tilde{\in}(F,A)$, $e\in A$, $F(e)\leq F(e)$

Since $(F,A)\tilde{\subseteq}(F,A) \Rightarrow A\tilde{\subseteq} A$

$eFq(F,A)$, $(F,A)\tilde{\in}\tau$ from definition of Q-IFSS nbd. We get

$\Rightarrow eFq(F,A)$.

Definition 3.14. A subfamily β of τ is called an intuitionistic fuzzy soft base of intuitionistic fuzzy soft topological space (X,τ,E) if the following conditions hold .

(1) $\Phi \tilde{\in} \beta$

(2) $\tilde{\cup}\beta=E$ i.e for each $e\in E$ and $x\in U$, there exists $(F,A)\in\beta$ such that $F(e)=\{<x,\mu F(e)(x) ,\nu F(e)(x) > ,x\in X \}$

$=\tilde{1}=\{<x ,0 ,1>,x\in X \}$

(3) If $(F,A),(G,B)\tilde{\in}\beta$ then for each $e\in E$ and $x\in U$ there exists $(H,C)\tilde{\in}\beta$ such that $(H,C)\tilde{\subseteq}(F,A)\cap(G,\beta)$ and $H(e)=\{\langle x,min\{\mu F(e)(x),\mu G(e)(x)\} ,max\{\mu F(e)(x),\nu G(e)(x)\}\rangle ,x\in X \}$ Where $C\subseteq A\cap B$.

Theorem 3.15. Let β be an intuitionistic fuzzy soft base for an intuitionistic fuzzy soft topology on (U,E) ,suppose τB consist of those intuitionistic fuzzy soft set (G,A)over (U,E) for wich corresponding to each $e\in E$ and $x\in U$, there exists $(F,B)\in B$ such that $(F,B)\subseteq(G,A)$ and $Fe(x)B=Ge(x)A$, where $B\subseteq A$ .Then τB is an intuitionistic fuzzy soft topology space on (U,E)

Proof : We have $\Phi\in \tau B$ by default .

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(1) $\forall e\in E$ and $x\in U$,$\exists(F,A)\tilde{\in}\beta$ $s.t$ $F(e)=\tilde{1}$ $E\tilde{\in}\tau\beta$

(2) Let $(F,A),(G,B)\in\tau\beta$

$\Rightarrow$ $\forall e\in E$ and $x\in U$ $\exists(H,C),(I,D)$ $\in B$ where $C\subseteq A$ and $D\subseteq B$ $s.t$ $(H,C)\tilde{\subseteq}(F,A),(I,D)\tilde{\subseteq}(G,B)$ and $H(e)(\tilde{H},A)=F(e)(\tilde{F},A)$,$I(x)(\tilde{I},D)=G(e)(\tilde{G},e)$

Let $(F,A)\tilde{\cap}(G,B)=(J,A\cap B)$

Since (H,C) ,$(I,D)\in\beta$ and $e\in E$,$x\in U$,$\exists(K,p)\in\beta$ such that $(K,p)\subseteq(H,C)\cap(I,D)$ and $K(e)(K,p)=\{\langle x,min\{\mu(x)e(H,C)$,$\mu(x)e(I,D)\}$,$max\{\nu(x)e(H,C),\nu(x)e(I,D)\}\rangle\}$ $i.e$ $p\subseteq C\cap D$

Let $a\in E$

Then $Ke(a)(K,P)\tilde{\subseteq}$ $H(a)e(\tilde{H},C)\tilde{\cap}I(a)e(I,D)\subseteq F(a)e(F,A)\cap GE(a)(G,B)=(J,A\cap B)$

Therefore $(K,P)\subseteq(J,A\cap B)$

(3)Let $(F\alpha,A\alpha)\in\tau\beta$ $for\ all$ $\alpha\in\wedge$ an index set and $e\in E$,$x\in U$,Let $(J,C)=\cup\alpha\in\wedge(F\alpha,A\alpha)$

Where $C=\cup\alpha\in\wedge A\alpha$

$(\tilde{J},C)=\{\langle x,\vee\mu F\alpha,\wedge\nu F\alpha\rangle$ $x\in X$ $\}$

