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On fuzzy soft $\mathcal{M}$-hyponormal operator

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Abstract:

The aim of this search, is to introduce some notions of fuzzy soft $\mathcal{M}$ hyponormal operator defined on fuzzy soft Hilbart space, denoted by $\mathcal{FSMh}$-operator ,also some important properties of this operator, as well as discussion for some theorems related to this operator, also in this paper we investigate another generalized of some types of hyponormal operator which as soft fuzzy soft $\mathcal{M}$ hyponormal operator, shortly ($\mathcal{SFSMh}$ $-$operator and some theorems of operations about this concepts have been given.

Keywords: fuzzy soft $\mathcal{M}$ hyponormal operator ,soft fuzzy soft $\mathcal{M}$ hyponormal operator.

Introduction

In 1965 Zadeh [1] proposed the theory of the fuzzy sets as an extension of regular sets to handle uncertainty. The fuzzy set is defined by the characteristic membership functions on domain of crisp set X to the interval [0,1]. In 1999 Molodtsov [2] first suggested the idea of soft set theory as an extension of regular sets to address complex problems and dispel uncertainty. The soft set is aparameterized set of universal set subsets. In 2001, Maji, Biswas, et al. [3] combined the fuzzy and soft concepts into a single idea that they called $\mathcal{FS}$ - set. The $\mathcal{FS}$- idea was then applied by other researchers to create concepts, such as the $\mathcal{FS}$ -point in 2012 [11] and $\mathcal{FS}$-normed spaces by T. Beaula and M.M. Priyanga in 2015 [10], as well as in 2020.The $\mathcal{FS}$ - inner product on fuzzy soft linear spaces was recently defined by N. Faried, M. S. Ali, and H H Sakr [6], who also presented its characteristics and other related studies. Next follows , In 2020, Faried, Ali, and Sakr [4] presented the $\mathcal{FS}$ Hilbert space definition, along with its characteristics and various new results. Moreover, the $\mathcal{FS}$ - linear operator in the fuzzy soft Hilbert space and related spectral theory concepts were defined by Faried, Ali, etc al. in 2020 [5]. In 2020 N. Faried, M. S. Ali, and H H Sakr [7] also presented the $\mathcal{FS}$- Hermitian operator , along with examples and related theorems linked to it.

The $\mathcal{FSM}$- hyponormal operator on fuzzy soft Hilbert space, another fuzzy soft bounded linear operator, is defined in this paper along with many theorems and theories concerning its characteristics and connections to other kinds of fuzzy soft bounded linear operators. an expanded version of this idea for the soft fuzzy soft hyponormal operator was also submitted.

1. Definition and concepts

In this section of the search, we will discuss some important definitions and theorems for the fundamental set, which is a fuzzy soft set with a fuzzy soft vector.

Definition (1) [1]

For a set U (called the universal set), a $\mathcal{F}$ -set $\tilde{\mathcal{A}}$ on U is defined in terms of a membership function $\mu_{\tilde{\mathcal{A}}}: \mathrm{U} \rightarrow I$, where I is the unit interval [0,1], by characterizing $\tilde{\mathcal{A}}$ as the set of all ordered pairs $\tilde{\mathcal{A}} = \{(x , \mu_{\tilde{\mathcal{A}}}\widetilde{(x): x} \in \mathrm{U}, \mu_{\tilde{\mathcal{A}}}(x) \in \mathrm{I}\}$. the set $\tilde{\mathcal{A}}$ is also sometimes be written as $\tilde{\mathcal{A}} = \{\frac{\mu_{\tilde{\mathcal{A}}(x)}}{x} : x \in \mathrm{U}\}$.

The real number, $\mu_{\tilde{\mathcal{A}}}(x)$, is called the membership of x in $\tilde{\mathcal{A}}$.

Definition (2) [2]

suppose $P(\mathrm{U}\ )$be the collection of the power set of universal U, E be a set of parameters, A pair $(\mathcal{F}, \mathrm{E})$ or $\mathcal{F}_E$ is known to as soft set over U if there is a function $\mathcal{F}: \mathrm{E} \rightarrow \mathrm{P(U)}$ where $\{\mathcal{F}_{\mathrm{E}}(\mathrm{e}) \in \mathrm{P(U)}: \mathrm{e} \in \mathrm{E}\}$

Definition (3) [3]

Assumption E be a set of parameters, a soft set $(\mathcal{F},\mathcal{A})$ is called a fuzzy soft set over a universal set U, whenever $\mathcal{F}$ is a mapping given by

$\mathcal{F}\colon \widetilde{\mathcal{A}} \to \widetilde{\mathrm{P}}(\mathrm{U})$, and $\{\mathcal{F}(\mathrm{e}) \in \widetilde{\mathrm{P}}(\mathrm{U})\colon \mathrm{e} \in \tilde{\mathcal{A}}\}$ where $\tilde{P}(U)$is the set of all fuzzy subsets of U , represented by $\mathcal{FS}-\mathrm{set}$.

