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About Stability and Data Dependence Results For The Multi_Explicit Four Step on Convex Metric Spaces

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Abstract

In this paper, a new type of contraction mapping is introduced, called generalized quasi like contractive mapping, and three types of new iterative schemes are presented as multi_L_explicit four step iteration, multi_ L_ explicit Noor iteration and multi_ L_Picard s-iteration. We study the rate of convergence and stability with generalized quasi like contractive mapping

Keywords:

Fixed Point, Convex Metric Space, Contractive Mapping, rate of convergence, Stability,and Suzuki generalized nonexpansive mappings.

1.Introduction:

Takahashi [1]established the idea of convexity in a metric space(m.s) and studied the features of the space known as a convex m.s in 1970. Furthermore, several fixed point(f-fixed) results for non-expansive maps were developed, he also mentioned that Banach spaces and their convex subsets are all convex m.s.On the other hands, Zamfirescu [2] introduced a more general mappings, and similar results were obtained for B-Theorem,concluding that all results of Kannan[3],Edelstein[4] and Singh [5]are in fact results of this new generalization, which Zamfirescu calls the operator if it satisfies the condition Z (Zamfirescu condetion). Suzuki[6] make some conditions on mapping called condition C, and proved the convergence theorem for mapping satisfying the condition C. Noor [7]introduces and discusses a novel class of three-step approximation algorithms for generic variances. His results cover Ishikawa and Mann iterations as special examples.He also investigated the schemes' convergence criteria. Asaduzzaman and Hossain[8] used the contractive-like operators to analyzed the f.point theorem for new iterative schemes called Four-step f.point scheme, and proved the convergence and data dependence.Olatinwo [9] introduced a definition of T-stably in convex.m.s and prove Mann and Ishikawa iterative is T-stable,The results obtained in stability are a generalization and expansion of the results obtained by each of Berinde [10], Harder and Hicks [11],Ostrowski [12],Shimizu and Takahashi [13].Berinde[14] created a new method in 2004 to compare the rate of convergence of two fixed point iterative algorithms. This method has become a typical tool for comparing the rate of convergence of two fixed point iterative systems in recent years. The researchers presented many studies in this field see [15-20].

2. Preliminaries: In this section, we will present some basic concepts and properties that we need in our work

Definition 2.1:[*1*] Let $Q: X \times X \times [0,1] \rightarrow$ be a mapping, then we say that Q is convex structure on metric space X if

$$d\big(z, Q(x,y,\xi)\big) \leq \xi d(z,x) + (1-\xi)d(z,y)$$

$\forall\, x,y,z \in X$ and $\xi \in [0,1]$. A metric space (X,d) with a convex structure Q define us convex metric structure and denoted by (X,d,Q).

Let E be a nonempty closed convex subset of a convex metric space X

Definition 2.2:[*7*] The explicit Noor iterative scheme is define as:$x_0 \in E$

$$x_n = (1-\alpha_n)x_{n-1} + \alpha_n Ŧ y_n$$

$$y_n = (1-\beta_n)x_{n-1} + \beta_n Ŧ z_n$$

$z_n = (1-\gamma_n)x_{n-1} + \gamma_n Ŧ x_{n-1}$; $n \in N$

Where $\langle\alpha_n\rangle$, $\langle\beta_n\rangle$ and $\langle\gamma_n\rangle$are real sequence in $[0,1]$. If $\gamma_n = 1$, we have explicit Ishikawa iterative scheme.

Definition 2.*3*:[2*1*] The Picard s-iterative scheme is define as:

$$x_0 \in E$$

$$x_n = Ŧ y_n$$

$$y_n = (1-\beta_n)Ŧ x_{n-1} + \beta_n Ŧ z_n$$

$z_n = (1-\gamma_n)x_{n-1} + \gamma_n Ŧ x_{n-1}$; $n \in N$, Where $\langle\beta_n\rangle$ and $\langle\gamma_n\rangle$are real sequence in $[0,1]$

Definition 2.*4*:[*8*] The four step explicit iterative scheme is define as:

$$x_0 \in E$$

$$x_n = (1-\alpha_n)x_{n-1} + \alpha_n Ŧ y_n$$

$$y_n = (1-\beta_n)x_{n-1} + \beta_n Ŧ z_n$$

$$z_n = (1-\gamma_n)x_{n-1} + \gamma_n Ŧu_n$$

$u_n = (1-\delta_n)x_{n-1} + \delta_n Ŧx_{n-1}\ ;\ n \in N$

Where $\langle\alpha_n\rangle$, $\langle\beta_n\rangle$, $\langle\gamma_n\rangle$ and $\langle\delta_n\rangle$ are real sequence in [0,1]. If $\delta_n = 1$, we have Noor type explicit iterative scheme.

Definition 2.5:[9] Suppose, (X, d, Q) be a convex metric space and $Ŧ: X \to X$ self mapping, $Ŧ\rho = \rho$. Let $\{x_n\}_{n=0}^{\infty} \subset X$ be the sequence produced by an iterative method hiring Ŧ, with the definition given by

$x_{n+1} = f_{Ŧ,a_n}^{x_n}$, n=0,1,2,…

A some function $f_{Ŧ,a_n}^{x_n}$ have convex structure with $a_n \in [0,1]$ and $x_0 \in X$ the initial approximation , $x_n \to \rho$. Let $\{y_n\}_{n=0}^{\infty} \subset X$ and $\epsilon_n = d\left(y_{n+1}, f_{T,a_n}^{y_n}\right), n = 0,1,2, \dots$. say $x_{n+1} = f_{T,a_n}^{x_n}$ is Ŧ-stable if and only if $\epsilon_n = 0$ implies $y_n = \rho$.

Lemma 2.6:[10] If $0 \le \sigma < 1$ and $\{\epsilon_n\}_{n=0}^{\infty}$ is sequence of positive number and $\epsilon_n \to 0$,if $\{x_n\}_{n=0}^{\infty}$ satisfying

$x_{n+1} \le \sigma x_n + \epsilon_n$, we have $x_n = 0$.

3. Main Results

In this section, we define generalized quasi like contractive mapping and introduce a new three iterative schemes from deferent type to study the convergent and convergence rate between of them.

Definition 3.1: The mapping $Ŧ: E \to E$ called generalized quasi like contractive when there exists mapping $\emptyset: R^+ \to R^+ with\ \emptyset(0) = 0$ and the constant $\xi \in [0,1], \lambda, \eta \ge 0$, then for each $x, y \in E$

$d(Ŧx, Ŧy) \le \xi d(x, y) + \lambda\emptyset\left(d(x, Ŧx)\right) + \eta min\{d(x, Ŧx), d(y, Ŧy)\}$

Remark 3.2: let Ŧ is generalized quasi like contractive

1. If $\xi \in [0,1], \lambda, \eta$=0, we get Ŧ is Zamfirescu mapping.
2. If $\xi = 0, \lambda = \frac{1}{2}, \eta = 0$and $\emptyset(x) = I_c x$,we get Ŧ is Suzuki generalized nonexpansive mapping.

