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The Mole Plough Domination in Graphs

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Abstract

In this paper, we initiate the study of mole plough domination , consider $G = (V, E)$ be a simple, finite, and undirected graph without isolated vertex. A mole plough dominating set is a set $D \subseteq V(G)$ where, every vertex in D dominates at least 2 and at most 3 vertices of $V - D$. The domination number of G, denotes $\Upsilon_{mf}(G)$ is the smallest cardinality of the minimum mole plough dominating set in G. We determine best possible upper and lower bounds for $\Upsilon_{mf}(G)$, discussed for several standard graphs such as : complete, complete bipartite and wheel graphs. Also, we study the inverse mole plough domination number.

Keywords: Dominating set; mole plough domination; minimum mole plough domination; inverse mole plough domination.

1.Introduction

Terms related to graph theory that are not covered here can be found in [1]. Let $G = (V, E)$ be a graph. For every vertex $v \in V$, The open neighborhood of v, denoted by $N(v)$, is defined by $\{u \in V, uv \in E\}$, and the set $N[v] = N[v] \cup \{v\}$ is a closed neighborhood. The complement graph $\bar{G}$ of a simple graph G If and only if two vertices in graph G are not neighboring, then they are adjacent in graph G . We use the terms from [2, 4], for graph terminology. One of the areas of graph theory that is expanding the fastest is the study of dominance problems. For a thorough analysis of dominance, see [5,6]. If every vertex in $V - D$ is adjcent to a vertex in D , then, a set $D \subseteq V(G)$ is a dominating set; if D has no dominating subset, then, it is a minimal dominating set. The minimum cardinality of a dominating set D of G is known as the domination number $\gamma(G)$.

Here, the mole plough domination model of graph is shown. There are limits on the mole plough domination number related to a graph's order, size, minimum degree, maximum degree, and other attributes. mole plough domination is applied for a few modified and known graphs.

2. Mole plough domination number

This section introduces the concept of mole plough domination and provides proofs for some of its properties and bounds.

Definition 2.1. If each vertex $v \in D$, at least dominates two and at most three vertices of $V - D$, then, a subset $D \subseteq V(G)$ is mole plough dominating set. Let G, non-trivial without isolated vertices, and a simple graph, see Figure 1.

Definition 2.2. A subset $D \subseteq V(G)$ is minimum mole plough if it has the smallest number of elements over all mole plough dominating sets.

Definition 2.3. If $D \subseteq V(G)$ is a minimal mole plough dominating set of G, if it has no proper mole plough dominating set.

Definition 2.4. The mole plough domination number denoted by $\Upsilon_{mf}(G)$ is a minimum cardinality over all mole plough dominating sets in G, such set denoted by $\Upsilon_{mf} -$ set.

(a) Min. dominating set (b) Min. mole plough dominating set

Figure1. The mole plough domination.

0bservation 2.5. Let $G(n, m)$ be a graph that has mole plough dominating set D and mole plough domination number $\Upsilon_{mf}(G)$, we have:

1- The order of G is ≥ 3 .
2- $\delta(G) \geq 1$ *and* $\Delta(G) \geq 2$.
3- Every $v \in D$, $\deg(v) \geq 2$.
4- $\gamma_{mf}(G) \geq \gamma(G)$.
5- Every support vertex v , $v \in D$.

Proposition 2.6. If G has a support vertex adjacent with more than three leaf vertices in a graph G, then G has no mole plough domination.

Proof: : Suppose that D is a γ_{mf} – set in G. Let v be a support vertex adjacent with four leaf vertices . If $v \in D$ then, v dominates four leaf vertices which is contradiction . If $v \notin D$, then every vertex of the four leaf vertices dominates v but has no neighborhoods in D . Thus, G has no mole plough dominating set .

Now, we give the relationship between a graph's size in the following theorem. and the mole plough domination number.

Theorem 2.7. Suppose that a graph $G(n,m)$ has mole plough domination number $\gamma_{mf}(G)$, then :

$$2\gamma_{mf}(G) \leq m \leq \binom{n}{2} + \gamma^2(G) + (3-n)\gamma_{mf}(G)$$

Proof: As evidence for the lower bound. Let G have the fewest edges feasible Given two null graphs, $G[D]$ and $[V-D]$ .. The value of D to $V-D$ edges is $2|D| = 2\,\gamma_{mf}(G)$ according to the definition of mole plough domination which states that there are at least two edges from each vertex of D to $V-D$. Then, $2\,\gamma_{mf}(G) \leq m$.

