Journal of Education for Pure Science- University of Thi-Qar
Vol.13, No.3 (Sept.2023)
Website: jceps.utq.edu.iq Email jceps@eps.utq.edu.iq

DOI: http://doi.org/10.32792/utq.jceps.10.01.01

Hesitant Fuzzy Soft HX Prime Ideal of HX Ring

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Received 14/8/2023, Accepted 11/9/2023, Published 21/9/2023

Abstract :

In this paper , we study hesitant fuzzy soft HX set and some of its properties . We introduce the notions of hesitant fuzzy soft HX ideal , hesitant fuzzy soft HX prime ideal of a HX ring and hesitant fuzzy soft HX strongly prime ideal with some results about the hesitant fuzzy soft HX prime ideal of a HX ring .

Key words : Hesitant fuzzy soft HX set (HFSHXS), hesitant fuzzy soft HX ideal of a HX ring (HFSHXI(M)), hesitant fuzzy soft HX prime ideal of a HX ring (HFSHXPI(M)).

1.Introduction.

In (1965), Zadeh [10] introduced the concept of fuzzy set of a set X as a function from X into [0,1]. The theory of soft set was introduced by Molodtsov [9] in (1999). In (2001), Maji et al. [8] introduced the concept of fuzzy soft set . Ahmed and kharal [2] presented some more properties of them. Babitha and John[3] in(2013), introduced the hesitant fuzzy soft sets (HFSSs) concept. In (2021) Ali Abbas J. and M. J. Mohammed [1], introduced the notions of hesitant fuzzy ideal of a ring R. In (1988), Li Hong Xing [5], [6] proposed the concept of HX ring and derived some of its properties. In (2016) R. Muthuraj and N. Ramila Gandhi [7], introduced the concept of fuzzy HX left ideal and fuzzy HX right ideal of a HX ring.

the remainder of the paper is organized as follows : in section two, we recall some definition along with some results about them. In section three, we study hesitant fuzzy soft HX ideal of a HX ring, hesitant fuzzy soft HX prime ideal and hesitant fuzzy soft HX strongly prime ideal of a HX ring and introduced homomorphism on hesitant fuzzy soft HX prime ideal of HX ring.

2. Definition and preliminaries

In this section, we will discuss the following definitions as well as some of the findings that will be expected in the following pages.

2.1 Definition [4]

Let R be a ring and $(\tilde{F},E)$ be a fuzzy soft set defined on R and $M \subset 2^R - \{\emptyset\}$ be HX ring on R, the fuzzy soft HX set (G^F,E) : $G^F: E \to I^M$ on M defined as follows :

For all $e \in E$, $G^F{}_e(A) = \max \{F_e(x) \,/\, \text{for all } x \in A \subseteq R\}$.

2.2 Definition

Let R be a ring and let $(H\tilde{F},E)$ be a hesitant fuzzy soft set defined on R, $M \subset 2^R - \{\emptyset\}$ be a HX ring of R, the hesitant fuzzy soft set is called hesitant fuzzy soft HX set on M, if for all $A \in M$.

$H^F{}_e(A) = \cup\{ F_e(x) \,/\, x \in A \subseteq R\}$.

2.3 Example

Let $(Z_2,+,\cdot)$ be a ring and $(H\tilde{F},E)$ be a hesitant fuzzy soft set on Z_2 , $E =\{e_1, e_2\}$, $M= \{\{0\},\{1\}\}$ clearly M is a HX ring.

$H\tilde{F}: E \longrightarrow HF(z_2)$, $\tilde{F}e_1(0)=[0.2 ,0.6]$, $\tilde{F}e_1(1) =[0.2 ,0.3]$

$\tilde{F}e_2(0)= (0.4,0.8]$,$\tilde{F}e_2(1)= [0.3,0.6]$

$(\tilde{H}^F{}_e ,E) :\tilde{H}^F{}_e :E\longrightarrow HF(M)$

$(\tilde{H}^F{}_e, E) = \{< e_1 , \{ \frac{\{0\}}{\{0.6\}} , \frac{\{1\}}{\{0.3\}} \}>, < e_2 , \{ \frac{\{0\}}{\{0.8\}} ,\frac{\{1\}}{\{0.6\}} \}>\}$

Thus, ($\tilde{H}^F{}_e$,E) is a hesitant fuzzy soft HX set of M.