Definition (4) [10]

The $\mathcal{FS}\mathfrak{T}$- space $(\tilde{U},\widetilde{<\cdot,\cdot>})$, if it is $\mathcal{FS}$-complete in the induced $\mathcal{FSN}$- space also known as a fuzzy soft Hilbert space ,and represented by $\mathcal{FSH}$ -space.and denoted by $(\widetilde{\mathcal{H}},\widetilde{<\cdot,\cdot>})$.

Definition (5) [10]

If $\widetilde{\mathcal{H}}$ is $\mathcal{FSH}$-space, and $\tilde{T}\colon \widetilde{\mathcal{H}} \to \widetilde{\mathcal{H}}$ be a $\mathcal{FS}$-operator is said to be $\mathcal{FS}$ - bounded operator ($\mathcal{FSB}$-operator) if, $\exists$ $\mathcal{FS}-$Real number $\tilde{k} \,\tilde{\in}\, \tilde{\mathcal{R}}^{+}_{\mathcal{A}}$, s.t. $\| \tilde{T}\left(\widetilde{\tilde{v}_{\mu_{\mathcal{F}(e)}}}\right) \| \leq \tilde{k} \| \widetilde{\tilde{v}_{\mu_{\mathcal{F}(e)}}} \|$. $\forall$ $\tilde{v}_{\mu_{\mathcal{F}(e)}} \,\tilde{\in}\, \widetilde{\mathcal{H}}$, and the set of all$\mathcal{FS}-$bounded linear operators symbolized by $\widetilde{\mathcal{B}}\,(\widetilde{\mathcal{H}}\,)$,

fact is also, true on the ($\mathcal{FSB}$-operator define on $\mathcal{FSH}$-space.)

Definition (6) [9]

If $\widetilde{\mathcal{H}}$ be $\mathcal{FSH}$-space, and $\tilde{T}\colon\widetilde{\mathcal{H}} \to \widetilde{\mathcal{H}}$ be $\mathcal{FSB}$- operator, then, $\tilde{T}^*$ is the $\mathcal{FS}-$adjoint operator and is denoted by $< \widetilde{\tilde{T}\tilde{v}^{1}_{\mu_{1\mathcal{F}(e1)}}, \tilde{v}^{2}_{\mu_{2\mathcal{F}(e2)}}} > \;=\; < \tilde{v}^{1}_{\mu_{1\mathcal{F}(e1)}}, \widetilde{\tilde{T}^*\,\tilde{v}^{2}_{\mu_{2\mathcal{F}(e2)}}} >$, $for\ all\ \tilde{v}^{1}_{\mu_{1\mathcal{F}(e1)}}, \tilde{v}^{2}_{\mu_{2\mathcal{F}(e2)}} \,\tilde{\in}\, \widetilde{\mathcal{H}}$.

Definition (7) [8], [11], [12]

Let $\widetilde{\mathcal{H}}$ be a $\mathcal{FSH}$-space, if $\tilde{T}^* \,\tilde{\in}\, \widetilde{\mathcal{B}}(\widetilde{\mathcal{H}})$ then

- **i.** $\tilde{T}$ is fuzzy soft Hermitian operator ($\mathcal{FS}$-self adjoint operator) if $\tilde{T} \cong \tilde{T}^*$.
- **ii.** $\tilde{T}$ is fuzzy soft Normal operator ($\mathcal{FSN}$-operatos) if $\tilde{T}\tilde{T}^* = \tilde{T}^*\tilde{T}$.
- **iii.** $\tilde{T}$ is fuzzy soft Quasi Normal operator shortly by ($\mathcal{FSQN}$-operatos) if $\tilde{T}(\tilde{T}^*\tilde{T}) = (\tilde{T}^*\tilde{T})\tilde{T}$.
- **iv.** $\tilde{T}$ is fuzzy soft hyponormal operator ($\mathcal{FS}h$-operator) if $\widetilde{\mathcal{T}^*\tilde{T}} \gtrsim \tilde{T}\tilde{T}^*$.

2. Main results

In this section, we will introduce a new type of fuzzy soft hyponormal operator define on fuzzy soft Hilbert space is given which we call fuzzy soft $\mathcal{M}$-hyponormal operator with some important theorems related on it.

4.2

4.3 Definition (1)

Assume $\widetilde{\mathcal{H}}$ is $\mathcal{FSH}$-space, and $\tilde{T}: \widetilde{\mathcal{H}} \to \widetilde{\mathcal{H}}$ be $\mathcal{FSB}$- operator on $\mathcal{FSH}$-space $\widetilde{\mathcal{H}}$, then $\tilde{T}$ namely fuzzy soft $\mathcal{M}$-hyponormal operator ($\mathcal{FSM}h$-operator) if existence a positive real number$\mathcal{M}$, where $\mathcal{M}^2\tilde{T}^*\tilde{T} \cong \tilde{T}\tilde{T}^*$.

4.4 Theorem (1)

Suppose $\widetilde{\mathcal{H}}$ be a $\mathcal{FSH}$-space, If $\tilde{T} \,\tilde{\in}\, \widetilde{\mathcal{B}}(\widetilde{\mathcal{H}})$, then $\tilde{T}$ is $\mathcal{FSM}h$-operator iff $\exists\ \mathcal{M} > 0$, satisfy $\mathcal{M}^2 \parallel \tilde{T}\widetilde{\tilde{v}_{\mu_{1\mathcal{F}(e1)}}} \parallel \tilde{\geq} \parallel \tilde{T}^*\widetilde{\tilde{v}_{\mu_{1\mathcal{F}(e1)}}} \parallel$, for every $\tilde{v}_{\mu_{\mathcal{F}(e)}} \,\tilde{\in}\, \widetilde{\mathcal{H}}$.

