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Stress - Strength Reliability Estimation for Parallel Redundant System Based on Weibull-Ryleigh Distribution

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Abstract:

Estimation for reliability of a parallel redundant system with independent stress and strength Weibull-Ryleigh probability density functions is considered. Estimation of the reliability parameters was conducted according to three methods, namely maximum likelihood, moments and percentiles methods. Finally, the reliability estimate was calculated and the best method for estimation for each case was given using the mean squared error criteria. It was found that the best estimation method is the percentiles method.

Keywords: Weibull-Ryleigh distribution, Reliability, Stress- Strength, Reliability Estimation.

Introduction

Several studies have been conducted in terms of the stress-strength reliability. In (2020) N. S. Karam et.al. [1] studied the reliability of a multicomponent system based on the Lomax stress-strength model. In (2021) F. GülceCüran [2], the reliability of a redundant system with exponentially distributed stress and strength variables.In (2021) N. S. Karam [3], estimated the reliability a stress-strength model based on the Generalized Inverted Kumaraswamy distribution. The same year, S. A. Jabr and N. S. Karam [4] discussed the estimation of the reliability for Gompertz Fréchet stress-strength model. E. Sh. M. Haddad and F. Sh. M. Batah (2021) [5] studied the reliability estimation of the stress-strength Rayleigh Pareto model. A. A. J. Ahmed and F. Sh. M. Batah (2023) [6] estimated the reliability of a stress strength Power Rayleigh model.

The subject of this work is the Weibull-Rayleigh distribution which is a continuous probability distribution found in life testing experiments, reliability analysis, applied statistics and clinical studies.

Several generalizations have been studied by authors for the Weibull-Rayleigh distribution, one of which is the subject of this work and is expressed as $X \sim WRD(a,k)$. The probability density is given by [7]:

$$f(x,a,k) = \begin{cases} \frac{2a}{k^a} x^{2a-1} e^{-\left(\frac{x^2}{k}\right)^a} & ,x > 0, a,k > 0 \\ 0 & o.w. \end{cases} \qquad (1)$$

Where, a and k are respectively shape and scale parameters which are real numbers greater than zero. The cumulative distribution is given by:

$$F(x,a,k)=\begin{cases}1-e^{-\left(\frac{x^2}{k}\right)^{a}} & ,x>0,a,k>0\\ 0 & o.w.\end{cases} \quad (2)$$

The System Stress- Strength Reliability

A parallel system (redundant system) is composed of r components which is the limit state does not necessarily indicate a system failure. Reliability and redundancy have been the subject of numerus studies such as [8].

Assuming that the two random variables X and Y represent the strength and the stress respectively are independent with $X \sim WRD(a_2,k_2)$ and $Y \sim WRD(a_1,k_1)$, then:

$$f_x(x,a_2,k_2)=\begin{cases}\frac{2a_2}{k_2^{a_2}}x^{2a_2-1}e^{-\left(\frac{x^2}{k_2}\right)^{a_2}} & ,x>0,a_2,k_2>0\\ 0 & o.w.\end{cases} \quad (3)$$

and

$$f_y(y,a_1,k_1)=\begin{cases}\frac{2a_1}{k_1^{a_1}}y^{2a_1-1}e^{-\left(\frac{y^2}{k_1}\right)^{a_1}} & ,y>0,a_1,k_1>0\\ 0 & o.w.\end{cases} \quad (4)$$

The cumulative distribution for each is then given by:

$$F_y(y,a_1,k_1)=\begin{cases}1-e^{-\left(\frac{y^2}{k_1}\right)^{a_1}} & ,y>0,a_1,k_1>0\\ 0 & o.w.\end{cases} \quad (5)$$

And

$$F_x(x,a_2,k_2)=\begin{cases}1-e^{-\left(\frac{x^2}{k_2}\right)^{a_2}} & ,x>0,a_2,k_2>0\\ 0 & o.w.\end{cases} \quad (6)$$

The stress-strength (s-s) reliability can be found if we consider the formula given in [9]:

$$R_0=\int_0^{\infty}R_p(y)f_y(y)dy \quad (7)$$

Where, $R_p(y)=1-[1-R_x(y)]^r$ is the parallel system reliability.

$$F_x(y)=1-e^{-\left(\frac{y^2}{k_2}\right)^{a_2}} \quad y>0, \quad a_2,k_2>0 \quad (8)$$