3. Hesitant fuzzy soft HX prime ideal.

3.1 Definition

Let R be a ring and let $(H\tilde{F}, E)$ be a hesitant fuzzy soft set defined on R, $M \subset 2^R - \{\emptyset\}$ be a HX ring on R, the hesitant fuzzy soft HX set is called

1) hesitant fuzzy soft HX subring (in short; HFSHXR) on a HX ring M if for all $A, B \in M$:

i. $\tilde{H}^F{}_e(A - B) \supseteq \tilde{H}^F{}_e(A) \cap \tilde{H}^F{}_e(B)$.
ii. $\tilde{H}^F{}_e(A B) \supseteq \tilde{H}^F{}_e(A) \cap \tilde{H}^F{}_e(B)$.

2) hesitant fuzzy soft HX ideal (in short ; HFSHXI) of M if for all A, B ϵ M:

i. $\widetilde{H}^{F}{}_{e}(A - B) \supseteq \widetilde{H}^{F}{}_{e}(A) \cap \widetilde{H}^{F}{}_{e}(B)$.
ii. $\widetilde{H}^{F}{}_{e}(A\ B) \supseteq \widetilde{H}^{F}{}_{e}(A)$.
iii. $\widetilde{H}^{F}{}_{e}(A\ B) \supseteq \widetilde{H}^{F}{}_{e}(B)$.

If (i) and (ii) are satisfied then it is called hesitant fuzzy soft HX right ideal of M, if (i) and (iii) are satisfied then it is called hesitant fuzzy soft HX left ideal of M.

Where $\widetilde{H}^{F}{}_{e}(A) = \cup \{ F_e(x) / x \in A \subseteq R\}$

3.2 Example

Let $(Z_2,+,\cdot)$ be a ring, M= {{0} ,{1}} be HX ring and $\widetilde{H}^{F}{}_{e} = < A, \widetilde{H}^{F}{}_{e}(A) >$ be a hesitant fuzzy soft HX set of M,

$$\widetilde{H}^{F}{}_{e}(A) = \begin{cases} [0,1] & \text{if } A = \{0\} \\ \emptyset & \text{if } A = \{1\} \end{cases}$$

Then $\widetilde{H}^{F}{}_{e}$ is a hesitant fuzzy soft HX ideal of HX ring.

3.3 Definition

A hesitant fuzzy soft HX ideal of a HX ring M is said to be hesitant fuzzy soft HX prime ideal of a HX ring M, if for all A, B ϵ M, then

$\widetilde{H}^{F}{}_{e}(AB) \subseteq \widetilde{H}^{F}{}_{e}(A) \cup \widetilde{H}^{F}{}_{e}(B)$

∗ will denote the set of all hesitant fuzzy soft HX prime ideal in M as HFSHXPI(M).

3.4 Example

Let $(Z_2,+,\cdot)$ be a ring and M = {{0},{1}} be HX ring

$\widetilde{H}^{F}{}_{e}(\{0\}) = \{0.6\}$, $\widetilde{H}^{F}{}_{e}(\{1\}) = \{0.3\}$

Then we can easily show that $\widetilde{H}^{F}{}_{e}$ ϵ HFSHXI(M).

Then $\widetilde{H}^{F}{}_{e}$ ϵ HFSHXPI(M).

3.5 Proposition

Let $\widetilde{H}^{F}_{e}$ be HFSHXS of a HX ring M, such that $(\widetilde{H}^{F}_{e})_{\alpha}$ is a prime ideal of M, for all $\alpha \subseteq [0, 1]$, then $\widetilde{H}^{F}_{e}$ is HFSHXPI of a HX ring M.