1) *Proof: By using* $\boldsymbol{\tilde{T}}$ *is* $\boldsymbol{\mathcal{FSM}h}$ *-operator then we obtain* $\boldsymbol{\mathcal{M}^2 \parallel \tilde{T}\widetilde{\tilde{v}_{\mu_{1\mathcal{F}(e1)}}} \parallel \tilde{\geq} \parallel \tilde{T}^*\widetilde{\tilde{v}_{\mu_{1\mathcal{F}(e1)}}} \parallel}$.

$$\mathcal{M}^2 \parallel \tilde{T}\widetilde{\tilde{v}_{\mu_{\mathcal{F}(e)}}} \parallel^2 \cong < \mathcal{M}^2\tilde{T}\tilde{v}_{\mu_{\mathcal{F}_e}}(e), \tilde{T}\tilde{v}_{\mu_{\mathcal{F}_e}}(e)) >$$
$$\tilde{\geq} < \tilde{v}_{\mu_{\mathcal{F}_e}}(e), \tilde{T}^*\tilde{T}\tilde{v}_{\mu_{\mathcal{F}_e}}(e)) >$$
$$\tilde{\geq} < \tilde{v}_{\mu_{\mathcal{F}_e}}(e), \tilde{T}\,\tilde{T}^*\tilde{v}_{\mu_{\mathcal{F}_e}}(e)) >$$
$$\tilde{=} < \tilde{T}^*\tilde{v}_{\mu_{\mathcal{F}_e}}(e), \tilde{T}^*\tilde{v}_{\mu_{\mathcal{F}_e}}(e)) >$$
$$\cong \parallel \tilde{T}^*\widetilde{\tilde{v}_{\mu_{\mathcal{F}(e)}}} \parallel^2 .$$

Hence, we get that $\mathcal{M}^2 \parallel \tilde{T}\widetilde{\tilde{v}_{\mu_{1\mathcal{F}(e1)}}} \parallel \tilde{\geq} \parallel \tilde{T}^*\widetilde{\tilde{v}_{\mu_{1\mathcal{F}(e1)}}} \parallel$.

Conversely, suppose that existence a positive real number $\mathcal{M}$ where $\mathcal{M}^2 \parallel \widetilde{\tilde{T}\tilde{v}_{\mu_{1\mathcal{F}(e1)}}} \parallel \geqq \parallel \widetilde{\tilde{T}^*\tilde{v}_{\mu_{1\mathcal{F}(e1)}}} \parallel$ for every $\tilde{v}_{\mu_{\mathcal{F}(e)}} \tilde{\in} \widetilde{\mathcal{H}}$, then we obtain that,

$$< \mathcal{M}^2\tilde{T}^*\tilde{T}\tilde{v}_{\mu_{\mathcal{F}_e}}(e), \tilde{v}_{\mu_{\mathcal{F}_e}}(e)) > \cong < \mathcal{M}^2\tilde{T}\tilde{v}_{\mu_{\mathcal{F}_e}}(e), \tilde{T}\tilde{v}_{\mu_{\mathcal{F}_e}}(e)) >$$
$$\cong \mathcal{M}^2 \parallel \widetilde{\tilde{T}\tilde{v}_{\mu_{\mathcal{F}(e)}}} \parallel^2$$
$$\geqq \parallel \widetilde{\tilde{T}^*\tilde{v}_{\mu_{\mathcal{F}(e)}}} \parallel^2$$
$$\cong < \tilde{T}^*\tilde{v}_{\mu_{\mathcal{F}(e)}}(e), \tilde{T}^*\tilde{v}_{\mu_{\mathcal{F}(e)}}(e)) >$$
$$\cong < \tilde{T}\tilde{T}^*\tilde{v}_{\mu_{\mathcal{F}(e)}}(e), \tilde{v}_{\mu_{\mathcal{F}(e)}}(e)) >$$

for every $\tilde{v}_{\mu_{\mathcal{F}(e)}} \tilde{\in} \widetilde{\mathcal{H}}$,then $< (\mathcal{M}^2\tilde{T}^*\tilde{T} - \tilde{T}\tilde{T}^*)\tilde{v}_{\mu_{\mathcal{F}(e)}}(e), \tilde{v}_{\mu_{\mathcal{F}(e)}}(e)) > \geqq 0$ and for every $\tilde{v}_{\mu_{\mathcal{F}(e)}} \tilde{\in} \widetilde{\mathcal{H}}$,thus $(\mathcal{M}^2\tilde{T}^*\tilde{T} - \tilde{T}\tilde{T}^*)\tilde{v}_{\mu_{\mathcal{F}(e)}} \geqq 0$, therefore $\mathcal{M}^2\tilde{T}^*\tilde{T} - \tilde{T}\tilde{T}^* \geqq 0$.