To demonstrate the higher bound. Assume are two complete sub graphs, $G[D]$ and $G[V-D]$ such that G has the highest number of edges. Assume that m_1 represents $G[D]$ edge count and m_2 represents $G[D-V]$ edge count. Then, $m_1 = \frac{|D|\,|D-1|}{2} = \frac{\gamma_{mf}(G)\,(\gamma_{mf}(G)-1)}{2}$, $m_2 = \frac{|V-D|\,|V-D-1|}{2} = \frac{(n-\gamma_{mf}(G))\,(n-\gamma_{mf}(G)-1)}{2}$ and

$m_3 = 3|D| = 3\,\gamma_{mf}(G)$ is the maximum number of edges between D and $V-D$.

Then,

$$m \leq m_1 + m_2 + m_3$$

$$m \leq \binom{n}{2} + (3-n)\gamma_{mf}(G) + \gamma^2{}_{mf}(G)$$

Theorem 2.8. Given a graph $G = (V,E)$ and a mole plough domination number $\gamma_{mf}(G)$, we can say that:

$$\left\lceil \frac{n}{3} \right\rceil \leq \gamma_{mf}(G) \leq n-2$$

Proof: To establish the lower bound. Let D be γ_{mf} –set of , let $v_i, v_j \in D$ such that $v_i \neq v_j$ then

1- If $N(v_i) \cap N(v_j) \cap (V-D) = \emptyset$. Then, $\gamma_{mf}(G) = \frac{n}{3}$ because every vertex in D dominates at least two vertices of $V-D$. Similarly, $\gamma_{mf}(G) = \frac{n}{4}$ because at most, every vertex in D dominates three vertices of $V-D$, hence, $\frac{n}{4} < \frac{n}{3} \leq \gamma_{mf}(G)$.

2- If $N(v_i) \cap N(v_j) \cap (V-D) \neq \emptyset$. Afterwards, two vertices of D, v_i and v_j, dominate one or more common vertices in $V-D$. subsequently, $\gamma_{mf}(G) \geq \left\lceil \frac{n}{3} \right\rceil$. After that,

$\left\lceil \frac{n}{3} \right\rceil \leq \gamma_{mf}(G)$.

For the maximum bound. There are at least two vertices in $V-D$, that are all other $n-2$ vertices of D dominate then, and there are at dominated by all other vertices of G, since each vertex in D dominates two vertices of $V-D$ at least and three vertices at most. Consequently,

$\gamma_{mf}(G) \leq n-2$.

Corollary 2.9. Considering a graph $G = (V,E)$ with a mole plough domination number $\gamma_{mf}(G)$, then:

1-$\gamma_{mf}(G) \geq \left\lceil \frac{n}{\delta+2} \right\rceil$, $\delta \geq 1$

2-$\gamma_{mf}(G) \geq \left\lceil \frac{n}{\Delta+1} \right\rceil$, $\Delta \geq 2$

3-$\gamma_{mf}(G) \geq \left\lceil \frac{n}{\frac{\Delta}{\delta}+1} \right\rceil$

4-$\gamma_{mf}(G) \geq \left\lceil \frac{n}{\Delta+\delta} \right\rceil$

Proof:1- From Theorem 2.8 we have $\gamma_{mf}(G) \geq \left\lceil \frac{n}{3} \right\rceil$ and since $\delta \geq 1$ by Observation 2.5. $\left\lceil \frac{n}{3} \right\rceil \geq \left\lceil \frac{n}{\delta+2} \right\rceil$, thus, $\gamma_{mf}(G) \geq \left\lceil \frac{n}{\delta+2} \right\rceil$.

2-Since $\Delta \geq 2$ by Observation 2.5. and $\left\lceil \frac{n}{3} \right\rceil \geq \left\lceil \frac{n}{\Delta+1} \right\rceil$ which gets $\gamma_{mf}(G) \geq \left\lceil \frac{n}{\Delta+1} \right\rceil$.

