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The Numerical Solutions of 3-Dimensional Fractional Differential Equations

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Abstract:

The variational iteration technique (VIT) is an excellent analytical tool employed in this study to solve the nonlinear 3D-fractional differential equations. The Antagana-Baleanu sense is used to characterize the fractional derivatives (ABFD). To show the applicability of the recommended technique, example is presented.

Keywords: 3D- Fractional differential equations, Fractional variational iteration method, Atangana-Baleanu fractional operator.

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1. Introduction

In recent years, fractional differential equations have sparked a lot of interest, and they've been studied and applied to a lot of real-world situations in a variety of fields. One reason for this unpopularity might be that fractional derivatives have numerous non-equivalent definitions [1]. Another issue is that, due to their nonlocal nature, fractional derivatives have no obvious geometrical meaning. However, in the last 12 years, scientists have begun to pay considerably more attention to fractional calculus. With the use of fractional derivatives, it was discovered that a variety of applications, particularly multidisciplinary applications [2-4], may be neatly described.

Many analytical and approximation approaches for solving fractional differential equations have been developed in recent years [5-76]. Our objective is to illustrate the FVIM and show how to use it with ABDO to solve the Navier-Stokes problem. The remainder of this work is broken down into the sections below. In section 2, you'll find some fractional calculus definitions. The FVIM analysis is carried out in Section 3 utilizing ABDO. Section 4 shows how FVIM may be use. Section 5 is where this effort ends.

2. Preliminaries

In this section, we'll go over some of the most important fractional calculus definitions and formulas [77-79].

Definition 1. The ABFD of order α is given as follows:

$$ {}^{AB}_{a}D^{\alpha}_{t} u(t) = \frac{M(\alpha)}{1-\alpha} \int_{a}^{t} E_{\alpha}\left(\frac{-\alpha(t-x)^{\alpha}}{\alpha-1}\right) u'(x)dx \qquad (2.1) $$

where $0 < \alpha < 1$ and $M(0) = M(1) = 1$.

Definition 2. The ABFI of order α defined as follows:

$$ {}^{AB}_{a}I^{\alpha}_{t}\, u(t) = \frac{1-\alpha}{M(\alpha)} u(t) + \frac{\alpha}{M(\alpha)} \frac{1}{\Gamma(\alpha)} \int_{a}^{t} (t-x)^{\alpha-1} u(x)\, dx. \qquad (2.2) $$

The properties of ABFI is defined as follows**:**

1. ${}^{AB}_{a}I^{\alpha}_{t}\, {}^{AB}_{a}D^{\alpha}_{t}\; u(t) = u(t) - u(0).$
2. ${}^{AB}_{a}I^{\alpha}_{t}\; c = \frac{c}{M(\alpha)} \left(1-\alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right).$
3. ${}^{AB}_{a}I^{\alpha}_{t}\; t^{k} = \frac{t^{k}}{M(\alpha)} \left(1-\alpha + \frac{\alpha\,\Gamma(k+1)\; t^{\alpha}}{\Gamma(\alpha+k+1)}\right).$

3. Analysis of FVIM

Let us consider the following partial differential equations

$$ {}^{AB}D^{\alpha}_{t}\, u_i(x,y,z,t) + R\, u_i(x,y,z,t) + N\, u_i(x,y,z,t) = g_i(x,y,z,t), \qquad 0 < \alpha \le 1 \quad (3.1) $$

with the initial conditions

$$ u_i(x,y,z,0) = f_i(x,y,z), $$

where ${}^{AB}D^{\alpha}_{t}\, u_i$ is ABFD, $i = 1,2,3$.

The correctional functional for (3.1) is approximately expressed as follows:

$$u_{i(n+1)}(x,y,z,t) = u_{in}(t) + {}^{AB}_{a}I^{\alpha}_{t} \left[\lambda_i(\xi)\left({}^{AB}D^{\alpha}_{\xi}u_{in}(\xi) + R\,\tilde{u}_{in}(\xi) + N\tilde{u}_{in}(\xi) - g_i(\xi)\right)\right], \quad (3.2)$$

where $\lambda_i(\xi)$ is general Lagrange's multiplier. $\tilde{u}_{in}$ and g_i are considered as restricted variations. Putting the relevant adjustment in place and making it functioning and noticing $\delta\tilde{u}_{in} = 0$ and $g_i = 0$, we obtain