Hence we have $\mathcal{M}^2\tilde{T}^*\tilde{T} \geqq \tilde{T}\tilde{T}^*$, then $\tilde{T}$ is $\mathcal{FSM}h$-operator.

4.5

4.6 Proposition (1)

Let $\tilde{T}$ be invertible $\mathcal{FSM}h$-operator on $\mathcal{FSH}$-space $\widetilde{\mathcal{H}}$,then $\tilde{T}^{-1}$ is $\mathcal{FSM}h$-operator.

1) Proof:

Since $\tilde{T}$ be $\mathcal{FSM}h - operator$, then $\mathcal{M}^2\tilde{T}^*\tilde{T} \geqq \tilde{T}\tilde{T}^*$.

And since $\tilde{T}$ invertible, then $\mathcal{M}^2(\tilde{T}^*\tilde{T})(\tilde{T}\tilde{T}^*)^{-1} \geqq I$, this implies

$\mathcal{M}^2(\tilde{T}\tilde{T}^*)^{-1} \geqq (\tilde{T}^*\tilde{T})^{-1}$ so, $\mathcal{M}^2(\tilde{T}^*)^{-1}\tilde{T}^{-1} \geqq \tilde{T}^{-1}(\tilde{T}^*)^{-1}$, so we have

$\mathcal{M}^2(\tilde{T}^{-1})^*\tilde{T}^{-1} \geqq \tilde{T}^{-1}(\tilde{T}^{-1})^*$.

Then $\tilde{T}^{-1}$ is $\mathcal{FSM}h$-operator.

4.7 Proposition (2)

Let $\tilde{T}: \widetilde{\mathcal{H}} \to \widetilde{\mathcal{H}}$ be a $\mathcal{FS}h$ -operator defined on $\mathcal{FSH}$ -space $\widetilde{\mathcal{H}}$, then $\mathcal{M}^2(\tilde{T}^*\tilde{T})^n \geqq (\tilde{T}\tilde{T}^*)^n$ is $\mathcal{FSM}h$-operator.

1) Proof:

We can prove by using mathematical induction .

Since $\tilde{T}$ is $\mathcal{FSM}h$-operator, then $\mathcal{M}^2(\tilde{T}^*\tilde{T})^n \gneqq (\tilde{T}\tilde{T}^*)^n$ $\mathcal{FSM}h$-operator for n=1 that is

$\mathcal{M}^2(\tilde{T}^*\tilde{T})^1 \gneqq (\tilde{T}\tilde{T}^*)^1$ ...(1)

Suppose that $the\ result\ is\ true\ for\ n = m, that\ is$

$\mathcal{M}^2(\tilde{T}^*\tilde{T})^m \gneqq (\tilde{T}\tilde{T}^*)^m$ ... (2)

Now, to prove the result is true for n= m+1, that is

$$\mathcal{M}^2\left(\tilde{T}^*\tilde{T}\right)^{m+1} \gneqq (\tilde{T}\tilde{T}^*)^m(\tilde{T}\tilde{T}^*)^1$$

Then from (1) and (2), we have

$$\mathcal{M}^2\left(\tilde{T}^*\tilde{T}\right)^{m+1} \gneqq (\tilde{T}\tilde{T}^*)^{m+1}$$

Therefore $\mathcal{M}^2(\tilde{T}^*\tilde{T})^n \gneqq (\tilde{T}\tilde{T}^*)^n$ is $\mathcal{FSM}h$-operator.

4.8

4.9 Theorem (2)

Let $\tilde{T}_1$ and $\tilde{T}_2$ be $\mathcal{FSM}h$-operators on $\mathcal{FSH}$-space $\tilde{\mathcal{H}}$. If $\tilde{T}_1\tilde{T}_2^* \cong \tilde{T}_2^*\tilde{T}_1$ and $\tilde{T}_2\tilde{T}_1^* \cong \tilde{T}_1^*\tilde{T}_2$, also there exist a positive real number $\mathcal{M}$ such that $\mathcal{M}^2 \geq \mathcal{M}_1^2, \mathcal{M}_2^2$ then $\tilde{T}_1 + \tilde{T}_2$ is $\mathcal{FSM}h$-operator

1) Proof :

Since $\tilde{T}_1$ and $\tilde{T}_2$ be $\mathcal{FSM}h$-operators, then $\tilde{T}_1\tilde{T}_1^* \cong \tilde{T}_1^*\tilde{T}_1$ and $\tilde{T}_2\tilde{T}_2^* \cong \tilde{T}_2^*\tilde{T}_2$.