Definition 3.3: The multi_L explicit four-step iteration is defined as follows:

Let $x_0 \in E$

$x_n = Q\left(V_{iE}x_{n-1}, Ŧ^0 y_n, \sum_{i=1}^{k} \quad \alpha_{ni}\right)$

$$y_n = Q\left(g x_{n-1}, Ŧ^0 z_n\,, (1-\beta_{n0})\right)$$

$z_n = Q(V_{0E}u_n, Ŧ^0 y_n, (1-\gamma_{n0}))$

$u_n = Q\left(V_{iE}x_{n-1}, T^0 x_{n-1}, \sum_{i=1}^{k} \delta_{n0}\right)$; $n \in N$

where V_{iE} contractive self mapping on E, g non-expansive self mapping on E and $\langle\alpha_{ni}\rangle$, $\langle\beta_{ni}\rangle$, $\langle\gamma_{ni}\rangle$ and $\langle\delta_{n0}\rangle$ are real sequence in [0,1], for all $i = 0,1,2,\dots,k$.

Remark 3.4: In the multi_L explicit four-step iterative if put $k = 1$ and $V_{iE}(x) = g(x) = I_E(x)$, we get the four step explicit iterative.

Definition 3.5: The multi_L explicit Noor iteration is defined as follows:

Let $x_0 \in E$

$x_n = Q\left(x_{n-1}, Ŧ^0 y_n, \sum_{i=1}^{k} \alpha_{ni}\right)$

$$y_n = Q\left(V_{0E}\, g x_{n-1}, Ŧ^0 z_n, (1-\beta_{n0})\right)$$

$z_n = Q\left(V_{iE}\, g x_{n-1}, V_{iE} Ŧ^0 x_{n-1}, \sum_{i=1}^{k} \gamma_{ni}\right)$; $n \in N$

where V_{iE} contractive self mapping on E , g non-expansive self mapping on E and $\langle\alpha_{ni}\rangle$, $\langle\beta_{ni}\rangle$ and $\langle\gamma_{ni}\rangle$ are real sequence in [0,1], for all $i = 0,1,2,\dots,k$.

Remark 3.6: In the multi_L explicit Noor iteration if put $k = 1$ and $V_{iE}(x) = g(x) = I_E(x)$, we get the explicit Noor iteration.

Definition 3.7: The multi_L Picard s-iteration is defined as follows

Let $x_0 \in E$

$$x_n = Q(V_{0E} y_n)$$

$y_n = Q\left(g\, x_{n-1}, V_{0c} Ŧ^i z_n, \alpha_{n0}\right)$

$z_n = Q\left(V_{iE}x_{n-1}, Ŧ^0 x_{n-1}, \sum_{i=1}^{k} \beta_{ni}\right)$ $n \in N$

where V_{iE} contractive self mapping on E, g non-expansive self mapping on E and $\langle\alpha_{ni}\rangle$, and $\langle\beta_{ni}\rangle$ are real sequence in [0,1], for all $i = 0,1,2,\dots,k$.

In the theorems below we denote E by a closed convex subset of the convex metric space.

Theorem 3.8: Let (X, d, Q) be a convex metric space and $Ŧ^0$ be a finite generalized quasi like contractive self mapping on E, with $\xi \in [0,1)$ and g is non-expansive self mapping on E, if $\rho \in CF(Ŧ^i, V_{iE}, g)$. Then, the multi_L explicit four-step iteration $\{x_n\}_{n=0}^{\infty}$ with $\Sigma(1-a_n) = \infty$, converges to the fixed point of $Ŧ^0$.

Proof: since $\rho \in CF(Ŧ^0, V_{iE}, g)$, then

$$d(x_n,\rho)=d\left(Q\left(V_{iE}x_{n-1},Ŧ^0y_n,\sum_{i=1}^{k}\alpha_{ni}\right),\rho\right)$$

$$\leq\sum_{i=1}^{k}\alpha_{ni}d(V_{iE}x_{n-1},\rho)+\alpha_{n0}d(Ŧ^0y_n,\rho)$$

$$\leq\sum_{i=1}^{k}L^i\alpha_{ni}d(x_{n-1},\rho)+\alpha_{n0}d(Ŧ^0y_n,\rho)$$

$$<\sum_{i=1}^{k}L^i\alpha_{ni}d(x_{n-1},\rho)+\alpha_{n0}(\xi_0d(y_n,\rho)+\lambda_0\emptyset_0(Ŧ^0\rho,\rho)\ +\eta_0min\{d(Ŧ^0y_n,y_n),d(Ŧ^0\rho,\rho)\})$$

$$<L^i\sum_{i=1}^{k}\alpha_{ni}d(x_{n-1},\rho)+\alpha_{n0}\xi_0d(y_n,\rho)$$

(3.1)

$$d(y_n,\rho)=d\left(Q\left(gx_{n-1},Ŧ^0z_n,(1-\beta_{n0})\right),\rho\right)$$

$$\leq(1-\beta_{n0})d(gx_{n-1},\rho)+\beta_{n0}d(Ŧ^0z_n,\rho)$$

$$\leq(1-\beta_{n0})d(x_{n-1},\rho)+\beta_{n0}d(Ŧ^0z_n,\rho)$$

$$\leq(1-\beta_{n0})d(x_{n-1},\rho)+\beta_{n0}(\xi_0d(z_n,\rho)+\lambda_0\emptyset_0(Ŧ^0\rho,\rho)\ +\eta_0min\{d(Ŧ^0z_n,z_n),d(Ŧ^0\rho,\rho)\})$$

$$\leq(1-\beta_{n0})d(x_{n-1},\rho)+\beta_{n0}\xi_0d(z_n,\rho)$$

(3.2)

$$d(z_n,\rho)=d(Q(V_{0E}u_n,Ŧ^0y_n,(1-\gamma_{n0})),\rho)$$

$$\leq(1-\gamma_{n0})d(V_{0E}x_{n-1},\rho)+\gamma_{n0}d(Ŧ^0u_n,\rho)$$

$$\leq L^0(1-\gamma_{n0})d(x_{n-1},\rho)+\gamma_{n0}(\xi_0d(u_n,\rho)+\lambda_0\emptyset_0(Ŧ^0\rho,\rho)\ +\eta_0min\{d(Ŧ^0u_n,u_n),d(Ŧ^0\rho,\rho)\})$$

$$\leq L^0(1-\gamma_{n0})d(x_{n-1},\rho)+\gamma_{n0}\xi_0d(u_n,\rho)$$

(3.3)

$$d(u_n,\rho)=d\left(Q\left(V_{iE}x_{n-1},T^0x_{n-1},\sum_{i=1}^{k}\delta_{n0}\right),\rho\right)$$

$$\leq\sum_{i=1}^{k}\delta_{ni}d(V_{iE}x_{n-1},\rho)+\delta_{n0}d(Ŧ^0x_{n-1},\rho)$$

$$\leq L^i\sum_{i=1}^{k}\delta_{ni}d(x_{n-1},\rho)+\delta_{n0}(\xi_0d(x_{n-1},\rho)+\lambda_0\emptyset_0(Ŧ^0\rho,\rho)\ +\eta_0min\{d(Ŧ^0x_{n-1},x_{n-1}),d(Ŧ^0\rho,\rho)\})$$

$$d(u_n,\rho)\leq\left(L^i\sum_{i=1}^{k}\delta_{ni}+\delta_{n0}\xi_0\right)d(x_{n-1},\rho)$$