$\Rightarrow$ $(J,C)=(F\alpha,A\alpha)$ for some $\alpha\in\wedge$

Since $(F\alpha,A\alpha)\in\tau$,$\exists(G,B)\in\beta$ such that $(G,B)\subseteq(F\alpha,A\alpha)$

and $(G,B)=(F\alpha,A\alpha)$

$\Rightarrow(G,B)\subseteq(J,C)\Rightarrow(J,C)\in\tau\beta$

Then $\tau\beta$ is topology

Definition 3.16. Suppose β is an Intuitionistic fuzzy soft base for an intuitionistic fuzzy soft topological space on (U,E) .Then $\tau\beta$, described in above theorem is called an intuitionistic fuzzy soft topology generated by β and β is called an intuitionistic fuzzy soft base for $\tau\beta$

Theorem3.17 Let(F,A) ∈ IFS (U,E) . Then $(F,A)\in\tau$ if and only if (F,A) is a neighborhood of each of its intuitionistic fuzzy soft point .

Proof : If $(F,A)\in\tau$, then obviously (F,A) is aneiborhood of each of its IFSP

Conversely, let (F,A) is a neighborhood of each of its IF soft points. Then for any $F\alpha(e)\in(F,A)\alpha\in\wedge$ there exists $(G,A)\in\tau$ $s.t$ $F\alpha(e)\in(G,A)\subseteq(F,A)$

So that $\cup F(e)\alpha\subseteq\cup(G,A)\subseteq(F,A)$……….(1)

We now show that $\cup F\alpha(e)$=(F,A) where $e\in E$ and $\alpha\in\wedge$ there exist $\alpha\in\wedge$ s.t $F\alpha e(a)=(F,A)(a)$ there for $\cup F\alpha e(a)=(F,A)(a)$

$\cup Fe(e)=(F,A)$……….(2)

From (1) and (2) we get

$(F,A)=\cup(G,A)$

$(G,A)\in\tau$, $\cup(G,A)\in\tau$

$\Rightarrow(F,A)\in\tau$

Definition 3.18. The collection of all neighborhoods of a point $F(e)$ over (U,E) is called the neighborhood system at $F(e)$ and it is denoted by $\mathscr{H}F(e)$.

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Theorem 3.19. The neighborhood system $\mathscr{H}F(e)$ at any point $F(e)$ over (U,E) satisfy the following properties :

1) $\mathscr{H}F(e)\neq\phi$

2)$(G,B)\tilde{\in}\mathscr{H}F(e)\Longrightarrow F(e)\tilde{\in}(G,B)$

3)(G,B) ,$(H,A)\tilde{\in}\mathscr{H}F(e)\Longrightarrow(G,B)\cap(H,A)\in\mathscr{H}F(e)$

4)$(G,B)\tilde{\in}\mathscr{H}F(e)$ and $(G,B)\tilde{\subseteq}(H,A)\Longrightarrow(H,A)\in\mathscr{H}F(e)$

Proof : (1) Since $E\in\tau$ and $F(e)\tilde{\in}E$,$E\tilde{\in}\mathscr{H}F(e)$

(2) from definition nbd. ⇒obvious

(3) Since (G,B) and $(H,A)\tilde{\in}\mathscr{H}F(e)$,∃ (V,A) and (W,B) in τ s.t $Fe\in(V,A1)\tilde{\subseteq}(H,A)$. and $F(e)\in(W,B1)\tilde{\subseteq}(G,B)$

F$(e)=x(\alpha,\beta)e\tilde{\in}(V,A1)\Rightarrow\alpha\leq\mu e(x)(V,A1)$ and $\beta\geq Ve(x)(V,A1)$ $\forall x\tilde{\in}U$

F$(e)=x(\alpha,\beta)e$ $\tilde{\in}(W,B1)\Rightarrow\alpha\leq\mu e(x)(W,B1)$ and $\beta\geq ve(x)(W,B1)$

⇒α≤$\min\{\mu e(x)(V,A1)$,$\mu e(x)(W,B1)\}$ and $\beta\geq\max\{Ve(V,A1)$,$Ve(x)(W,B1)\}$

$\Rightarrow F(e)=x(\alpha,\beta)e\in(V,A1)\tilde{\cap}(W,B1)$

Since $F(e)\in(V,A1)\tilde{\cap}(W,B1)\subseteq(G,B)\cap(H,A)$

again since $(V,A1)\cap(W,B1)\in\tau$, $(G,B)\cap(H,A)\in\mathscr{H}F(x)$

(4) Obvious

Theorem 3.20 Let β be an intuitionistic fuzzy soft base for an intuitionistic fuzzy topology $\tau\beta$ on (U,E) then $(F,A)\in\tau\beta$ if and only if $(F,A)=\cup\alpha\in\wedge(B\alpha,A\alpha)$, where $(B\alpha,A\alpha)\in\beta$ for each $\alpha\in\wedge$,∧ an index set .