Now to show that $\tilde{T}_1 + \tilde{T}_2$ is $\mathcal{FSM}h$-operator , we have

$$\begin{aligned}
\left(\tilde{T}_1 + \tilde{T}_2\right)\left(\tilde{T}_1 + \tilde{T}_2\right)^* &\cong (\tilde{T}_1 + \tilde{T}_2)(\tilde{T}_1^* + \tilde{T}_2^*) \\
&\cong \tilde{T}_1\tilde{T}_1^* + \tilde{T}_1\tilde{T}_2^* + \tilde{T}_2\tilde{T}_1^* + \tilde{T}_2\tilde{T}_2^* \\
&\lneqq \mathcal{M}_1^2\, \tilde{T}_1^*\tilde{T}_1 + \tilde{T}_2^*\tilde{T}_1 + \tilde{T}_1^*\tilde{T}_2 + \mathcal{M}_2^2\tilde{T}_2^*\tilde{T}_2 \text{ so one get} \\
&\lneqq \mathcal{M}^2\, \tilde{T}_1^*\tilde{T}_1 + \mathcal{M}^2\tilde{T}_2^*\tilde{T}_1 + \mathcal{M}^2\tilde{T}_1^*\tilde{T}_2 + \mathcal{M}^2\tilde{T}_2^*\tilde{T}_2 \text{ also have} \\
&\cong \mathcal{M}^2(\tilde{T}_1\tilde{T}_1^* + \tilde{T}_1\tilde{T}_2^* + \tilde{T}_2\tilde{T}_1^* + \tilde{T}_2\tilde{T}_2^*) \\
&\cong \mathcal{M}^2\ \left(\tilde{T}_1^* + \tilde{T}_2^*\right)\widetilde{T}_1 + \left(\tilde{T}_1^* + \tilde{T}_2^*\right)\ \tilde{T}_2 \text{ thus} \\
&\cong \mathcal{M}^2\left(\tilde{T}_1^* + \tilde{T}_2^*\right)\left(\tilde{T}_1 + \tilde{T}_2\right)\text{, therefore,} \\
&\cong \mathcal{M}^2\left(\tilde{T}_1 + \tilde{T}_2\right)^*\left(\tilde{T}_1 + \tilde{T}_2\right),\ \text{ and we get}
\end{aligned}$$

$\tilde{T}_1 + \tilde{T}_2$ is $\mathcal{FSM}h$-operator

4.10 Theorem (3)

Let $\tilde{T}_1$ and $\tilde{T}_2$ be $\mathcal{FSM}h$-operators on $\mathcal{FSH}$-space $\widetilde{\mathcal{H}}$. Then $\tilde{T}_1\tilde{T}_2$ is $\mathcal{FSM}h$-operator If the conditions $\tilde{T}_1\tilde{T}_2^* \cong \tilde{T}_2^*\,\tilde{T}_1$ and $\tilde{T}_2\tilde{T}_1^* \cong \tilde{T}_1^*\,\tilde{T}_2$, are satisfied. also there exist a positive real number $\mathcal{M}$ such that $\mathcal{M}^2 = \mathcal{M}_1^2\mathcal{M}_2^2$

1) Proof :

Since $\tilde{T}_1$ and $\tilde{T}_2$ be $\mathcal{FSM}h$-operators, then $\tilde{T}_1\tilde{T}_1{}^* \cong \tilde{T}_1{}^*\tilde{T}_1$ and $\tilde{T}_2\tilde{T}_2{}^* \cong \tilde{T}_2{}^*\tilde{T}_2$.

Now to show that $\tilde{T}_1\tilde{T}_2$ is $\mathcal{FSM}h$ -operator , we have

$$\begin{aligned}
(\tilde{T}_1\tilde{T}_2)(\tilde{T}_1\tilde{T}_2)^* &\cong (\tilde{T}_1\tilde{T}_2)\ (\tilde{T}_2{}^*\tilde{T}_1{}^*)\\
&\cong \tilde{T}_1(\tilde{T}_2\tilde{T}_2{}^*)\tilde{T}_1{}^*\\
&\lesssim \mathcal{M}_2^2\,\tilde{T}_1\,(\tilde{T}_2{}^*\tilde{T}_2)\,\tilde{T}_1{}^*\\
&\cong \mathcal{M}_2^2(\tilde{T}_2{}^*\tilde{T}_2)\ (\tilde{T}_1\tilde{T}_1{}^*) \text{ thus}\\
&\lesssim \mathcal{M}_1^2\mathcal{M}_2^2(\tilde{T}_2{}^*\tilde{T}_2)\,(\tilde{T}_1\tilde{T}_1{}^*)\\
&\cong \mathcal{M}_1^2\mathcal{M}_2^2(\tilde{T}_2{}^*\,\tilde{T}_1{}^*)\tilde{T}_1\tilde{T}_2\\
&\lesssim \mathcal{M}^2\,(\tilde{T}_1\tilde{T}_2)^*(\tilde{T}_1\tilde{T}_2)
\end{aligned}$$

Hence, $\tilde{T}_1\tilde{T}_2$ is $\mathcal{FSM}h$-operator

4.11

4.12 Remark (1)

$\mathcal{FS}$-self adjoint operators are $\mathcal{FS}h$-operators. The converse, however, need not hold, for if $\tilde{T}:\widetilde{\mathcal{H}} \to \widetilde{\mathcal{H}}$ defined by

$\tilde{T} = \begin{bmatrix} (0.3,0) & (0.2,0) \\ (0.1,0) & (0.1,0) \end{bmatrix}$ is $\mathcal{FSM}h$-operator but not $\mathcal{FS}$-self adjoint operator.