3-Since $\delta \geq 1$ and $\Delta \geq 2$ by Theorem 2.8. we have $\gamma_{mf}(G) \geq \left\lceil \frac{n}{3} \right\rceil$, thus, $\left\lceil \frac{n}{3} \right\rceil \geq \left\lceil \frac{n}{\frac{\Delta}{\delta}+1} \right\rceil$.

4-Similer to the proof in above point 3.

Theorem 2.10. Let D be a mole plough domination set of a graph G. If one of the conditions is met, then D is a minimal mole plough dominating set.

1-$|N(v) \cap V - D| = 3 \ \forall \ v \in D$

2-$G\ [D]$ is a null graph.

Proof: Let D be any mole plough dominating set of graph G. Let that D is not minimal mole plough dominating set, and $D - \{v\}$ is minimal mole plough domination set such that $v \in D$.

Case 1: suppose that $|N(v) \cap V - D| = 3 \ \forall \ v \in D$. If one or more of the three vertices is dominated by only v, then $D - \{v\}$ is not mole plough domination set.

When another vertices in $D - \{v\}$ dominate the three vertices that are dominated by v. $D - \{v\}$ is not a mole plough dominating set if there isn't a vertex in $D - \{v\}$ that dominates v. If there exist $w \in D - \{v\}$ adjacent with v then w will dominates four vertices which is contradiction.

Case 2: If the second condition is true, then, v is not adjacent to any vertex in D because of the null graph is $G\ [D]$, Consequently, none of the vertices from $D - \{v\}$. Dominate . Therefore, the mole plough dominating set is not $D - \{v\}$.

3. Mole plough domination of some graphs.

The mole plough domination number will be investigated for several known graphs.

Theorem 3.1. let (n, m) , be complete graph K_n then,

$$\gamma_{mf}(K_n) = \begin{cases} 1 & if\ n = 3 \\ n-3 & if\ \ n > 3 \end{cases}$$

Proof: If $n = 2$ then, , K_2 has no mole plough dominating set. Because of every vertex in the graph is joined to one vertex. If $n = 3$ every vertex in D dominates two vertices in $V - D$. See Figure2.

In this case, if $n > 3$, then, each vertex in D dominates three vertices ; hence, $V - D$ has only three vertices that are dominated by the $n - 3$ vertices of D. Then, D is mole plough dominating set.

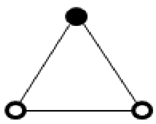


Figure 2. A minimum mole plough dominating set in K_3.

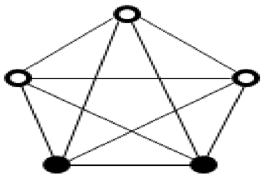


Figure 3. A minimum mole plough dominating set in K_5

Proposition 3.2. Let $K_{1,m}$ be a star graph $m > 3$, then, $K_{1,m}$ have no mole plough dominating set.

Proof: It's clear that star graph $K_{1,m}$ have no mole plough dominating set where $m > 3$, because doesn't definition 2.1.

Theorem 3.3 : let $G(n, m)$, be complete bipartite graph $K_{n,m}$ then

$$\gamma_{mf}(K_{n,m}) = \begin{cases} 1 \quad if\ n = 1 \ \ , m = 2,3 \\ \quad 2 \quad if\ n\ , m = 2,3 \\ n + m - 6 \quad if\ \ n\ , m > 3 \end{cases}$$

Proof: Given a pair of $K_{n,m}$ vertices, where, $|V_1| = n$ and $|V_2| = m$ we have V_1 then:

Case 1: Let $G(1, m)$ be a star graph when the vertex of V_1dominates the two or three vertices of V_2 then, $\gamma_{mf}(K_{1,m}) = 1\ \ if\ and\ only\ if\ m = 2, m = 3$.