Proof:

Let $\widetilde{H}^{F}_{e}$ is HFSHXs of M such that $(\widetilde{H}^{F}_{e})_{\alpha}$ is a prime ideal of a HX ring M.

Suppose that $\widetilde{H}^{F}_{e}$ is not HFSHXPI of a HX ring M.

Then there exist A, B ϵ M such that

$\widetilde{H}^{F}_{e}(AB) \supset \widetilde{H}^{F}_{e}(A) \cup \widetilde{H}^{F}_{e}(B)$

Choose $\alpha \subseteq [0,1]$ such that $\widetilde{H}^{F}_{e}(AB) \supset \alpha \supset \widetilde{H}^{F}_{e}(A) \cup \widetilde{H}^{F}_{e}(B)$

This implies $AB \in (\widetilde{H}^{F}_{e})_{\alpha}$ but $A \notin (\widetilde{H}^{F}_{e})_{\alpha}$ and $B \notin (\widetilde{H}^{F}_{e})_{\alpha}$

Which is contradiction with $(\widetilde{H}^{F}_{e})_{\alpha}$ is a prime ideal of M

Hence, $\widetilde{H}^{F}_{e}$ is HFSHXPI of a HX ring M.

3.6 Definition

A hesitant fuzzy soft HX ideal of a HX ring M is said to be a hesitant fuzzy soft HX strongly prime ideal of M (in short, HFSHXSPI) if for all A, B ϵ M, such that

$\widetilde{H}^{F}_{e}(AB) = \widetilde{H}^{F}_{e}(A)$ or $\widetilde{H}^{F}_{e}(AB) = \widetilde{H}^{F}_{e}(B)$

$*$ Will denote the set of all hesitant fuzzy soft HX strongly prime ideal in M as HFSHXSPI(M).

3.7 Example

Let $(Z_2, +, \cdot)$ be a ring and M = {{0}, {1}} is a HX ring

$\widetilde{H}^{F}_{e}(\{0\}) = \{0.7\}$, $\widetilde{H}^{F}_{e}(\{1\}) = \{0.5\}$

Then we easily show that $\widetilde{H}^{F}_{e}$ is HFSHXI (M)

Thus, $\widetilde{H}^{F}_{e} \epsilon$ HFSHXSPI (M).

3.8 Theorem

Every hesitant fuzzy soft HX strongly prime ideal of M is a hesitant fuzzy soft HX prime ideal of M.

Proof :

Suppose that $\widetilde{H}^{F}_{e}$ is a hesitant fuzzy soft HX strongly prime ideal of M. Let A, B ϵ M, since $\widetilde{H}^F{}_e$ is a hesitant fuzzy soft HX strongly prime ideal of M, then

$\widetilde{H}^F{}_e\,(AB) = \widetilde{H}^F{}_e\,(A)$ or $\widetilde{H}^F{}_e\,(AB) = \widetilde{H}^F{}_e\,(B)$

If $\widetilde{H}^F{}_e\,(AB) = \widetilde{H}^F{}_e\,(A) \subseteq \widetilde{H}^F{}_e\,(A) \cup \widetilde{H}^F{}_e\,(B)$

If $\widetilde{H}^F{}_e\,(AB) = \widetilde{H}^F{}_e\,(B) \subseteq \widetilde{H}^F{}_e\,(A) \cup \widetilde{H}^F{}_e\,(B)$

Then, $\widetilde{H}^F{}_e\,(AB) \subseteq \widetilde{H}^F{}_e\,(A) \cup \widetilde{H}^F{}_e\,(B)$

Hence, $\widetilde{H}^F{}_e$ ϵ HFSHXPI (M).