And

$$R_x(y)=1-F_x(y)=e^{-\left(\frac{y^2}{k_2}\right)^{a_2}} \quad (9)$$

$$R_p(y)=1-[1-R_x(y)]^r=1-\left[1-e^{-\left(\frac{y^2}{k_2}\right)^{a_2}}\right]^r \quad (10)$$

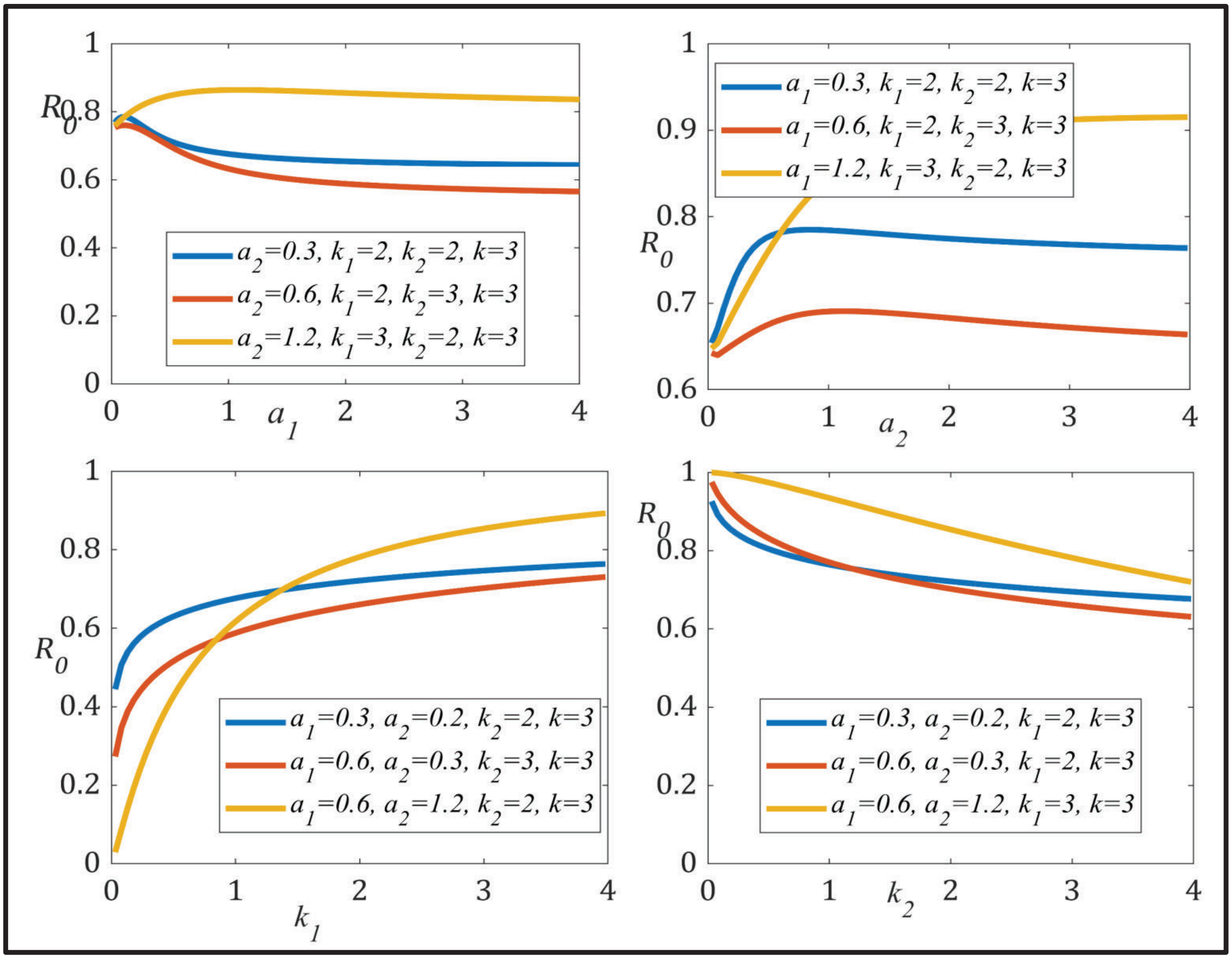

Figure 1: R_0 as plotted against different values of the parameters a_1, a_2, k_1, k_2.