(3.4)

$$L=\{L^i,i=0,1,2,\dots,k\}$$

$$\xi=\xi_0$$

From (3.1),(3.2),(3.3) and (3.4) we have

$$d(x_n,\rho) < \left(\sum_{i=1}^{k} \alpha_{ni} + \alpha_{n0}\xi\left((1-\beta_{n0}) + \beta_{n0}\xi\left((1-\gamma_{n0}) + \gamma_{n0}\xi\left(\sum_{i=1}^{k} \delta_{ni} + \delta_{n0}\xi\right)\right)\right)\right) d(x_{n-1},\rho)$$

$$= (1-\alpha_{n0} + \alpha_{n0}\xi(1-\beta_{n0}) + \alpha_{n0}\xi^2\beta_{n0}(1-\gamma_n) + \alpha_{n0}\xi^3\beta_{n0}\gamma_{n0}(1-\delta_{n0} + \delta_{n0}\xi)\ )d(x_{n-1},\rho)$$

$$= (1-\alpha_{n0} + \alpha_{n0}\xi - \alpha_{n0}\xi\beta_{n0} + \alpha_{n0}\xi^2\beta_{n0} - \alpha_{n0}\xi^2\beta_{n0}\gamma_n + \alpha_{n0}\xi^3\beta_{n0}\gamma_{n0} - \alpha_{n0}\xi^3\beta_{n0}\gamma_{n0}\delta_{n0} + \alpha_{n0}\xi^4\beta_{n0}\gamma_{n0}\delta_{n0}\ )d(x_{n-1},\rho)$$

$$= \left(1-\alpha_{n0}(1-\xi+\xi\beta_{n0} - \xi^2\beta_{n0} + \xi^2\beta_{n0}\gamma_{n0} - \xi^3\beta_{n0}\gamma_{n0} + \xi^3\beta_{n0}\gamma_{n0}\delta_{n0} - \xi^4\beta_{n0}\gamma_{n0}\delta_{n0}\ )\right)\ d(x_{n-1},\rho)$$

$$= (1-\alpha_{n0}(1-\xi+\xi\beta_{n0}(1-\xi) + \xi^2\beta_{n0}\gamma_{n0}(1-\xi) + \xi^3\beta_{n0}\gamma_{n0}\delta_{n0}(1-\xi)\ )\ )d(x_{n-1},\rho)$$

$$< (1-\alpha_{n0}(1-\xi)(1+\xi\beta_{n0} + \xi^2\beta_{n0}\gamma_{n0} + \xi^3\beta_{n0}\gamma_{n0}\delta_{n0})\ )\ d(x_{n-1},\rho)$$

:

$$d(x_n,\rho) < \prod_{j=1}^{n} (1-\alpha_{j0}(1-\xi)(1+\xi\beta_{j0} + \xi^2\beta_{j0}\gamma_{j0} + \xi^3\beta_{j0}\gamma_{j0}\delta_{j0}))d(x_0,\rho)$$

Take limit for two sides; we have $d(x_n,\rho)=0$ then $\{x_n\}_{n=0}^{\infty}$ define by multi_explicit four step-iteration convergent to fixed point ρ

Corollary 3.9: Under the same condition in theorem (3.8) ,if $₮^0$ is Suzuki generalized nonexpansive self mapping on E then, the multi_L explicit four-step iteration , converges to the fixed point of $₮^0$.

Corollary 3.10: Under the same condition in theorem (3.8) ,if $\{x_n\}_{n=0}^{\infty}$ defined by the four-step explicit iteration then, its converges to the fixed point of ₮.

Theorem 3.11: Let (X,d,Q) be a convex metric space and $₮^i$ be a finite generalized quasi like contractive self mapping on E for all $i=0,1,2,\dots,k$, and g is non-expansive self mapping on E, if $\rho \in CF(₮^i, V_{iE}, g)$. Then, the multi_L explicit Noor iteration and multi_L Picards-iteration are converges to the fixed point of $₮^i$.

Theorem 3.12: Let (X,d,Q) be a convex metric space and $₮^0$ be finite generalized quasi like contractive self mapping on E, for all $i=1,2,\dots,k$ with $\xi \in [0,1)$ and g is non-expansive self mapping on E with $\rho \in CF(₮^0, V_{iE}, g)$. Then, for $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by the multi_L explicit four-step iterative is $₮^0$-stable.

Proof: Let $\{h_n\}_{n=0}^{\infty} \subset E$ is an arbitrary sequence $\epsilon_n = d(h_n, Q(V_{iE}h_{n-1}, ₮^0 e_n, \sum_{i=1}^{k} \alpha_{ni}))$ where

$e_n = Q(gh_{n-1}, ₮^0 f_n, (1-\beta_{n0}))$, $f_n = Q(V_{0E}h_{n-1}, ₮^0 k_{n-1}, (1-\gamma_{n0}))$ and

$k_n = Q(V_{iE}h_{n-1}, Ŧ^0 h_{n-1}, \sum_{i=1}^{k} \delta_{ni})$,$\epsilon_n = 0$ $d(h_n,\rho) \le d\left(h_n, Q\left(V_{iE}h_{n-1}, Ŧ^0 e_n, \sum_{i=1}^{k} \alpha_{ni}\right)\right) + d\left(Q\left(V_{iE}h_{n-1}, Ŧ^0 e_n, \sum_{i=1}^{k} \alpha_{ni}\right), \rho\right)$

$$\le \epsilon_n + \sum_{i=1}^{k} \alpha_{ni} d(V_{iE}h_{n-1},\rho) + \alpha_{n0} d(Ŧ^0 e_n, \rho)$$

$$\le \epsilon_n + \sum_{i=1}^{k} L^i \alpha_{ni} d(h_{n-1},\rho) + \alpha_{n0}\left[\xi_0 d(e_n,\rho) + \lambda_0 \emptyset_0\left(d(Ŧ^0\rho,\rho)\right) + \eta_0 min\{d(e_n, Ŧ^0 e_n), d(Ŧ^0\rho,\rho)\}\right]$$

$$\le \epsilon_n + \sum_{i=1}^{k} L^i \alpha_{ni} d(h_{n-1},\rho) + \alpha_{n0}\xi_0 d(e_n,\rho)$$