4.13

4.14 Remarks and examples (2)

i) $\mathcal{FSQN}$-operatos are $\mathcal{FSM}h$-operators, the converse need not hold, as the example below shows

The $\mathcal{FS}$- operator $\tilde{T}=\begin{bmatrix}(0.3,0) & (0.2,0)\\(0.1,0) & (0.1,0)\end{bmatrix}$ is $\mathcal{FSM}h$-operator, where $\mathcal{M}^2$=4 ,but not $\mathcal{FSQN}$-operator

ii) $\mathcal{FS}h$-operators are $\mathcal{FSM}h$-operators. The converse holds if $\mathcal{M}=1$. For values of $\mathcal{M}\geq 1$, the converse no longer hold as shown in the counter example below:

The $\mathcal{FS}$- operator $\tilde{T}=\begin{bmatrix}(0.3,0) & (0.2,0)\\(0.1,0) & (0.1,0)\end{bmatrix}$ is $\mathcal{FSM}h$-operator , where $\mathcal{M}^2$=4, but not $\mathcal{FS}h$-operator.

4.15 Definition (2)

Assume $\widetilde{\mathcal{H}}$ be a fuzzy soft Hilbert space and $\tilde{T}_S: SE(\widetilde{\mathcal{H}})\rightarrow SE(\widetilde{\mathcal{H}})$ be a bounded $\mathcal{SFS}$-operator, if $\tilde{T}_S\tilde{T}_S^* \lesseqqgtr \mathcal{M}^2\tilde{T}_S^*\tilde{T}_S$ is said to be soft fuzzy soft $\mathcal{M}$ hyponormal operator. And shortly ($\mathcal{SFSM}h$-operator)

4.16

4.17 Theorem (3)

Let $(\widetilde{\mathcal{H}},E)$ be $soft\ complex\ Hilbert$-space, and $\{\tilde{T}_e: e\ \tilde{\in}\ E\}$ be a collection of $\mathcal{FS}$-operator in $\mathcal{FSH}$-space $\widetilde{\mathcal{H}}$. if $\tilde{T}_S: SE(\widetilde{\mathcal{H}})\rightarrow SE(\widetilde{\mathcal{H}})$ defined by $\left(\tilde{T}_S\left(\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}\right)\right)(e)=\tilde{T}\left(\tilde{v}_{\mu_{\mathcal{F}(e)}}(e)\right)$,$\forall\ e\ \tilde{\in}\ E$,$and\ \forall\ \tilde{v}_{\mu_{\mathcal{F}(e)}}\ \tilde{\in}\ \widetilde{\mathcal{H}}$, is $\mathcal{SFSM}h$-operator, then $\tilde{T}$ is $\mathcal{FSM}h$-operator, for all $e\ \tilde{\in}\ E$.

1) Proof:

Let $\tilde{T}_S$ be $\mathcal{SFSM}h$-operator, then $\tilde{T}_S\tilde{T}_S^* \lesseqqgtr \mathcal{M}^2\tilde{T}_S^*\tilde{T}_S$. If $\tilde{T}_S\left(\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}\right)=\tilde{w}_{\rho_{\mathcal{F}(e)}}$, then,

$$<(\tilde{T}\tilde{T}^*)(\tilde{v}_{\mu_{\mathcal{F}(e)}}(e),\tilde{v}_{\mu_{\mathcal{F}_e}}(e))>\cong<\tilde{T}((\tilde{T}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}(e)),\tilde{v}_{\mu_{\mathcal{F}_e}}(e)>$$
$$\cong<\tilde{T}((\tilde{u}_{\sigma_{\mathcal{F}(e)}}(e)),\tilde{v}_{\mu_{\mathcal{F}_e}}(e)>$$
$$\cong<((\tilde{T}_S\tilde{T}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}(e)),\tilde{v}_{\mu_{\mathcal{F}_e}}(e)>$$
$$\lesseqqgtr<\mathcal{M}^2(\tilde{T}_S^*\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}(e)),\tilde{v}_{\mu_{\mathcal{F}_e}}(e)>$$

$$\begin{aligned}
&\cong< \mathcal{M}^2(\tilde{T}_S^*\tilde{w}_{\rho_{\mathcal{F}_e}}(e)), \tilde{v}_{\mu_{\mathcal{F}_e}}(e) > \\
&\cong< \mathcal{M}^2\tilde{T}_S^*(\tilde{w}_{\rho_{\mathcal{F}_e}}(e)), \tilde{v}_{\mu_{\mathcal{F}_e}}(e) > \\
&\cong< \mathcal{M}^2\tilde{T}^*(\tilde{w}_{\rho_{\mathcal{F}_e}}(e)), \tilde{v}_{\mu_{\mathcal{F}_e}}(e)) > \\
&\cong< \mathcal{M}^2\tilde{T}^*\left(\left(\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}\right)(e)\right), \tilde{v}_{\mu_{\mathcal{F}(e)}}(e) > \\
&\cong< \mathcal{M}^2(\tilde{T}^*\tilde{T})(\tilde{v}_{\mu_{\mathcal{F}(e)}}(e), \tilde{v}_{\mu_{\mathcal{F}(e)}}(e)) > .
\end{aligned}$$

Thus $\tilde{T}$ is $\mathcal{FSMh}$-operator.