$$*\,\delta u_{i(n+1)}(t) = \delta u_{in}(t) + {}^{AB}_{a}I^{\alpha}_{t} \left[\delta\lambda_i(\xi)\left({}^{AB}D^{\alpha}_{\xi}u_{in}(\xi)\right)\right],$$

or

$$\delta u_{i(n+1)}(t) = \delta u_{in}(t) + \lambda_i(\xi)\,\delta u_{in}(t) - {}^{AB}_{a}I^{\alpha}_{t} \left[\lambda'_i(\xi)\,\delta u_{in}(\xi)\right],$$

which produces the stationary conditions

$$\lambda'_i(\xi) = 0,$$
$$1 + \lambda_i(\xi) = 0$$

Therefore, we identified $\lambda_i = -1$ and obtain the following variational iteration formula:

$$u_{i(n+1)}(t) = u_{in}(t) - {}^{AB}_{a}I^{\alpha}_{t} \left[{}^{AB}D^{\alpha}_{\xi}u_{in}(\xi) + R\,u_{in}(\xi) + Nu_{in}(\xi) - g_i(\xi)\right]. \quad (3.3)$$

Finally, we obtain the solution of (3.1) as follows:

$$u_i(x,t) = \lim_{n\to\infty} u_{in}.$$

4. Illustrative example

Consider the time fractional-order three- dimensional Navier–Stokes equation

$$\begin{aligned} {}^{AB}D^{\alpha}_{t}u + uu_x + vu_y + wu_z &= p\left[u_{xx} + u_{yy} + u_{zz}\right] + q_1 \\ {}^{AB}D^{\alpha}_{t}v + uv_x + vv_y + wv_z &= p\left[v_{xx} + v_{yy} + v_{zz}\right] + q_2 \\ {}^{AB}D^{\alpha}_{t}w + uw_x + vw_y + ww_z &= p\left[w_{xx} + w_{yy} + w_{zz}\right] + q_3, \quad 0 < \alpha \le 1 \end{aligned} \quad (4.1)$$

with initial conditions

$$u(x,y,z,0) = -0.5x + y + z$$

$$v(x,y,z,0) = x - 0.5y + Z$$

$$w(x,y,z,0) = -x + y - 0.5y$$

In view of (3.3) , (4.1) and let $q_1 = q_2 = q_3 = 0$.

$$u_{n+1}(x,y,t) = u_n - {}^{AB}I^{\alpha}\left[Lu_n + (u_n)(u_n)_x + (v_n)(u_n)_y + (w_n)(u_n)_z - p\left[u_{nxx} + u_{nyy} + u_{nzz}\right] - q\right]d\xi$$

$$v_{n+1}(x\,,y,t) = v_n - {}^{AB}I^{\alpha}\left[Lu_n + (u_n)(v_n)_x + (v_n)(v_n)_y + (w_n)(v_n)_z - p\left[(v_n)_{xx} + v_{nyy} + v_{nzz}\right] + q\right]d\xi$$

$$w_{n+1}(x\,,y,t) = w_n{}^{AB}I^{\alpha}\left[Lu_n + (u_n)(w_n)_x + (v_n)(w_n)_y + (w_n)(w_n)_z - p\left[(w_n)_{xx} + v_{nyy} + v_{nzz}\right] + q\right]d\xi$$

Therefore, we obtain the successive approximations as follows:

$$u_0(x,y,z,t) = -0.5x + y + Z$$

$$v_0(x,y,z,t) = x - 0.5y + Z$$

$$w_0(x,y,z,t) = x + y - 0.5Z$$

$$u_1 = -0.5x + y + Z + {}^{AB}I^{\alpha}\{-2.25x\}$$

$$= -0.5x + y + Z + (-2.25x)\left(1 - \alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right)$$

$$v_1 = x - 0.5y + Z + {}^{AB}I^{\alpha}\{-2.25y\}$$

$$= x - 0.5y + Z + (-2.25y)\left(1 - \alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right)$$

$$w_1 = x + y - 0.5\,Z + {}^{AB}I^{\alpha}\{-2.25Z\}$$

$$= x + y - 0.5\,Z + (-2.25Z)\left(1 - \alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right)$$

$$u_2 = x + y - 0.5\,Z + (-2.25Z)\left(1 - \alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right) + {}^{AB}I^{\alpha}\Big(-2.25x + 2.25x + 2x + 2(2.25)[-0.5x + y + Z]\left(1 - \alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right) + (2.25)^2 x\left(1 - \alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right)^2 \Big\}$$