So, using the binomial expansion

$$(x+y)^r = \sum_{i=0}^{r} C_i^r x^{r-i} y^i \qquad (11)$$

Then

$$R_p(y) = 1 - \sum_{i=0}^{r} C_i^r \left(-\exp\left[-\frac{y^{2a_2}}{k_2^{a_2}}\right]\right)^i = 1 - \sum_{i=0}^{r} C_i^r (-1)^i \exp\left[\frac{-i}{k_2^{a_2}} y^{2a_2}\right] \qquad (12)$$

Where, r is the number of components. The overall reliability of parallel redundant system under stress can be found using equations (4) and (12) by [10]:

$$R_0 = \int_0^\infty \left(R_P(y).f_y(y)\right)dy = \int_0^\infty \left(1 - \sum_{i=0}^{r} C_i^r (-1)^i \exp\left[\frac{-i}{k_2^{a_2}} y^{2a_2}\right]\right) f_y(y)dy$$

$$R_0 = 1 - \sum_{i=0}^{r} C_i^r (-1)^i \int_0^\infty \exp\left[\frac{-i}{k_2^{a_2}} y^{2a_2}\right] \frac{2a_1}{k_1^{a_1}} y^{2a_1-1} \exp - \left(\frac{y^2}{k_1}\right)^{a_1} dy$$

So that:

$$R_0 = 1 - \sum_{i=0}^{r} C_i^r (-1)^i \frac{2a_1}{k_1^{a_1}} \int_0^\infty y^{2a_1-1} \exp\left[\frac{-i}{k_2^{a_2}} y^{2a_2}\right] \exp - \left(\frac{y^2}{k_1}\right)^{a_1} dy \qquad (13)$$

Where $a_i, k_i > 0$ and $k \in \mathbb{Z}^+$, and the integration above can be calculated using the MATLAB command "integral". The behavior if the system reliability R_0 is plotted against each of the parameters in Figure 1.

Estimation Methods

In this section, three estimators for R_0 are discussed. The estimators are the maximum likelihood (MLE), moment (MO) and the Percentiles estimators. As the three derived estimators require initial values, the regression method is used to acquire them.

1.1 Regression Method

If the cumulative function given in 5 is considered, then

$$1 - F = e^{-\frac{x^{2a}}{k^a}} \Rightarrow k^{-a} x^{2a} = -\ln(1-F) \qquad (14)$$

Taking the natural logarithm of 14 and solving for $\ln x$ give:

$$\ln x = \frac{1}{2}\ln k + \frac{1}{2a}\ln(-\ln(1-F)) \qquad (15)$$

To acquire an approximate value for the parameters a, k it is possible to compare equation 15 to the linear regression equation[11]:

$$A = \Phi + \theta B + \epsilon \qquad (16)$$

The problem here is to estimate F. To that end, consider random sample $X_i \sim WRD(a,k)$ $i = 1, .., n$ and consider y_i to be the order statistic. Then a good estimation for $F(y_i)$ is the plot position $p_i = \frac{i}{n+1}$ especially since $E(F(y_i)) = p_i$ [12]. Considering this, then

$$\ln y_i = \frac{1}{2}\ln k + \frac{1}{2a}\ln(-\ln(1-p_i)) \qquad (17)$$

And if we compare this to

$$A_i = \Phi + \theta B_i + \epsilon \qquad (18)$$

We have $A_i = \ln y_i$, $\Phi = \frac{1}{2}\ln k$, $\theta = \frac{1}{2a}$ and $B_i = \ln(-\ln(1-p_i))$ so that

$$\hat{a}_0 = \frac{1}{2\hat{\theta}} \qquad (19)$$

$$\hat{k}_0 = e^{2\hat{\Phi}} \qquad (20)$$

Such that

$$\hat{\Phi} = \sum_{i=1}^{n} \frac{A_i}{n} - \hat{\theta} \sum_{i=1}^{n} \frac{B_i}{n} \qquad (21)$$

$$\hat{\theta} = \frac{n\sum_{i=1}^{n} A_i B_i - \sum_{i=1}^{n} A_i \sum_{i=1}^{n} B_i}{n\sum_{i=1}^{n} B_i^2 - (\sum_{i=1}^{n} B_i)^2} \qquad (22)$$

1.2 Maximum Likelihood Estimator (MLE)

For a random sample $X_i \sim WRD(a,k)$ $i = 1,..,n$, then the likelihood function is given by:

$$L = 2^n a^n k^{-na} \prod_{i=1}^{n} x_i^{2a-1} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{2a}}{k^a}\right) \qquad (23)$$