(3.5)

$$d(e_n,\rho) \le (1-\beta_{n0}) d(gh_{n-1},\rho) + \beta_{n0} d(Ŧ^0 f_n, \rho)$$

$$\le (1-\beta_{n0}) d(h_{n-1},\rho) + \beta_{n0}\left[\xi_0 d(f_n,\rho) + \lambda_0 \emptyset_0\left(d(Ŧ^0\rho,\rho)\right) + \eta_0 min\{d(f_n, Ŧ^0 f_n), d(Ŧ^0\rho,\rho)\}\right]$$

$$\le (1-\beta_{n0}) d(h_{n-1},\rho) + \beta_{n0}\xi_0 d(f_n,\rho)$$

(3.6)

$$d(f_n,\rho) \le (1-\gamma_{n0}) d(V_{0E}h_{n-1},\rho) + \gamma_{n0} d(Ŧ^0 k_{n-1}, \rho)$$

$$\le L^0(1-\gamma_{n0}) d(h_{n-1},\rho) + \gamma_{n0}\left[\xi_0 d(k_n,\rho) + \lambda_0 \emptyset_0\left(d(Ŧ^0\rho,\rho)\right) + \eta_0 min\{d(k_n, Ŧ^0 k_n), d(Ŧ^0\rho,\rho)\}\right]$$

$$\le (1-\gamma_{n0}) d(h_{n-1},\rho) + \gamma_{n0}\xi_0 d(k_n,\rho)$$

(3.7)

$$d(k_n,\rho) \le \sum_{i=1}^{k} \delta_{ni} d(V_{iE}h_{n-1},\rho) + \delta_{n0} d(Ŧ^0 h_{n-1}, \rho)$$

$$\le \sum_{i=1}^{k} L^i \delta_{ni} d(h_{n-1},\rho) + \delta_{n0}\left[\xi_0 d(h_{n-1},\rho) + \lambda_0 \emptyset_0\left(d(Ŧ^0\rho,\rho)\right) + \eta_0 min\{d(h_{n-1}, Ŧ^0 h_{n-1}), d(Ŧ^0\rho,\rho)\}\right]$$

$$\le \sum_{i=1}^{k} L^i \delta_{ni} d(h_{n-1},\rho) + \delta_{n0}\xi_0 d(h_{n-1},\rho)$$

$$\le \left(\sum_{i=1}^{k} L^i \delta_{ni} + \delta_{n0}\xi_0\right) d(h_{n-1},\rho)$$

(3.8)

$$L = \{L^i, i = 0,1,2,\dots,k\}$$

From (3.5),(3.6),(3.7) and (3.8) we have

$$d(h_n,\rho) \le \epsilon_n + \sum_{i=1}^{k} \alpha_{ni} d(h_{n-1},\rho) +$$

$$\left(+\alpha_{n0}\xi_0\left((1-\beta_{n0})+\beta_{n0}\xi_0\left((1-\gamma_{n0})\ +\gamma_{n0}\xi_0\left(\left(\sum_{i=1}^{k}\quad\delta_{ni}+\delta_{n0}\xi_0\ \right)\right)\right)\right)\right)d(h_{n-1},\rho)$$

Take $\xi=\xi_0$

$$d(h_n,\rho)\le\epsilon_n+(1-\alpha_{n0}+\alpha_{n0}\xi-\alpha_{n0}\xi\beta_{n0}\ +\alpha_{n0}\beta_{n0}\xi^2-\alpha_{n0}\beta_{n0}\gamma_n\xi^2+\alpha_{n0}\beta_{n0}\gamma_{n0}\xi^3$$
$$-\alpha_{n0}\beta_{n0}\gamma_{n0}\delta_{n0}\xi^3+\alpha_{n0}\beta_{n0}\gamma_{n0}\delta_{n0}\xi^4\ )d(h_{n-1},\rho)$$

$$\le\epsilon_n+(1-\alpha_{n0}(1-\xi)(1+\alpha_{n0}\beta_{n0}+\beta_{n0}\gamma_{n0}\xi^2+\beta_{n0}\gamma_{n0}\delta_{n0}\xi^3)$$

but

$$(1-\alpha_{n0}(1-\xi)(1+\alpha_{n0}\beta_{n0}+\beta_{n0}\gamma_{n0}\xi^2+\beta_{n0}\gamma_{n0}\delta_{n0}\xi^3)\le 1-\alpha_{n0}(1-\xi)$$

$$d(h_n,\rho)\le\epsilon_n+(1-\alpha_{n0}(1-\xi))d(h_{n-1},\rho)$$

$(1-\alpha_{n0}(1-\xi))<1$ and $\epsilon_n=0$

by lemma (2.13)$d(h_{n-1},\rho)\to 0\ as\ n\to\infty$ then

$$d(h_n,\rho)\to 0\ as\ n\to\infty$$

Conversely If $h_n\ =\rho$

$\epsilon_n=d\left(h_n,Q\left(V_{iE}h_{n-1},Ŧ^0e_n,\sum_{i=1}^{k}\quad\alpha_{ni}\right)\right)$

$\le d(h_n,\rho)+d\left(\rho,Q\left(V_{iE}h_{n-1},Ŧ^0e_n,\sum_{i=1}^{k}\quad\alpha_{ni}\right)\right)$

$\le d(h_n,\rho)+\sum_{i=1}^{k}\quad\alpha_{ni}d(V_{iE}h_{n-1},\rho)+\alpha_{n0}d(Ŧ^0e_n,\rho)$

$\le$
$d(h_n,\rho)+\sum_{i=1}^{k}\quad L^i\alpha_{ni}d(h_{n-1},\rho)+$
$\alpha_{n0}\left[\xi_0d(e_n,\rho)+\lambda_0\emptyset_0\left(d(Ŧ^0\rho,\rho)\right)\ +\eta_0min\{d(e_n,Ŧ^0e_n),d(Ŧ^0\rho,\rho)\}\right]$