The converse of theorem above may not to be true. As seen the following example

3.9 Example

Let $(Z_3,+,\cdot)$ be a ring and M= {{0} ,{1} ,{2}} be a HX ring

Let $\widetilde{H}^F{}_e$ be a hesitant fuzzy soft HX set of M such that

$$\widetilde{H}^F{}_e\,(T) = \begin{cases} [0,1] & \text{if } T=\{0\} \\ \emptyset & \text{if } T=\{1\},\{2\} \end{cases}$$

We can easily prove that $\widetilde{H}^F{}_e$ ϵ HFSHXI(M)

Hence, $\widetilde{H}^F{}_e$ ϵ HFSHXPI(M)

But $\widetilde{H}^F{}_e\,(\{2\}.\{2\}) = \widetilde{H}^F{}_e\,(\{1\})$, hence $\{2\} \neq \{1\}$

Then $\widetilde{H}^F{}_e \notin$ HFSHXSPI(M).

3.10 Theorem

Let M be a HX ring and $\widetilde{H}^F{}_e$, $\widetilde{H}^G{}_e$ ϵ HFSHXSPI(M) then $\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e$ ϵ HFSHXSPI(M)

Proof :

Suppose that $\widetilde{H}^F{}_e$, $\widetilde{H}^G{}_e$ ϵ HFSHXSPI(M) and A, B ϵ M

$(\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e)\,(AB) = \cup_{S_1\epsilon\ \widetilde{H}^F{}_{e\,(AB)}\,,\ S_2\epsilon\ \widetilde{H}^G{}_{e(AB)}} \min\{S_1, S_2\}$

$= \cup_{S_1\epsilon\ \widetilde{H}^F{}_{e\,(A)}\,,\ S_2\epsilon\ \widetilde{H}^G{}_{e(A)}} \min\{S_1, S_2\}$

$= (\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e)(A)$

Then, $(\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e)(AB) = (\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e)(A)$

Thus, $(\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e) \in$ HFSHXSPI(M).

or

$(\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e)(AB) = \cup_{S1\in\, \widetilde{H}^F{}_{e\,(AB)}\,,\, S2\in\, \widetilde{H}^G{}_{e(AB)}} \min\{S_1, S_2\}$

$= \cup_{S1\in\, \widetilde{H}^F{}_{e\,(B)}\,,\, S2\in\, \widetilde{H}^G{}_{e(B)}} \min\{S_1, S_2\}$

$= (\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e)(B)$

Then $(\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e)(AB) = (\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e)(B)$

Thus, $(\widetilde{H}^F{}_e \cap \widetilde{H}^G{}_e)(AB) \in$ HFSHXSPI(M).

3.11 Proposition

Let $\widetilde{H}^F{}_e \in$ HFSHXSPI (M) and $(\widetilde{H}^F{}_e)_* = \{ A \in M : \widetilde{H}^F{}_e(A) = \widetilde{H}^F{}_e(Q) \}$. Then $(\widetilde{H}^F{}_e)_*$ is strongly prime ideal of M .

Proof :

suppose that $\widetilde{H}^F{}_e \in$ HFSHXSPI (M), since $Q \in M$, then $\widetilde{H}^F{}_e(Q) = \widetilde{H}^F{}_e(Q)$

this means $Q \in (\widetilde{H}^F{}_e)_*$. Thus $(\widetilde{H}^F{}_e)_* \neq \emptyset$

Let A, B $\in$ M, A $(\widetilde{H}^F{}_e)_*$ B $\subseteq (\widetilde{H}^F{}_e)_*$ and AB $\in (\widetilde{H}^F{}_e)_*$ implies