So, the log-likelihood become

$$\ln L = n \ln 2 + n \ln a - na \ln k + (2a-1)\sum_{i=1}^{n} x_i - \frac{\sum_{i=1}^{n} x_i^{2a}}{k^a} \qquad 24$$

Differentiating equation 24 with respect to a and equating to zero we obtain:

$$\frac{\partial \ln L}{\partial a} = \frac{n}{a} - n \ln k + 2\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\left(\frac{x_i^2}{k}\right)^a \ln \frac{\sum_{i=1}^{n} x_i^2}{k}$$

$$\frac{n}{a} = n \ln k - 2\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\left(\frac{x_i^2}{k}\right)^a \ln \frac{\sum_{i=1}^{n} x_i^2}{k}$$

and

$$\hat{a}_{MLE} = n\left[n \ln \hat{k}_0 - 2\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\left(\frac{x_i^2}{\hat{k}_0}\right)^{\hat{a}_0} \ln \frac{\sum_{i=1}^{n} x_i^2}{\hat{k}_0}\right]^{-1} \qquad (25)$$

Where $\hat{a}_0, \hat{k}_0$ are as given in equations 19 and 20. On the other hand, differentiating equation 24 with respect to k and equating to zero gives

$$\frac{\partial \ln L}{\partial k} = -\frac{na}{k} - (-ak^{-a-1})\sum_{i=1}^{n} x_i^{2a}$$

$$\frac{na}{k} = \frac{a}{k^{a+1}}\sum_{i=1}^{n} x_i^{2a} \Rightarrow n = \frac{1}{k^a}\sum_{i=1}^{n} x_i^{2a} \Rightarrow k^a = \frac{1}{n}\sum_{i=1}^{n} x_i^{2a}$$

$$\hat{k}_{MLE} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^{2a_0}\right)^{\frac{1}{a_0}} \qquad (26)$$

And the estimator for k is

Where $\hat{a}_0$ is as given in equation 19

1.3 Moments Method (MO)

The r^{th} moment of a random variable $X \sim WRD(a,k)$ is given by [7]

$$E(x^r) = k^{\frac{r}{2}}\,\Gamma\left(\frac{r}{2a}+1\right) = k^{\frac{r}{2}}\frac{r}{2a}\,\Gamma\left(\frac{r}{2a}\right)$$

$$E(x) = k^{\frac{1}{2}}\frac{1}{2a}\,\Gamma\left(\frac{1}{2a}\right) \qquad (27)$$

So, the first and second moments are given by:

$$E(x^2) = \frac{k}{a}\,\Gamma\left(\frac{1}{a}\right) \tag{28}$$

$$\bar{x} = k^{\frac{1}{2}}\frac{1}{2a}\,\Gamma\left(\frac{1}{2a}\right) \tag{29}$$

$$\frac{\sum_{i=1}^{n} x_i^2}{n} = \frac{k}{a}\,\Gamma\left(\frac{1}{a}\right) \tag{30}$$

$$\hat{k}_{MO} = \frac{\hat{a}_0 \sum_{i=1}^{n} x_i^2}{n\Gamma\left(\frac{1}{\hat{a}_0}\right)} \tag{31}$$

Equating equations 27 and 28 with $\bar{x}$ and $\frac{\sum_{i=1}^{n} x_i^2}{n}$ we obtain:

Form equation 30 we get

Also form equation 29

$$\hat{a}_{MO} = \left(\hat{k}_{MO}\right)^{\frac{1}{2}}\frac{\Gamma\left(\frac{1}{2\hat{a}_0}\right)}{2\bar{x}} \tag{32}$$

Again, $\hat{a}_0$ and $\hat{k}_0$ are as given in equations 19 and 20.

1.4 Percentiles (PE)

The method was first implemented 1959 by Kao, J [13]. The aim is to minimize P where:

$$P = \sum_{i=1}^{n}\left[\ln y_i - \frac{1}{2}\ln k - \frac{1}{2a}\ln(-\ln(1-p_i))\right]^2 \tag{33}$$

Here, $y_i,\ i = 1,2,\dots,n$ is the order statistic and $p_i = \frac{i}{n+1}$ is the plot position as discussed in section 3.1.