Proof :

Since every member of β is also a member of $\tau\beta$, any union of members of β is a member of $\tau\beta$

Conversely , Let $(F,A)\in\tau\beta$, where (F,A) is not equal to ϕ . Then for each $e\in E$ and $x\in U$, there exists $(Xe,Be)\in\beta$ such that $(Xe,Be)\subseteq(F,A)$, where $Bxe\subseteq A$

Let $B=\cup e\in E,x\in UBxe$ and $(G,B)=\cup e\in E,x\in U(Xe,Be)$

We now show that $(G,B)=(F,A)$.

Obviously , $(G,B)=(F,A)$,let $a\in E$ and $y\in U$,$GaGB(y)=$ $(y,\vee\mu aXBx(y),\wedge vaXBx(y))\tilde{\supseteq}$ $FFAa(y)$ $\forall a\in E$,and $y\in U$

So $(F,A)\tilde{\subseteq}(G,B)$

Hence $(F,A)\tilde{=}(G,B)$

The case $(F,A)=\Phi$ is obvious as $\Phi\in\beta$

(F,A) is the union of some members of β

Theorem3.21. A sub family β of an intutionistic fuzzy set topology τ over (U,E) is abase for τ if and only if for each intuitionistic fuzzy set point eG and for each Q-nbd . (F,A) of eG , there exist a member $(H,C)\tilde{\in}B$ such that $eG\ q(H,C)$ and $(H,C)\tilde{\subseteq}(F,A)$.

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Proof : (⇒) suppose that β is abase for τ .Let eG be an intuitionistic fuzzy set point and (F,A) be an Q-nbd. of eG .

Then ∃ $(I,W)\tilde{\in}\tau$ s.t $eGq(I,W)$ and $(I,W)\tilde{\subseteq}(F,A)$ since (I,W)$\tilde{\in}\tau$ and β is a base for τ By theorem 3.8

(I,W) can be expressed as $(I,W)=\cup j\in\tilde{J}(H,CJ)$ where $(H,Cj)\tilde{\in}\beta$ for all $j\in J$.

$\Rightarrow eGq\cup j\in J(H,Cj)$. so there exist some (H,Cj) such that $eGq\tilde{\cup} j\in J(H,Cj)$, and $(H,Cj)\tilde{\subseteq}(F,A)$.

(⇐) Let β is not a base for , Then $\exists(F,A)\tilde{\in}\tau$ such that let $M=\tilde{\cup}\{\ (H,C)\tilde{\in}\beta : (H,C)\tilde{\subseteq}(F,A)\ \}\neq(F,A)$,$\exists e\in E$ such that $G(e)\subseteq F(e)$ for some $x\in U$,$Mc=(I,W)$,$(I,W)q(F,A)$,$\exists(H,c)\tilde{\in}\beta$

$(I,W)q(H,C)$,$(H,C)\tilde{\subseteq}(F,A) \Rightarrow(H,B)\tilde{q}(F,A)c$, since $(H,C)\tilde{\in}M$ cont. # Such that

Then β is abase for τ

Definition 3.22. An intuitionistic fuzzy soft point eF is called an accumulation point of an intuitionistic fuzzy soft set (F,A) if eF is an adherence point of (F,A)and every Q-neighborhood of eF and (F,A) are quasi-coincident at some intuitionistic fuzzy soft point different from e , whenever $eF\in(F,A)$.

The union of all accumulation points of (F,A) is called the derived set of (F,A) , denoted by $(F,A)d$.

Theorem 3.23. $\overline{(F,A)}=(F,A)\tilde{\cup}(F,A)d$

Proof

Let $L=\{eG\ is\ an\ adherent\ point\ of\ (F,A)\}$

Then by theorem 3.11 $\overline{(F,A)}=\cup L$

Now $eG\tilde{\in}L$ if and only if $eG\tilde{\in}(F,A)\ or\ eG\tilde{\in}(F,A)d$

Hence $\overline{(F,A)}=\cup L=(F,A)\tilde{\cup}(F,A)d$

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