4.18

4.19 Theorem (4)

Let $\{\tilde{T}_e : e\ \tilde{\in}\ E\}$ be a collection of $\mathcal{FSMh}$-operator defined on $\mathcal{FSH}$-space $\tilde{\mathcal{H}}$. Then we can determine an operator $\tilde{T}_S: SE(\tilde{\mathcal{H}}) \to SE(\tilde{\mathcal{H}})$,$\mathcal{SFSMh}$- operator and defined by $\left(\tilde{T}_S\left(\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}\right)\right)(e) = \tilde{T}\left(\tilde{v}_{\mu_{\mathcal{F}(e)}}(e)\right)$,$\forall\ e\ \tilde{\in}\ E$, and $\forall\ \tilde{v}_{\mu_{\mathcal{F}(e)}}\ \tilde{\in}\ \tilde{\mathcal{H}}$, and $\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}\ \tilde{\in}\ \mathcal{SFS}(\tilde{\mathcal{H}})$

1) Proof:

Let $\tilde{T}$ be $\mathcal{FSMh}-$operator , then $\tilde{T}\tilde{T}^* \leq \mathcal{M}^2\,\tilde{T}^*\tilde{T}$ $for\ all\ e\ \tilde{\in}\ E$, and $\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} \in \mathcal{SFS}(\tilde{\mathcal{H}})$. If $\tilde{T}_S\left(\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}\right) = \tilde{u}_{\sigma_{\mathcal{F}(e)}}$, and $\tilde{T}_S\left(\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}\right) = \tilde{w}_{\rho_{\mathcal{F}(e)}}$ then:

$$\begin{aligned}
< ((\tilde{T}_S\tilde{T}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}), \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} > (e) &\cong< \tilde{T}_S((\tilde{T}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}(e)), \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}(e) > \\
&\cong < \tilde{T}_S((\tilde{u}_{\sigma_{\mathcal{F}(e)}}(e)), \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}(e) > \\
&\cong < \tilde{T}((\tilde{u}_{\sigma_{\mathcal{F}(e)}}(e)), \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}(e) > \\
&\cong< \tilde{T}(\tilde{T}_S^*(\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}})(e)), \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}(e) > \\
&\cong< (\tilde{T}\tilde{T}^*)\left(\tilde{v}_{\mu_{\mathcal{F}_e}}(e), \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}(e)\right) > \\
&\lesseqq < \mathcal{M}^2\,(\tilde{T}^*\tilde{T})\left(\tilde{v}_{\mu_{\mathcal{F}(e)}}(e), \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}(e)\right) > \\
&\cong < \mathcal{M}^2\,\tilde{T}^*(\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}})(e)), \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}(e) > \\
&\cong< \mathcal{M}^2\,\tilde{T}_S^*(\tilde{w}_{\rho_{\mathcal{F}(e)}}(\tilde{\lambda})), \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}(e) > \\
&\cong< \mathcal{M}^2\,(\tilde{T}_S^*(\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}))(\tilde{\lambda})), \tilde{v}_{\mu_{\mathcal{F}(e)}}(e) > \\
&\cong< \mathcal{M}^2\,(\tilde{T}_S^*\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}), \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}(e) >
\end{aligned}$$

Thus $\tilde{T}_S$ is $\mathcal{SFSMh}$-operator.

4.20 Theorem (5)

Let $(\tilde{\mathcal{H}}, E)$ be $soft\ complex\ Hilbert$ -space ,and $\tilde{S}_S, \tilde{T}_S: SE(\tilde{\mathcal{H}}) \to SE(\tilde{\mathcal{H}})$ is $\mathcal{SFSMh}-$operators. If $\tilde{S}_S\ and\ \tilde{T}_S$ are commute and if the conditions

$\tilde{S}_S\tilde{T}_S \cong \tilde{T}_S\tilde{S}_S$, with $\tilde{S}_S\tilde{T}_S^* \cong \tilde{T}_S^*\tilde{S}_S$ and $\tilde{T}_S\tilde{S}_S^* \cong \tilde{S}_S^*\tilde{T}_S$

Are holds, so one can have $\tilde{S}_S\tilde{T}_S$ is $\mathcal{SFSMh}-$operator, also there exist a positive real number $\mathcal{M}$ such that $\mathcal{M}^2 = \mathcal{M}_1^2\mathcal{M}_2^2$

1) Proof :

Since $S_S, T_S \in \mathcal{B}(\widetilde{\mathcal{H}})$ are commutative soft fuzzy soft $\mathcal{M}$ hyponormal operators then $\tilde{S}_S\tilde{S}_S^* \leqq \mathcal{M}_1^2\tilde{S}_S^*\tilde{S}_S$ and $\tilde{T}_S\tilde{T}_S^* \leqq \mathcal{M}_2^2\tilde{T}_S^*\tilde{T}_S$.