Now differentiating equation 33 with respect to k and equating to 0 gives:

$$0 = -\frac{2}{2k}\sum_{i=1}^{n}\left[\ln y_i - \frac{1}{2}\ln k - \frac{1}{2a}\ln(-\ln(1-p_i))\right] \tag{34}$$

To solve 34 for k we have

$$0 = \sum_{i=1}^{n}\ln y_i - \frac{n}{2}\ln k - \frac{1}{2a}\sum_{i=1}^{n}[\ln(-\ln(1-p_i))]$$

$$\ln k = \frac{2}{n}\sum_{i=1}^{n}\ln y_i - \frac{1}{na}\sum_{i=1}^{n}[\ln(-\ln(1-p_i))]$$

And so, by exponentiating both sides

$$\hat{k}_{PE} = \exp\left[\frac{2}{n}\sum_{i=1}^{n}\ln y_i - \frac{1}{n\hat{a}_0}\sum_{i=1}^{n}[\ln(-\ln(1-p_i))]\right] \tag{35}$$

Again differentiating equation 33 with respect to a and equating to 0 gives:

$$0=\left(\frac{2}{2a^2}\right)\sum_{i=1}^{n}\left[\ln y_i-\frac{1}{2}\ln k-\frac{1}{2a}\ln(-\ln(1-p_i))\right]\ln(-\ln(1-p_i)) \quad (36)$$

Solving 36 for a gives

$$0=2\sum_{i=1}^{n}[\ln y_i\ln(-\ln(1-p_i))]-\sum_{i=1}^{n}[\ln k\ln(-\ln(1-p_i))]-\frac{1}{a}\sum_{i=1}^{n}[\ln(-\ln(1-p_i))]^2$$

$$\frac{1}{a}\sum_{i=1}^{n}[\ln(-\ln(1-p_i))]^2=2\sum_{i=1}^{n}[\ln y_i\ln(-\ln(1-p_i))]-\sum_{i=1}^{n}[\ln k\ln(-\ln(1-p_i))]$$

Dividing and taking the reciprocal give:

$$\hat{a}_{PE}=\frac{\sum_{i=1}^{n}[\ln(-\ln(1-p_i))]^2}{2\sum_{i=1}^{n}[\ln y_i\ln(-\ln(1-p_i))]-\sum_{i=1}^{n}\left[\ln\hat{k}_0\ln(-\ln(1-p_i))\right]} \quad (37)$$

Where $\hat{a}_0$ and $\hat{k}_0$ are as given in equations 19 and 20.

Simulation

A simulation study of size (1000) is used to compare the reliability estimators. To that end, MATLAB(2018b) is used to generate a complete type data the data was then used to acquire the reliability estimates based on methods given in section 3. A comparison was then made to test for performance using the mean square error (MSE) criteria. The procedure was done as follows:

- From Equation 5 and 6, we let $U_x=F_x(x)$,$U_y=F_y(y)$ where U_x,U_y are uniformly distributed over $(0,1)$. And the random sample is generated by:

$$U_x=1-e^{-\left(\frac{x^2}{k_1}\right)^{a_1}}\Rightarrow\ln(1-U_x)=-\left(\frac{x^2}{k_1}\right)^{a_1}\Rightarrow\frac{x^2}{k_1}=(-\ln(1-U_x))^{\frac{1}{a_1}}$$

Therefore, we get

$$x=k_1^{\frac{1}{2}}(-\ln(1-U_x))^{\frac{1}{2a_1}} \quad (38)$$

A similar argument gives:

$$y=k_2^{\frac{1}{2}}\left(-\ln\left(1-U_y\right)\right)^{\frac{1}{2a_2}} \quad (39)$$

- A random sample of size m,n are generated for x_i and y_j using equations 38 and 39 where $m=15{,}30{,}90$ and $n=15{,}30{,}90$. The real values of the parameters a_1,a_2,k_1,k_2 were taken to be $(a_1,a_2,k_1,k_2)=(0.3,0.2,2,2)$, $(0.2,0.3,2,2)$, $(0.6,1.2,2,3)$, $(1.2,0.6,3,2)$, $(1.2,1.2,3,2)$, $(1.2,1.2,2,3)$, $(2,1.5,3,2)$ and $(2,1.5,2,3)$ the resulting data sets became 72 data sets for each x and y.
- The real values of the reliability R_0 was calculated according to equation 13 with $r=3$.
- Parametric estimation was then conducted for each data sets according to equations 24 and 25 for the MLE method, equations 31 and 32 for the moments method and equations 35 and 37 for

the percentiles method. For each case the reliability of the system was estimated according to equation 13 resulting in 72 system reliability data sets of size 1000.