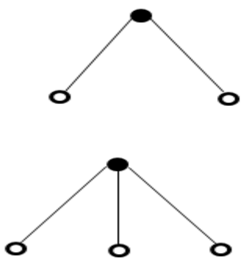
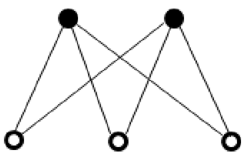


Figure 4. A minimum mole plough dominating set in $K_{1,m}$

Case 2: Let v be a vertex in $\gamma_{mf\_}$ set D. Suppose that $v \in V_1$, then v is adjacent to exactly two vertices from the set V_2. If we take any vertex from these vertices again this vertex is adjacent to three vertices from V_2 and other vertices in V_1 must belong to set D. Therefore, we get the result.

Case 3: Let D be the dominating set of the mole plough and let v be a vertex in a $\gamma_{mf\_}$ set D, if $n, m > 3$. Assuming $v \in V_1$, v is adjacent to at least three and more than three vertices in the set V_2. Subsequently, D has $m-3$ vertices in V_2 and $n-3$ vertices in V_1, meaning that all $n-3$ vertices dominate the two vertices in V_2. Furthermore, the three vertices of the set V_1 that are part of $V-D$ are subordinated to all $m-3$ vertices of the set V_2 that are in D. $\gamma_{mf}(K_{n,m}) = n-3+m-3 = m+n-6$ as a result. In order to demonstrate that D is a minimum mole plough dominating set, let's say that $D^` \subseteq D$ and $|D^`| <^` |D|$. In this case, there must be two or more vertices $V-D$ that are not dominated by $D^`$ or there are vertices of $D^`$ which dominate more than three vertices $V-D$, which is contradict definition. Thus, D is a minimal mole plough dominating set.



$K_{2,3}$

$K_{3,4}$

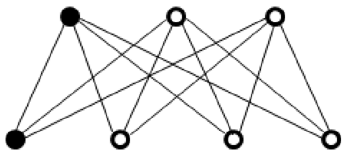


Figure 5. At the very least the mole plough prevailing set in $K_{n,m}$

Theorem 3.4: Taking the wheel graph $W_n (n \geq 3)$ as (n, m), we get $\gamma_{mf}(W_n) = \left\lceil \frac{n}{3} \right\rceil$.

Proof: We will identify the vertices of W_n as: $v_1, v_2, \dots, v_{n+1}$ where for every i between 1 and n, $\deg(v_i) = 3$ and $\deg(v_{n+1}) = n$. Where W_n be the wheel graph $W_n = C_n + K_1$.

According to n, two cases can be obtained in order to select a set D:

Case1: If $n \equiv 0 \ (mod\ 3)$, then D should consist of one vertex from each three vertices of C_n. The dominating set is then, $D = \{ v_{3i-2}, i = 1,2, \dots, \frac{n}{3}\}$. Each vertex in D dominates three vertices ,v_{n+1} and another two vertices adjacent to it.

When $n \equiv 2 (mod 3)$, there are two vertices v_1 and v_{n-1} of D dominate v_n, v_{n+1} and yet another vertex while the other vertices of D dominate threevertices. Thus, the D is $\gamma_{mf} - set$ and $\gamma_{mf} = |D| = \left\lceil \frac{n}{3} \right\rceil$.

Case 2: if $n \equiv 1 \ (mod 3)$, then we can take $D = \left\{ v_{3i-2},\ i = 1,2, \dots, \left\lceil \frac{n}{3} \right\rceil - 1 \right\} \cup \{v_{n-1}\}$. Hence , D is a $\gamma_{mf} - set$ and $\gamma_{mf} = |D| = \left\lceil \frac{n}{3} \right\rceil$. To demonstrate that set D is always the minimal mole plough dominating set in all cases, if we assume that a set $D^` \subset D$ and $|D^`| < |D|$, then there exist at least one vertex in $V-D$ don't dominated by any vertex of $D^`$. Hence, $D^`$ is not mole plough dominating set and D is minimum Mole plough dominating set of wheel. (See Fig. 6.).

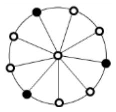
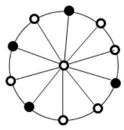
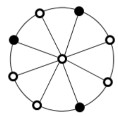

$n \equiv 0(mod3)$

$n \equiv 1(mod3)$

$n \equiv 2(mod3)$

Figure 6. A minimum mole plough dominating set of wheel graphs.