Let $\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} \in SE(\widetilde{\mathcal{H}})$, then

$$
\begin{aligned}
< (\tilde{S}_S\tilde{T}_S)(\tilde{S}_S\tilde{T}_S)^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}(e) > &\cong < \tilde{S}_S\tilde{T}_S\tilde{T}_S^*\tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}(e) > \\
&\cong < \tilde{T}_S\tilde{T}_S^*(\tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}), \tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} > (e) \\
&\leqq < \mathcal{M}_2^2\tilde{T}_S^*\tilde{T}_S(\tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}), \tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} > (e) \\
&\cong < \mathcal{M}_2^2\tilde{S}_S^*\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}, \tilde{T}_S\tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} > (e) \\
&\cong < \mathcal{M}_2^2\tilde{S}_S^*\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}, \tilde{S}_S^*\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} > (e) \\
&\cong < \mathcal{M}_2^2\tilde{S}_S\tilde{S}_S^*(\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}), \tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} > (e) \\
&\leqq \mathcal{M}_1^2\mathcal{M}_2^2\ \tilde{S}_S^*\tilde{S}_S(\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}), \tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} > (e) \\
&\cong < \mathcal{M}^2(\tilde{S}_S\tilde{T}_S)^*(\tilde{S}_S\tilde{T}_S)\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} > (e)
\end{aligned}
$$

then $\tilde{S}_S\tilde{T}_S$ is $\mathcal{SFSM}h$ −operator.

4.21

4.22 Theorem (6)

Let $\tilde{S}_S, \tilde{T}_S: SE(\widetilde{\mathcal{H}}) \to SE(\widetilde{\mathcal{H}})$ be $\mathcal{SFSM}h$ −operators. If $\tilde{T}_S\tilde{S}_S^* \cong \tilde{S}_S^*\tilde{T}_S$, then $\tilde{S}_S + \tilde{T}_S$ is $\mathcal{SFSM}h$ −operator , where $\mathcal{M}^2 \geq \mathcal{M}_1^2, \mathcal{M}_2^2$

1) Proof :

From assumption this $\tilde{T}_S\tilde{S}_S^* \cong \widetilde{S}_S^*\tilde{T}_S$, with $\tilde{S}_S\tilde{T}_S^* \cong \tilde{T}_S^*\tilde{S}_S$. Also since $\tilde{S}_S, \tilde{T}_S$ are $\mathcal{SFSM}h$ −operators, then

$< \tilde{S}_S\tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) \leqq \mathcal{M}_1^2 < \tilde{S}_S^*\tilde{S}_S\ \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e)\ and\ <$
$\tilde{T}_S\tilde{T}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) \leqq \mathcal{M}_2^2 < \tilde{T}_S^*\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e)$

Let $\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} \widetilde{\in} SE(\widetilde{\mathcal{H}})$, then

$$
\begin{aligned}
&< (\tilde{S}_S + \tilde{T}_S)(\tilde{S}_S + \tilde{T}_S)^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) \\
&\cong < \tilde{S}_S\tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} + \tilde{S}_S\tilde{T}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} + \tilde{T}_S\tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} + \tilde{T}_S\tilde{T}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) \\
&\cong < \tilde{S}_S\tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) + < \tilde{S}_S\tilde{T}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) + < \tilde{T}_S\tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} \\
&\qquad > (e) + < \tilde{T}_S\tilde{T}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) \\
&\leqq \mathcal{M}_1^2 < \tilde{S}_S^*\tilde{S}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) + < \tilde{S}_S\tilde{T}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) + \\
&\qquad < \tilde{T}_S\tilde{S}_S^*\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) + \mathcal{M}_2^2 < \tilde{T}_S^*\tilde{T}_S\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) \\
&\cong < \mathcal{M}^2(\tilde{S}_S^*\tilde{S}_S + \tilde{S}_S\tilde{T}_S^* + \tilde{T}_S\tilde{S}_S^* + \tilde{T}_S^*\tilde{T}_S)\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) \\
&\cong < \mathcal{M}^2((\tilde{S}_S^*\tilde{S}_S + \tilde{S}_S^*\tilde{T}_S) + (\tilde{T}_S^*\tilde{S}_S + \tilde{T}_S^*\tilde{T}_S))\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}(e)}} > (e) \\
&\cong < \mathcal{M}^2(\tilde{S}_S^*(\tilde{S}_S + \tilde{T}_S) + \tilde{T}_S^*(\tilde{S}_S + \tilde{T}_S))\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) \\
&\cong < \mathcal{M}^2(\tilde{S}_S^* + \tilde{T}_S^*)(\tilde{S}_S + \tilde{T}_S))\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e) \\
&\cong < \mathcal{M}^2((\tilde{S}_S + \tilde{T}_S)^*(\tilde{S}_S + \tilde{T}_S))\tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}}, \tilde{v}_{\mathcal{G}_{(e)}\mu_{\mathcal{F}_e}} > (e)
\end{aligned}
$$

Hence $(\tilde{S}_S + \tilde{T}_S)(\tilde{S}_S + \tilde{T}_S)^* \leqq (\tilde{S}_S + \tilde{T}_S)^*(\tilde{S}_S + \tilde{T}_S)$

Thus $\tilde{S}_S + \tilde{T}_S$ is $\mathcal{SFSM}h$ −operator.

Conclusions: In this work, we obtain some main conclusions such as the addition of fuzzy soft $\mathcal{M}$ hyponormal operators is not necessary to being fuzzy soft $\mathcal{M}$ hyponormal and may be can when we add some grantee conditions, also we get another consequence is some types of fuzzy soft operator.

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