- The mean of the data sets for each case was calculated and is given for each case in the tables (1-8).
- The mean squared error (MSE) was also calculated according to the relation MSE= $\frac{1}{N}\sum_{i=1}^{N}(\hat{R}_i - R_0)^2$, where $N = 1000$. The values are also given in the tables (1-8) along with the best method corresponding to the minimum value of the MSE.

Table 1: Estimation results with $a_1 = 0.3, a_2 = 0.2, k_1 = 2, k_2 = 2, R_0 = 0.72137$

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n = 15$	Mean	0.725427723	0.685266775	0.716997959	PE
$m = 15$	MSE	0.011599541	0.024536106	0.006931972	
$n = 15$	Mean	0.736146331	0.729956546	0.717753536	PE
$m = 30$	MSE	0.010727076	0.025682836	0.007080227	
$n = 15$	Mean	0.738242384	0.818832564	0.706009726	PE
$m = 90$	MSE	0.010085151	0.029505895	0.007018601	
$n = 30$	Mean	0.70852365	0.622347177	0.715410347	PE
$m = 15$	MSE	0.009104063	0.027065504	0.005902622	
$n = 30$	Mean	0.721387009	0.659187367	0.716748472	PE
$m = 30$	MSE	0.00661468	0.022046637	0.004897644	
$n = 30$	Mean	0.730231856	0.733861917	0.713706039	PE
$m = 90$	MSE	0.00502794	0.019370446	0.004060103	
$n = 90$	Mean	0.699525486	0.610961124	0.724282167	PE
$m = 15$	MSE	0.006127226	0.018820701	0.004234506	
$n = 90$	Mean	0.709785948	0.603026364	0.720967376	PE
$m = 30$	MSE	0.003169194	0.021947457	0.002536999	
$n = 90$	Mean	0.725321633	0.635277861	0.720585552	PE
$m = 90$	MSE	0.001864314	0.018004095	0.001537166	

Table 2: Estimation results with $a_1 = 0.2, a_2 = 0.3, k_1 = 2, k_2 = 2, R_0 = 0.77272$

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n = 15$	Mean	0.742904996	0.765343513	0.761155887	PE
$m = 15$	MSE	0.016244867	0.01553541	0.009049138	
$n = 15$	Mean	0.760372446	0.839319733	0.761717762	PE
$m = 30$	MSE	0.008654624	0.014278443	0.005560738	
$n = 15$	Mean	0.780205537	0.919711233	0.765174988	PE
$m = 90$	MSE	0.005449882	0.025034133	0.004176806	
$n = 30$	Mean	0.73766086	0.730887084	0.762232566	PE
$m = 15$	MSE	0.014708028	0.016497485	0.008141508	
$n = 30$	Mean	0.758790775	0.798704453	0.765913642	PE
$m = 30$	MSE	0.007050964	0.010271753	0.004888321	

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n=30$	Mean	0.771539828	0.8852018	0.766144389	PE
$m=90$	MSE	0.003193065	0.016864612	0.002514476	
$n=90$	Mean	0.736984319	0.692683356	0.768821566	PE
$m=15$	MSE	0.012080289	0.017035419	0.006925474	
$n=90$	Mean	0.75517679	0.742025757	0.772516863	PE
$m=30$	MSE	0.005471655	0.008039589	0.003861285	
$n=90$	Mean	0.766273924	0.822098504	0.770702919	PE
$m=90$	MSE	0.002096387	0.006413651	0.001538846	

Table 3: Estimation results with $a_1 = 0.6, a_2 = 1.2, k_1 = 3, k_2 = 2, R_0 = 0.85396$

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n=15$	Mean	0.826425091	0.712812326	0.830317301	PE
$m=15$	MSE	0.01054421	0.031125267	0.006055252	
$n=15$	Mean	0.846847684	0.827018489	0.838683979	PE
$m=30$	MSE	0.00512953	0.008874599	0.003881895	
$n=15$	Mean	0.856232854	0.975615693	0.842911397	PE
$m=90$	MSE	0.002354445	0.015284516	0.002007658	
$n=30$	Mean	0.823285017	0.641045371	0.834477307	PE
$m=15$	MSE	0.01087478	0.052887585	0.005950361	
$n=30$	Mean	0.844125329	0.722312293	0.844268731	PE
$m=30$	MSE	0.004607594	0.024192644	0.003038611	
$n=30$	Mean	0.854598705	0.911742528	0.846350696	PE
$m=90$	MSE	0.001719305	0.00529187	0.001513335	
$n=90$	Mean	0.81661566	0.589320949	0.835707148	PE
$m=15$	MSE	0.010894013	0.073055122	0.005595336	
$n=90$	Mean	0.834049334	0.60549	0.840513329	PE
$m=30$	MSE	0.004178154	0.064029992	0.002821787	
$n=90$	Mean	0.848548811	0.727522298	0.846611717	PE
$m=90$	MSE	0.00130747	0.01880961	0.001107668	