$\le d(h_n,\rho)+\sum_{i=1}^{k}\quad L^i\alpha_{ni}d(h_{n-1},\rho)+\alpha_{n0}\xi_0d(e_n,\rho)$
(3.9)

$$d(e_n,\rho)\le(1-\beta_{n0})d(gh_{n-1},\rho)+\beta_{n0}d(Ŧ^0f_n,\rho)$$

$$\le(1-\beta_{n0})d(h_{n-1},\rho)$$
$$+\beta_{n0}\left[\xi_0d(f_n,\rho)+\lambda_0\emptyset_0\left(d(Ŧ^0\rho,\rho)\right)\ +\eta_0min\{d(f_n,Ŧ^0f_n),d(Ŧ^0\rho,\rho)\}\right]$$

$$\le(1-\beta_{n0})d(h_{n-1},\rho)+\beta_{n0}\xi_0d(f_n,\rho)$$
(3.10)

$$d(f_n,\rho)\le(1-\gamma_{n0})d(V_{0E}h_{n-1},\rho)+\gamma_{n0}d(Ŧ^0k_{n-1},\rho)$$

$$\le L^0(1-\gamma_{n0})d(h_{n-1},\rho) + \gamma_{n0}\left[\xi_0 d(k_n,\rho) + \lambda_0\emptyset_0\left(d(Ŧ^0\rho,\rho)\right) + \eta_0 min\{d(k_n,Ŧ^0k_n), d(Ŧ^0\rho,\rho)\}\right]$$

$$\le (1-\gamma_{n0})d(h_{n-1},\rho) + \gamma_{n0}\xi_0 d(k_n,\rho)$$

(3.11)

$$d(k_n,\rho) \le \sum_{i=1}^{k} \delta_{ni} d(V_{iE}h_{n-1},\rho) + \delta_{n0} d(Ŧ^0h_{n-1},\rho)$$

$$\le \sum_{i=1}^{k} L^i\delta_{ni} d(h_{n-1},\rho) + \delta_{n0}\left[\xi_0 d(h_{n-1},\rho) + \lambda_0\emptyset_0\left(d(Ŧ^0\rho,\rho)\right) + \eta_0 min\{d(h_{n-1},Ŧ^0h_{n-1}), d(Ŧ^0\rho,\rho)\}\right]$$

$$\le \sum_{i=1}^{k} L^i\delta_{ni} d(h_{n-1},\rho) + \delta_{n0}\xi_0 d(h_{n-1},\rho)$$

$$\le \left(\sum_{i=1}^{k} L^i\delta_{ni} + \delta_{n0}\xi_0\right) d(h_{n-1},\rho)$$

(3.12)

$$L = \{L^i, i = 0,1,2,\dots,k\}$$

From (3.9), (3.10), (3.11) and (3.12) we have

$$\epsilon_n \le d(h_n,\rho) + \left(\sum_{i=1}^{k} \alpha_{ni} + \alpha_{n0}\xi_0\left((1-\beta_{n0}) + \beta_{n0}\xi_0\left((1-\gamma_{n0}) + \gamma_{n0}\xi_0\left(\sum_{i=1}^{k} \delta_{ni} + \delta_{n0}\xi_0\right)\right)\right)\right) d(h_{n-1},\rho)$$

Take limit for two sides with $d(h_n,\rho) = 0$, we have $d(h_n, Q(V_{iE}h_{n-1}, Ŧ^0e_n, \alpha_{ni})) = 0$

By definition (2.12), the multi_L explicit four-step iterative is $Ŧ^0$-stable

Corollary 3.13: Under the same condition in theorem (3.12) ,if $Ŧ^0$ is Suzuki generalized nonexpansive self mapping on E then, the multi_L explicit four-step iteration , is $Ŧ^0$ stable

Corollary 3.14: Under the same condition in theorem (3.12) ,if $\{x_n\}_{n=0}^{\infty}$ defined by the four-step explicit iteration then, it is Ŧ stable.

Theorem 3.15: Let (X,d,Q) be a convex metric space and $Ŧ^i$ be a finite generalized quasi like contractive self mapping on E for all $i = 0,1,2,\dots,k$ with $\xi \in [0,1)$,and g be a non-expansive self mapping on E, If $\rho \in CF(Ŧ^i, V_{iE}, g)$. Then, for $x_0 \in E$, the multi_L explicit four-step iterative convergence faster than multi_L explicit Noor iteration and multi_L Picard s-iteration

proof.

$\{x_n\}_{n=0}^{\infty}$ multi_L explicit four-step iterative

$d(x_n,\rho) = d\big(Q\big(V_{iE}x_{n-1}, Ŧ^0 y_n, \sum_{i=1}^{k} \alpha_{ni}\big), \rho\big)$

$\le \sum_{i=1}^{k} \alpha_{ni} d(V_{iE}x_{n-1},\rho) + \alpha_{n0} d(Ŧ^0 y_n,\rho)$

$\le \sum_{i=1}^{k} L^i \alpha_{ni} d(x_{n-1},\rho) + \alpha_{n0} d(Ŧ^0 y_n,\rho)$

$\le$
$\sum_{i=1}^{k} L^i \alpha_{ni} d(x_{n-1},\rho) + \alpha_{n0}(\xi_0 d(y_n,\rho) + \lambda_0 \emptyset_0(Ŧ^0\rho,\rho) + \eta_0 min\{d(Ŧ^0 y_n, y_n), d(Ŧ^0\rho,\rho)\})$

$\le \sum_{i=1}^{k} L^i \alpha_{ni} d(x_{n-1},\rho) + \alpha_{n0}\xi_0 d(y_n,\rho)$
(3.13)

$$d(y_n,\rho) = d\big(Q\big(gx_{n-1}, Ŧ^0 z_n, (1-\beta_{n0})\big),\rho\big)$$

$$\le (1-\beta_{n0}) d(gx_{n-1},\rho) + \beta_{n0} d(Ŧ^0 z_n,\rho)$$

$$\le (1-\beta_{n0}) d(x_{n-1},\rho) + \beta_{n0} d(Ŧ^0 z_n,\rho)$$

$$\le (1-\beta_{n0}) d(x_{n-1},\rho) + \beta_{n0}(\xi_0 d(z_n,\rho) + \lambda_0 \emptyset_0(Ŧ^0\rho,\rho) + \eta_0 min\{d(Ŧ^0 z_n, z_n), d(Ŧ^0\rho,\rho)\})$$

$\le (1-\beta_{n0}) d(x_{n-1},\rho) + \beta_{n0}\xi_0 d(z_n,\rho)$
(3.14)

$$d(z_n,\rho) = d(Q(V_{0E}u_n, Ŧ^0 y_n, (1-\gamma_{n0})),\rho)$$

$$\le (1-\gamma_{n0}) d(V_{0E}x_{n-1},\rho) + \gamma_{n0} d(Ŧ^0 u_n,\rho)$$

$$\le L^0(1-\gamma_{n0}) d(x_{n-1},\rho) + \gamma_{n0}(\xi_0 d(u_n,\rho) + \lambda_0 \emptyset_0(Ŧ^0\rho,\rho) + \eta_0 min\{d(Ŧ^0 u_n, u_n), d(Ŧ^0\rho,\rho)\})$$