Theorem 3.5. let $G(n,m) = C_n,\ n \geq 3$, be a cycle, graphs then, $\gamma_{mf}(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

Proof. Let the vertices of C_n be $v_1, v_2, \dots, v_n$ and D is the mole plough dominating set. There exists at least one vertex from any three consecutive vertices that belongs to D. Thus, we can choose the first vertex from any three consecutive vertices, then,

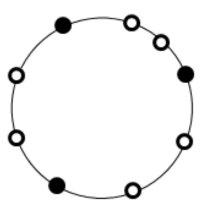
$$D = \begin{cases} \{v_{3i-2}, i = 1, 2, \dots, \lceil n/3 \rceil\} if\ n \equiv 0, 2\ (mod\ 3) \\ \{ v_{3i-2}, i = 1, 2, \dots, \lceil n/3 \rceil - 1\} \cup \{v_{n-1}\} if\ n \equiv 1\ (mod3) \end{cases}$$

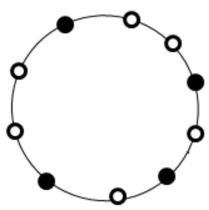
So there are two cases as follows.

Case 1.If $n \equiv 0, 2\ (\ mod\ 3)$, then the vertices chosen above represent all vertices in mole plough dominating set, $D = \{ v_{3i-2}, i = 1, 2, \dots, \lceil n/3 \rceil \}$. Thus, $\gamma_{mf}(C_n) \leq |D| = \left\lceil \frac{n}{3} \right\rceil$.

Case 2. If $n \equiv 1\ (\ mod\ 3)$, tnen we can take D in the same manner in case 1. Except the last step, the last step result in the vertex v_n belongs to D since this vertex dominates only to v_{n-1}, then v_{n-1} is better candidate to have mole plough domination. Therefore, the mole plough set in this case is $D = \{ v_{3i-2}, i = 1, 2, \dots, \lceil n/3 \rceil - 1\} \cup \{v_{n-1}\}$, thus, $\gamma_{mf}(C_n) \leq |D| = \left\lceil \frac{n}{3} \right\rceil$.

To prove the reverse inequalities of the two above cases, we use the induction method on the number of vertices n. The result is obvious if $n = 3$. Suppose that the result is true for all cycles of number of vertices less than n. Now, let M be any $\gamma_{mf} - set$. Then, $M = M_1 \cup M_2$ where M_1 is the minimum mole plough dominating set that dominates the cycle C_{n1} where n_1 is the greatest number less than n and $n_1 \equiv 0\ (mod\ 3)$. By induction $|M_1| \geq \left\lceil \frac{n-3}{3} \right\rceil$, and hence, $|M| \geq \left\lceil \frac{n-3}{3} \right\rceil + 1 = \left\lceil \frac{n}{3} \right\rceil$, since $|M_2| = 1$. Therefore, $\gamma_{mf}(C_n) = \left\lceil \frac{n}{3} \right\rceil$.


(a)$n \equiv 0(mod3)$


(b)$n \equiv 1(mod3)$

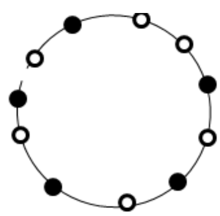

(c)$n \equiv 2(mod3)$

Figure 7. A minimum mole plough dominating set of cycle graphs.

Theorem 3.6. let $G(n,m) = P_n$, $n \geq 3$, be a path graphs then, $\gamma_{mf}(P_n) = \left\lceil \frac{n}{3} \right\rceil$.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of P_n and D the set $\subset V(P_n)$ such that

$$D = \begin{cases} \{v_{3i-1}, i = 1,2,\ldots,\lceil n/3 \rceil\} \, if \; n \equiv 0 \; (mod \; 3) \\ \{v_{3i-1}, i = 1,2,\ldots,\lceil n/3 \rceil - 2\} \cup \{v_{n-1}, v_{n-3}\} \, if \; n \equiv 1,2 \; (mod 3) \end{cases}$$

The maximum number of vertices that can be mole plough dominated by one vertex is three. Thus, we can choose the middle vertex from any three consecutive vertices. Thus, there are two cases as follows:

Case 1. If $n \equiv 0 \; (mod \; 3)$, then, the vertices chose above represent all vertices in mole plough dominating set and $D = \{v_{3i-1}, i = 1,2,\ldots,\lceil n/3 \rceil\}$. Thus,$|D| \leq \frac{n}{3}$.