Table 4: Estimation results with $a_1 = 1.2, a_2 = 0.6, k_1 = 2, k_2 = 3, R_0 = 0.6181$

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n=15$	Mean	0.612163539	0.725408676	0.619473515	PE
$m=15$	MSE	0.015198416	0.017887647	0.009589614	
$n=15$	Mean	0.625515617	0.810371749	0.621429266	PE
$m=30$	MSE	0.0148263	0.041473865	0.008730195	
$n=15$	Mean	0.649759682	0.923524262	0.62823111	PE
$m=90$	MSE	0.01390237	0.094974841	0.008010675	
$n=30$	Mean	0.610048054	0.646448027	0.628548041	MOM
$m=15$	MSE	0.008781301	0.005832998	0.006324822	
$n=30$	Mean	0.617643491	0.726487679	0.61999902	PE
$m=30$	MSE	0.00693799	0.01612949	0.004817349	

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n = 30$	Mean	0.629079982	0.862434536	0.620743315	PE
$m = 90$	MSE	0.00718289	0.061481297	0.004892279	
$n = 90$	Mean	0.59907573	0.542270366	0.626373522	PE
$m = 15$	MSE	0.004628496	0.008680029	0.00345837	
$n = 90$	Mean	0.610964958	0.599469044	0.62360424	MOM
$m = 30$	MSE	0.003207352	0.002653597	0.002933778	
$n = 90$	Mean	0.617710036	0.729818257	0.618468118	PE
$m = 90$	MSE	0.002107393	0.013833818	0.001794575	

Table 5: Estimation results with $a_1 = 1.2, a_2 = 1.2, k_1 = 3, k_2 = 2, R_0 = 0.86387$

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n = 15$	Mean	0.853309601	0.817182358	0.836508333	PE
$m = 15$	MSE	0.00900365	0.01741069	0.006760778	
$n = 15$	Mean	0.869334094	0.952357167	0.841460726	PE
$m = 30$	MSE	0.005167792	0.010371861	0.004658162	
$n = 15$	Mean	0.882041747	0.998944159	0.845918512	MLE
$m = 90$	MSE	0.003336358	0.018254459	0.003398954	
$n = 30$	Mean	0.853038132	0.672986075	0.84690759	PE
$m = 15$	MSE	0.005983375	0.049465143	0.004638418	
$n = 30$	Mean	0.860137998	0.840844006	0.847628452	PE
$m = 30$	MSE	0.003484878	0.00765907	0.002782311	
$n = 30$	Mean	0.873639621	0.992158863	0.85097136	MLE
$m = 90$	MSE	0.002008538	0.016551882	0.002059506	
$n = 90$	Mean	0.845608764	0.482753327	0.854060678	PE
$m = 15$	MSE	0.004817874	0.150760052	0.003102701	
$n = 90$	Mean	0.854080171	0.581361231	0.853898782	PE
$m = 30$	MSE	0.002649958	0.086083904	0.002010359	
$n = 90$	Mean	0.865244778	0.859275259	0.857196735	PE
$m = 90$	MSE	0.001075093	0.002646136	0.000990661	

Table 6: Estimation results with $a_1 = 1.2, a_2 = 1.2, k_1 = 2, k_2 = 3, R_0 = 0.60686$

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n = 15$	Mean	0.576595217	0.599370904	0.62082802	PE
$m = 15$	MSE	0.021116208	0.028023029	0.011330403	
$n = 15$	Mean	0.593732483	0.797573784	0.613241449	PE
$m = 30$	MSE	0.014256942	0.05080499	0.009323139	
$n = 15$	Mean	0.61578725	0.987152757	0.613368482	PE
$m = 90$	MSE	0.010913067	0.144859798	0.006928251	
$n = 30$	Mean	0.576403407	0.444672568	0.62425912	PE
$m = 15$	MSE	0.013980277	0.042916607	0.008641735	
$n = 30$	Mean	0.589930901	0.604317338	0.617374229	PE
$m = 30$	MSE	0.008751599	0.015486118	0.00687228	