$\le (1-\gamma_{n0}) d(x_{n-1},\rho) + \gamma_{n0}\xi_0 d(u_n,\rho)$
(3.15)

$d(u_n,\rho) = d\big(Q\big(V_{iE}x_{n-1}, T^0 x_{n-1}, \sum_{i=1}^{k} \delta_{n0}\big),\rho\big)$

$\le \sum_{i=1}^{k} \delta_{ni} d(V_{iE}x_{n-1},\rho) + \delta_{n0} d(Ŧ^0 x_{n-1},\rho)$

$\le$
$\sum_{i=1}^{k} L^i \delta_{ni} d(x_{n-1},\rho) +$
$\delta_{n0}(\xi_0 d(x_{n-1},\rho) + \lambda_0 \emptyset_0(Ŧ^0\rho,\rho) + \eta_0 min\{d(Ŧ^0 x_{n-1}, x_{n-1}), d(Ŧ^0\rho,\rho)\})$

$d(u_n,\rho) \le \big(\sum_{i=1}^{k} L^i \delta_{ni} + \delta_{n0}\xi_0\big) d(x_{n-1},\rho)$
(3.16)

$$L = \{L^i, i = 0,1,2,\ldots,k\}$$

$$\xi = \xi_0$$

From (3.13),(3.14),(3.15) and (3.16) we have

$d(x_n,\rho) \le \left(\sum_{i=1}^{k} \alpha_{ni} + \alpha_{n0}\xi\left((1-\beta_{n0}) + \beta_{n0}\xi\left((1-\gamma_{n0}) + \gamma_{n0}\xi\left(\sum_{i=1}^{k} \delta_{ni} + \delta_{n0}\xi\right)\right)\right)\right)$
$d(x_{n-1},\rho)$

$$= (1-\alpha_{n0} + \alpha_{n0}\xi(1-\beta_{n0}) + \alpha_{n0}\xi^2\beta_{n0}(1-\gamma_n) + \alpha_{n0}\xi^3\beta_{n0}\gamma_{n0}(1-\delta_{n0} + \delta_{n0}\xi)\ )d(x_{n-1},\rho)$$

$$= (1-\alpha_{n0} + \alpha_{n0}\xi - \alpha_{n0}\xi\beta_{n0} + \alpha_{n0}\xi^2\beta_{n0} - \alpha_{n0}\,\xi^2\beta_{n0}\,\gamma_n + \alpha_{n0}\xi^3\beta_{n0}\,\gamma_{n0} - \alpha_{n0}\,\xi^3\beta_{n0}\,\gamma_{n0}\delta_{n0} + \alpha_{n0}\xi^4\beta_{n0}\,\gamma_{n0}\delta_{n0}\ )d(x_{n-1},\rho)$$

$$\le (1-\alpha_{n0}(1-\xi\ ))\,d(x_{n-1},\rho)$$

:

$d(x_n,\rho)\ \le \prod_{j=1}^{n} \left(1-\alpha_{j0} + \alpha_{j0}\xi - \alpha_{j0}\xi\beta_{j0} + \alpha_{j0}\xi^2\beta_{j0} - \alpha_{j0}\xi^2\beta_{j0}\gamma_j + \alpha_{j0}\xi^3\beta_{j0}\gamma_{j0} - \alpha_{j0}\,\xi^3\beta_{j0}\,\gamma_{j0}\delta_{j0} + \alpha_{j0}\xi^4\beta_{j0}\gamma_{j0}\delta_{j0}\ \right) d(x_0,\rho)$

$d(x_n,\rho)\ \le \prod_{j=1}^{n} \left(1-\alpha_{j0}(1-\xi\ )\right) d(x_0,\rho)$

Now to get $\{x_n\}_{n=0}^{\infty}$ multi_L explicit Noor iteration

$d(x_n,\rho) = d\left(Q\left(x_{n-1}, Ŧ^0 y_n, \sum_{i=1}^{k} \alpha_{ni}\right),\rho\right)$

$\le \sum_{i=1}^{k} \alpha_{ni}\, d(x_{n-1},\rho) + \alpha_{n0}\, d(Ŧ^0 y_n,\rho)$

$\le$
$\sum_{i=1}^{k} \alpha_{ni}\, d(x_{n-1},\rho) + \alpha_{n0}\left[\xi_0 d(y_n,\rho) + \lambda_0\emptyset_0\left(d(Ŧ^0\rho,\rho)\right) + \eta_0 min\{d(y_n,Ŧ^0 y_n), d(Ŧ^0\rho,\rho)\}\ \right]$

$\le \sum_{i=1}^{k} \alpha_{ni}\, d(x_{n-1},\rho) + \alpha_{n0}\xi_0\, d(y_n,\rho)$
(3.17)

$$d(y_n,\rho) = d\left(Q\left(V_{0E}\, g x_{n-1}, Ŧ^0 z_n, (1-\beta_{n0})\right),\rho\right)$$

$$\le (1-\beta_{n0})d(V_{0E}\, g x_{n-1},\rho) + \beta_{n0} d(Ŧ^0 z_n,\rho)$$

$$\le L^0(1-\beta_{n0})d(x_{n-1},\rho) + \beta_{n0}\left[\xi_0 d(z_n,\rho) + \lambda_0\emptyset_0\left(d(Ŧ^0\rho,\rho)\right) + \eta_0 min\{d(z_n,Ŧ^0 z_n), d(Ŧ^0\rho,\rho)\}\ \right]$$

$\le L^0(1-\beta_{n0})d(x_{n-1},\rho) + \xi_0\beta_{n0} d(z_n,\rho)$
(3.18)

$d(z_n,\rho) = d\left(Q\left(V_{iE}\, g x_{n-1}, V_{0E}Ŧ^0 x_{n-1}, \sum_{i=1}^{k} \gamma_{ni}\right),\rho\right)$

$\le \sum_{i=1}^{k} \gamma_{ni} d(V_{0E}\, g x_{n-1},\rho) + \gamma_{n0}\, d(V_{iE}Ŧ^0 x_{n-1},\rho)$

$\leq$
$\sum_{i=1}^{k} L^i \gamma_{ni} d(x_{n-1}, \rho) +$
$L^0 \gamma_{n0} \left[\xi_0 d(x_{n-1}, \rho) + \lambda_0 \emptyset_0 \big(d(Ŧ^0 \rho, \rho)\big) + \eta_0 min\{d(x_{n-1}, Ŧ^0 x_{n-1}), d(Ŧ^0 \rho, \rho)\} \right]$

$\leq \sum_{i=1}^{k} L^i \gamma_{ni} d(x_{n-1}, \rho) + L^0 \gamma_{n0} \xi_0 d(x_{n-1}, \rho)$