Case 2. If $n \not\equiv 0 \; (mod \; 3)$, then we can take D in the same manner in case 1. Except the last step, where two sub cases as follows:

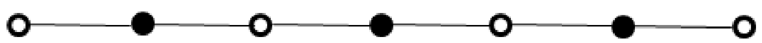
(i) If $n \equiv 1 \; (mod \; 3)$, the last two steps imply v_{n-2} and v_{n+1} belong to D. The vertex v_{n+1} does not exist in the path and the vertex v_{n-2} does not dominate the vertex v_n, so the chosen vertices in these steps must be changed. First, the vertex v_{n-1} is chosen according to Observation 2.5(5). Therefore, the vertex v_{n-2} is excluded from D, since this vertex is adjacent to vertex v_{n-1}. Thus, vertex v_{n-3} is replacement. This means $D = \{v_{3i-1}, i = 1,2,\ldots,\lceil n/3 \rceil - 2\} \cup \{v_{n-1}, v_{n-3}\}$. Thus, $|D| \leq \frac{n}{3}$.

(ii) If $n \equiv 2 \; (mod \; 3)$, the last result in that vertex v_n belong to D. Since, this vertex dominates only v_{n-1}, then, v_{n-1} is better candidate to have mole plough domination. Therefore, the mole plough dominating set in this sub case is $D = \{v_{3i-1}, i = 1,2,\ldots,\lceil n/3 \rceil - 1\} \cup \{v_{n-1}\}$. Thus, $|D| \leq \frac{n}{3}$.

Moreover, the vertex $v_{3(\lceil n/3 - 1) - 1]}$ is by itself the vertex v_{n-3}, so we can write the set D as formula in case (i). Thus, we get the result.

To prove the reverse inequalities of the two above cases, we use the same procedure in Theorem 3.5 .

$(a) n \equiv 0 (mod3)$



$(b) n \equiv 1 (mod3)$

$(c) n \equiv 2(mod3)$

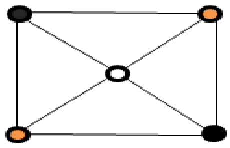
Figure 8. A minimum mole plough dominating set of path graphs.

4. Inverse mole plough domination in graphs.

In this section we discuss the problem to choose another mole plough dominating set disjoint from the first one. The inverse mole plough domination is introduced as a new dominating model,.

Definition 4.1. Let D be minimum mole plough dominating set in G. If $V - D$ contains a mole plough dominating set, then it is called inverse mole plough dominating set of G with respect to D and denoted by D^{-1}.

Definition 4.2. If there is no valid mole plough dominating subset in a subset $D^{-1} \subseteq V(G)$, then it is called a minimal inverse mole plough dominating set. for instance (Fig.9).



A minimum inverse mole plough dominant set is shown on Figure 9.

Definition 4.3. As stated in a minimum inverse mole plough dominating set is one whose cardinality is the lowest of all the inverse mole plough dominating sets in G.

Definition 4.4. The cardinality of the smallest inverse mole plough dominating set is represented by the inverse mole plough domination number, $\gamma_{mf}^{-1}(G)$. The term "γ_{mf}^{-1}_ set" refers to such set.

Observation 4.5. Assume that G is a graph with an inverse mole plough dominating set. We have:

1. $\gamma_{mf}^{-1}(G) \geq 1$
2. $\gamma_{mf}^{-1}(G) = 1$ if and only if $G \in \{ C_3, C_4, K_4 \}$.
3. $\gamma_{mf}^{-1}(G) \geq \gamma_{mf}(G)$.

Remark 4.6. If $\gamma_{mf}(G) \geq \frac{n}{2}$, then G does not have an inverse mole plough dominating set.

5.Conclusion

Anew type of domination " mole plough domination" is introduced here. The relation between mole plough domination number and the order, size, minimum degree and maximum degree is determined. The domination number can be evaluated for several standard graphs and some modified graphs formed in this paper.

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