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n = 30$	Mean	0.607272782	0.918979618	0.612279103	PE
$m = 90$	MSE	0.006625888	0.099793877	0.0045877	
$n = 90$	Mean	0.568429039	0.313655187	0.621998843	PE
$m = 15$	MSE	0.010281187	0.093086992	0.00743745	
$n = 90$	Mean	0.583488326	0.353369201	0.614492018	PE
$m = 30$	MSE	0.00610869	0.070342069	0.004315439	
$n = 90$	Mean	0.596905763	0.597498181	0.612877187	PE
$m = 90$	MSE	0.002820167	0.007361019	0.002200717	

Table 7: Estimation results with $a_1 = 2, a_2 = 1.5, k_1 = 3, k_2 = 2, R_0 = 0.88568$

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n = 15$	Mean	0.892401207	0.906945167	0.863236499	PE
$m = 15$	MSE	0.005620087	0.007839361	0.004891529	
$n = 15$	Mean	0.902817734	0.995903044	0.866095421	MLE
$m = 30$	MSE	0.004196372	0.01222198	0.004234508	
$n = 15$	Mean	0.907328501	0.999968713	0.863410204	MLE
$m = 90$	MSE	0.003091883	0.013062101	0.003521717	
$n = 30$	Mean	0.882777828	0.657255068	0.866813981	PE
$m = 15$	MSE	0.003438798	0.069519978	0.003417402	
$n = 30$	Mean	0.891710462	0.927882348	0.872349429	PE
$m = 30$	MSE	0.002798417	0.004507294	0.002626945	
$n = 30$	Mean	0.89895694	0.999926801	0.870716167	MLE
$m = 90$	MSE	0.001985264	0.013052499	0.002174852	
$n = 90$	Mean	0.873873459	0.299452693	0.876116435	PE
$m = 15$	MSE	0.002757303	0.35185998	0.002069535	
$n = 90$	Mean	0.884130781	0.497805145	0.877391835	PE
$m = 30$	MSE	0.001623176	0.159618302	0.001382318	
$n = 90$	Mean	0.888945034	0.943119918	0.878822802	PE
$m = 90$	MSE	0.000958345	0.003898799	0.0008167	

Table 8: Estimation results with $a_1 = 2, a_2 = 1.5, k_1 = 2, k_2 = 3, R_0 = 0.531997$

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n = 15$	Mean	0.50590801	0.57944516	0.550437265	PE
$m = 15$	MSE	0.019542376	0.036281064	0.011334603	
$n = 15$	Mean	0.51224202	0.887219323	0.536811541	PE
$m = 30$	MSE	0.015802702	0.132785144	0.009224038	
$n = 15$	Mean	0.526933331	0.999399924	0.536988123	PE
$m = 90$	MSE	0.012273367	0.218476945	0.008554037	
$n = 30$	Mean	0.498928784	0.307767657	0.547641853	PE
$m = 15$	MSE	0.011883189	0.069372146	0.008187573	
$n = 30$	Mean	0.513898161	0.585464044	0.543155661	PE
$m = 30$	MSE	0.008984441	0.021394788	0.005692368	

Method		$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{PE}$	Best
$n = 30$ $m = 90$	Mean	0.533094974	0.985378984	0.544020037	PE
	MSE	0.006250318	0.205766133	0.004512653	
$n = 90$ $m = 15$	Mean	0.503580099	0.107162184	0.552086771	PE
	MSE	0.0073134	0.183698489	0.005788737	
$n = 90$ $m = 30$	Mean	0.512920049	0.186333714	0.542326788	PE
	MSE	0.004876205	0.125083702	0.003665313	
$n = 90$ $m = 90$	Mean	0.52153936	0.577460818	0.53518021	PE
	MSE	0.002764141	0.009909256	0.002291603	

Result Discussion

For the majority of the results, the PE method seems to perform better in terms of the MSE criterion. Some rare exception can be found however in Table 4 with the MO method performed better in two occasions. Again in Table 5 and Table 7 we see better performance for the MLE method in 2 and 3 occasions respectively.

Conclusion and Recommendation

Estimation of the reliability parameters was conducted according MLE, MO and PE methods for a parallel redundant system based on Weibull-Ryleigh distribution. The data was generated using a size 1000 simulation with a different value for the parameters a_1, k_2 and a_2, k_2 and components $k = 3$. The result shown PE method performed better in the majority of the experiments. It is therefore recommended to use the PE method for this particular distribution.

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