$d(z_n, \rho) \leq (\sum_{i=1}^{k} L^i \gamma_{ni} + L^0 \gamma_{n0} \xi_0)\, d(x_{n-1}, \rho)$
(3.19)

$$L = \{L^i, i = 0,1,2, \dots, k\}$$

Take $\xi = \xi_0$

From (3.17),(3.18), and (3.19) we get

$d(x_n, \rho) \leq \big(\sum_{i=1}^{k} \alpha_{ni} + \alpha_{n0} \xi \left[(1 - \beta_{n0}) + \xi \beta_{n0} \big(\sum_{i=1}^{k} \gamma_{ni} + \gamma_{n0} \xi \big)\right]\big) d(x_{n-1}, \rho)$

$= (1 - \alpha_{n0} + \alpha_{n0} \xi - \alpha_{n0} \xi \beta_{n0} + \alpha_{n0} \xi^2 \beta_{n0} - \alpha_{n0} \xi^2 \beta_{n0} \gamma_{n0} + \alpha_{n0} \xi^3 \beta_{n0} \gamma_{n0})\, d(x_{n-1}, \rho)$

:

$d(x_n, \rho) \leq \prod_{j=1}^{n} \left(1 - \alpha_{j0} + \alpha_{j0} \xi - \alpha_{j0} \xi \beta_{j0} + \alpha_{j0} \xi^2 \beta_{j0} - \alpha_{j0} \xi^2 \beta_{j0} \gamma_{j0} + \alpha_{j0} \xi^3 \beta_{j0} \gamma_{j0} \right) d(x_0, \rho)$

Now we will compare the rate of convergence between $\{x_n\}_{n=0}^{\infty}$ multi_L explicit four-step iterative and $\{x_n\}_{n=0}^{\infty}$ multi_explicit Noor iterative

$\frac{MEFSI}{MENI} =$
$\lim_{n\to\infty} \frac{\prod_{j=1}^{n} \left(1-\alpha_{j0}+\alpha_{j0}\xi-\alpha_{j0}\xi\beta_{j0}+\alpha_{j0}\xi^2\beta_{j0}-\alpha_{j0}\xi^2\beta_{j0}\gamma_j+\alpha_{j0}\xi^3\beta_{j0}\gamma_{j0}-\alpha_{j0}\xi^3\beta_{j0}\gamma_{j0}\delta_{j0}+\alpha_{j0}\xi^4\beta_{j0}\gamma_{j0}\delta_{j0}\right) d(x_0,\rho)}{\prod_{j=1}^{n} \left(1-\alpha_{j0}+\alpha_{j0}\xi-\alpha_{j0}\xi\beta_{j0}+\alpha_{j0}\xi^2\beta_{j0}-\alpha_{j0}\xi^2\beta_{j0}\gamma_{j0}+\alpha_{j0}\xi^3\beta_{j0}\gamma_{j0}\right) d(x_0,\rho)}$
=0

By using definitions (2.11), we have multi_L explicit four step iterative faster than multi_L explicit Noor iteration when

Now to get $\{x_n\}_{n=0}^{\infty}$ multi_L Picard s-iterative

$$d(x_n, \rho) = d(Q(V_{0E} y_n), \rho)$$

$\leq L^0 d(y_n, \rho)$
(3.20)

$$d(y_n, \rho) = d\big(Q\big(g\, x_{n-1}, V_{0c} Ŧ^i z_n, \alpha_{n0}\big), \rho\big)$$

$\leq \alpha_{n0} d(g\, x_{n-1}, \rho) + \sum_{i=1}^{k} \alpha_{ni} d(V_{0S} Ŧ^i z_n, \rho)$

$\leq \alpha_{n0} d(x_{n-1}, \rho) + L^0 \sum_{i=1}^{k} \alpha_{ni} d(Ŧ^i z_n, \rho)$

$\leq$
$\alpha_{n0} d(x_{n-1}, \rho) + L^0 \sum_{i=1}^{k} \alpha_{ni} \left[\xi_i d(z_n, \rho) + \lambda_i \emptyset_i \left(d(Ŧ^i \rho, \rho)\right) + \eta_i min\{d(z_n, Ŧ^i x_n), d(Ŧ^i \rho, \rho)\}\right]$

$\leq \alpha_{n0} d(x_{n-1}, \rho) + L^0 \sum_{i=1}^{k} \alpha_{ni} \xi_i d(z_n, \rho)$
(3.21)

$d(z_n, \rho) = d\left(Q\left(V_{iE} x_{n-1}, Ŧ^0 x_{n-1}, \sum_{i=1}^{k} \beta_{ni}\right), \rho\right)$

$\leq \sum_{i=1}^{k} \beta_{ni} d(V_{iE} x_{n-1}, \rho) + \beta_{n0} d(Ŧ^0 x_{n-1}, \rho)$

$\leq$
$\sum_{i=1}^{k} L^i \beta_{ni} d(x_{n-1}, \rho) +$
$\beta_{n0}\left[\xi_0 d(x_{n-1}, \rho) + \lambda_0 \emptyset_0\left(d(Ŧ^0 \rho, \rho)\right) + \eta_0 min\{d(x_{n-1}, Ŧ^0 x_{n-1}), d(Ŧ^0 \rho, \rho)\}\right]$

$\leq \sum_{i=1}^{k} L^i \beta_{ni} d(x_{n-1}, \rho) + \beta_{n0} \xi_0 d(x_{n-1}, \rho)$

$\leq (\sum_{i=1}^{k} L^i \beta_{ni} + \beta_{n0} \xi_0) d(x_{n-1}, \rho)$
(3.22)

$$L = \{ L^i, i = 1,2, \dots, k\}$$

$$\xi = \{ \xi_i, i = 1,2, \dots, k\}$$

From (3.20),(3.21)and(3.22) we have

$d(x_n, \rho) \leq \alpha_{n0} d(x_{n-1}, \rho) + \sum_{i=1}^{k} \alpha_{ni} \xi d(z_n, \rho)$

$= \alpha_{n0} d(x_{n-1}, \rho) + \sum_{i=1}^{k} \alpha_{ni} \xi \left((\sum_{i=1}^{k} \beta_{ni} + \beta_{n0} \xi) d(x_{n-1}, \rho)\right)$

$= \left(\alpha_{n0} + \sum_{i=1}^{k} \alpha_{ni} \xi\left(\sum_{i=1}^{k} \beta_{ni} + \beta_{n0} \xi\right)\right) d(x_{n-1}, \rho)$

$\leq (\alpha_{n0} + (1 - \alpha_{n0}) \xi) d(x_{n-1}, \rho)$

$\leq 1 - (1 - \alpha_{n0})(1 - \xi) d(x_{n-1}, \rho)$

$\vdots$

$d(x_n, \rho) \leq \prod_{j=1}^{n} 1 - (1 - \alpha_{j0})(1 - \xi) d(x_0, \rho)$

Now, we will compare the rate of convergence between $\{x_n\}_{n=0}^{\infty}$ multi_L implicit Noor iterative and $\{x_n\}_{n=0}^{\infty}$ multi_L Picard s-iterative

$$\frac{MIFSI}{MPSI} = \lim_{n\to\infty} \frac{\prod_{j=1}^{n} (1-\alpha_{j0}(1-\xi)) d(x_0,\rho)}{\prod_{j=1}^{n} 1-(1-\alpha_{j0})(1-\xi) d(x_0,\rho)} = 0$$

By using definitions (2.11), we have multi_L explicit four step iteration faster than multi_L Picard s-iterative .