$\widetilde{H}^F{}_e(AB) = \widetilde{H}^F{}_e(Q)$

since $\widetilde{H}^F{}_e \in$ HFSHXSPI (M), then

$\widetilde{H}^F{}_e(AB) = \widetilde{H}^F{}_e(A) = \widetilde{H}^F{}_e(Q)$, then $\widetilde{H}^F{}_e(A) = \widetilde{H}^F{}_e(Q)$

implies that A $\in (\widetilde{H}^F{}_e)_*$

If $\widetilde{H}^F{}_e(AB) = \widetilde{H}^F{}_e(B) = \widetilde{H}^F{}_e(Q)$, then $\widetilde{H}^F{}_e(B) = \widetilde{H}^F{}_e(Q)$

implies that B $\in (\widetilde{H}^F{}_e)_*$

So , A, B $\in$ M, A $(\widetilde{H}^F{}_e)_*$ B $\subseteq (\widetilde{H}^F{}_e)_*$ and AB $\in (\widetilde{H}^F{}_e)_*$

imply A $\in (\widetilde{H}^F{}_e)_*$ or B $\in (\widetilde{H}^F{}_e)_*$

Then $(\widetilde{H}^F{}_e)_*$ is strongly prime ideal of M.

3.12 Proposition

Let $f : M \longrightarrow M^{*}$ be an onto homomorphism of a HX rings and $\widetilde{H}^{F}{}_{e} \in$ HFSHXPI(M), then $f(\widetilde{H}^{F}{}_{e}) \in$ HFSHXPI(M^{*}).

Proof:

Let $\widetilde{H}^{F}{}_{e} \in$ HFSHXPI(M)

Let A, B $\in M^{*}$, since f onto homomorphism, then there exist C, D $\in$ M such that f(C) = A, f (D) = B

$$f(\widetilde{H}^{F}{}_{e})(AB) = \cup_{f(C)=A,\, f(D)=B} \widetilde{H}^{F}{}_{e}(CD)$$

$$\subseteq \cup_{f(C)=A,\, f(D)=B} \{ \widetilde{H}^{F}{}_{e}(C) \cup \widetilde{H}^{F}{}_{e}(D) \}$$

$$= \{ \cup_{f(C)=A} \widetilde{H}^{F}{}_{e}(C)\} \cup \{ \cup_{f(D)=B} \widetilde{H}^{F}{}_{e}(D) \}$$

$$= \{ \cup_{C \in f^{-1}(A)} \widetilde{H}^{F}{}_{e}(C)\} \cup \{ \cup_{D \in f^{-1}(B)} \widetilde{H}^{F}{}_{e}(D) \}$$

$$= f(\widetilde{H}^{F}{}_{e})(A) \cup f(\widetilde{H}^{F}{}_{e})(B)$$

Hence, $f(\widetilde{H}^{F}{}_{e}) \in$ HFSHXPI(M^{*}).

3.13 Proposition

Let $f : M \longrightarrow M^{*}$ be a homomorphism of a HX rings and $\widetilde{H}^{F}{}_{e} \in$ HFSHXPI(M^{*}), then $f^{-1}(\widetilde{H}^{F}{}_{e}) \in$ HFSHXPI(M).

Proof:

Let $\widetilde{H}^{F}{}_{e} \in$ HFSHXPI(M^{*}) and A, B $\in$ M, thus

$$f^{-1}(\widetilde{H}^{F}{}_{e})(AB) = \widetilde{H}^{F}{}_{e}(f(AB)) = \widetilde{H}^{F}{}_{e}(f(A)\, f(B)) \quad [\text{ f hom. }]$$

$$\subseteq \widetilde{H}^{F}{}_{e}(f(A)) \cup \widetilde{H}^{F}{}_{e}(f(B))$$

$$= f^{-1}(\widetilde{H}^{F}{}_{e})(A) \cup f^{-1}(\widetilde{H}^{F}{}_{e})(B)$$

Then, $f^{-1}(\widetilde{H}^{F}{}_{e})(AB) \subseteq f^{-1}(\widetilde{H}^{F}{}_{e})(A) \cup f^{-1}(\widetilde{H}^{F}{}_{e})(B)$

Hence, $f^{-1}(\widetilde{H}^{F}{}_{e}) \in$ HFSHXPI(M).

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