Corollary 3.16: Under the same condition in theorem (3.15) ,if $Ŧ^0$ is Suzuki generalized nonexpansive self mapping on E then, the multi_L explicit four-step iteration convergence faster than multi_L explicit Noor iteration and multi_L Picards-iteration.

Corollary3.17: Under the same condition in theorem(3.15), if $\{x_n\}_{n=0}^{\infty}$ defined by the four-step explicit iteration then, it is convergence faster than explicit Noor iteration.

Conclusion

1- We conclude that generalized quasi like contractive mapping more general from Suzuki generalized nonexpansive mapping and Zamfirescu mapping.
2- The multi_L explicit four-step iteration, multi_L explicit Noor iteration and multi_L Picards-iteration convergence to fixed point of generalized quasi like contractive mapping.
3- The multi_L explicit four-step iteration, multi_L explicit Noor iteration and multi_L Picards-iteration are $Ŧ^i$- stable .

The multi_L explicit four-step iteration convergence faster than multi_L explicit Noor iteration and multi_L Picard s-iteration with generalized quasi like contractive mapping. We will study these concepts in the future using spaces and other mapping, see([23-26]

References

1. Takahashi, Wataru. "A convexity in metric space and nonexpansive mappings, I." *Kodai mathematical seminar reports*. Vol. 22. No. 2. Department of Mathematics, Tokyo Institute of Technology, 1970.
2. Zamfirescu, Tudor. "Fix point theorems in metric spaces." *Archiv der Mathematik* 23 (1972): 292-298.
3. Kannan, Rangachary. "Some results on fixed points." *Bull. Cal. Math. Soc.* 60 (1968): 71-76.
4. Edelstein, Michael. "On fixed and periodic points under contractive mappings." *Journal of the London Mathematical Society* 1.1 (1962): 74-79.
5. Singh, S. P. "Some theorems on fixed points." *Yokohama Math. J* 18 (1970): 23-25.
6. Suzuki, Tomonari. "Fixed point theorems and convergence theorems for some generalized nonexpansive mappings." *Journal of mathematical analysis and applications* 340.2 (2008): 1088-1095.
7. Noor, Muhammad Aslam. "New approximation schemes for general variational inequalities." *Journal of Mathematical Analysis and applications* 251.1 (2000): 217-229.
8. Asaduzzaman, Md, and Altaf Hossain. "Data dependence for four-step fixed point iterative scheme allocating via contractive-like operators." *Adv. Fixed Point Theory* 8.2 (2018): 188-204.
9. Olatinwo, Memudu Olaposi. "Stability results for some fixed point iterative processes in convex metric spaces." *Annals of the Faculty of Engineering Hunedoara* 9.3 (2011): 103.
10. BERINDE, Vasile. "On the stability of some fixed point procedures." *Buletinul ştiinţific al Universitatii Baia Mare, Seria B, Fascicola matematică-informatică* (2002): 7-14.

11. Harder, Alberta M., and Troy L. Hicks. "Stability results for fixed point iteration procedures." *Math. Japonica* 33.5 (1988): 693-706.
12. Ostrowski, A. M. "The Round‐off Stability of Iterations." *ZAMM‐Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik* 47.2 (1967): 77-81.
13. Shimizu, T., and W. Takahashi. "Fixed point theorems in certain convex metric spaces." *Math. Japon* 37.5 (1992): 855-859.
14. Berinde, Vasile. "Picard iteration converges faster than Mann iteration for a class of quasi-contractive operators." *Fixed Point Theory and Applications* 2004.2 (2004): 1-9.
15. Jamil, Zeana Z., and Moamer B. Abed. "On a modified SP-iterative scheme for approximating fixed point of a contraction mapping." *Iraqi J. Sci.* 56.4B (2015): 3230-3239.
16. Maibed, Zena Hussein. "Strong Convergence of Iteration Processes for Infinite Family of General Extended Mappings." *Journal of Physics: Conference Series*. Vol. 1003. No. 1. IOP Publishing, 2018.
17. Beg, Ismat, Mujahid Abbas, and Jong Kyu Kim. "Convergence theorems of the iterative schemes in convex metric spaces." *Nonlinear Funct. Anal. Appl* 11.3 (2006): 421-436.
18. Abed, Salwa Salman, and Karrar Emad Abdul Sada. "Common fixed points in modular spaces." *Ibn Al-Haitham Journal for Pure and Applied science* (2018): 500-509.
19. Malih, Sabah Hassan. "Fixed point theorems of modified Mann and Ishikawa iterations." *Journal of Interdisciplinary Mathematics* 24.5 (2021): 1093-1097.
20. Maibed, Zena Hussein. " Generalized tupled common fixed point theorems for weakly compatible mappings in fuzzymetric space." *International Journal of Civil Engineering* 10.1 (2019).
21. Gürsoy, Faik, and Vatan Karakaya. "A Picard-S hybrid type iteration method for solving a differential equation with retarded argument." *arXiv preprint arXiv:1403.2546* (2014).
22. Abed, Salwa S., and Rafah S. Abed Ali. "Fixed Point for Asymptotically Non-Expansive Mappings in 2-Banach Space." *Ibn AL-Haitham Journal For Pure and Applied Science* 27.1 (2017): 343-350.
23. Hamed, Mustafa Mohamed, and Zeana Zaki Jamil. "Stability And Data Dependence Results For The Mann Iteration Schemes on n-Banach Space." *Iraqi Journal of Science* (2020): 1456-1460.
24. Thajil, A. Q., and Z. H. Maibed. "On The Convergence Speediness of K* and D-Iterations." *Journal of Physics: Conference Series*. Vol. 1897. No. 1. IOP Publishing, 2021.
25. Hazim, Revan I., and Saba N. Majeed. "Quasi Semi and Pseudo Semi (p, E)-Convexity in Non-Linear Optimization Programming." *Ibn AL-Haitham Journal For Pure and Applied Sciences* 36.1 (2023): 355-366.