$$= x + y - 0.5\,Z + (-2.25Z)\left(1 - \alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right) + (2\,x)\left(1 - \alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right) + 2.(-2.25)[-0.5x + y + Z]\left((1-\alpha)^2 + 2(1-\alpha)\frac{t^{\alpha}}{\Gamma(\alpha)} + \frac{\alpha^2 t^{\alpha}}{\Gamma(2\alpha+1)}\right) + (2.25)^2 x\left((1-\alpha)^2\left(1 - \alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right) + 2\,(1-\alpha)\frac{t^{\alpha}}{\Gamma(\alpha)}\left(1 - \alpha + \frac{\alpha\Gamma(\alpha+1)\,t^{\alpha}}{\Gamma(2\alpha+1)}\right) + \frac{t^{2\alpha}}{(\Gamma(\alpha))^2}\left(1 - \alpha + \frac{\alpha\Gamma(2\alpha+1)t^{\alpha}}{\Gamma(3\alpha+1)}\right)\right)$$

$$v_2 = x - 0.5y + Z + (-2.25y)\left(1 - \alpha + \frac{t^{\alpha}}{\Gamma\alpha}\right) + {}^{AB}I^{\alpha}\{\ (-2.25y + 2.25y + 2y +$$

$$2(2.25)[x-0.5\,y+Z]\left(1-\alpha+\frac{t^{\alpha}}{\Gamma(\alpha)}\right)+\ (2.25)^2y\ \left(1-\alpha+\frac{t^{\alpha}}{\Gamma(\alpha)}\right)^2\ \}$$

$$=x-0.5y+Z+(-2.25y)\left(1-\alpha+\frac{t^{\alpha}}{\Gamma\alpha}\right)+\ (2\,y)\left(1-\alpha+\frac{t^{\alpha}}{\Gamma(\alpha)}\right)$$

$$2.(-2.25)[x-0.5y+Z]\left((1-\alpha)^2+2(1-\alpha)\frac{t^{\alpha}}{\Gamma(\alpha)}+\frac{\alpha^2t^{\alpha}}{\Gamma(2\alpha+1)}\right)+$$

$$(2.25)^2y\left((1-\alpha)^2\left(1-\alpha+\frac{t^{\alpha}}{\Gamma(\alpha)}\right)+2\,(1-\ \alpha)\frac{t^{\alpha}}{\Gamma(\alpha)}\left(1-\alpha+\frac{\alpha\Gamma(\,\alpha+1)\,t^{\alpha}}{\Gamma(2\alpha+1)}\right)\right.$$

$$\left.+\frac{t^{2\alpha}}{\left(\Gamma(\alpha)\right)^2}\left(1-\alpha+\frac{\alpha\Gamma(2\alpha+1)t^{\alpha}}{\Gamma(3\alpha+1)}\right)\right)$$

$$w_2=x+y-0.5Z+(-2.25z)\left(1-\alpha+\frac{t^{\alpha}}{\Gamma(\alpha)}\right)+{}^{AB}I^{\alpha}\{\ -2.25z+2.25z+\ 2z+$$

$$2(2.25)[x+\ y-0.5Z]\left(1-\alpha+\frac{t^{\alpha}}{\Gamma(\alpha)}\right)+\ (2.25)^2z\ \left(1-\alpha+\frac{t^{\alpha}}{\Gamma(\alpha)}\right)^2\ \}$$

$$=x+y-0.5Z+(-2.25Z)\left(1-\alpha+\frac{t^{\alpha}}{\Gamma(\alpha)}\right)+(2\,Z)\left(1-\alpha+\frac{t^{\alpha}}{\Gamma(\alpha)}\right)+$$

$$2.(-2.25)[x+y-0.5Z]\left((1-\alpha)^2+2(1-\alpha)\frac{t^{\alpha}}{\Gamma(\alpha)}+\frac{\alpha^2t^{\alpha}}{\Gamma(2\alpha+1)}\right)+$$

$$(2.25)^2Z\left((1-\alpha)^2\left(1-\alpha+\frac{t^{\alpha}}{\Gamma(\alpha)}\right)+2\,(1-\ \alpha)\frac{t^{\alpha}}{\Gamma(\alpha)}\left(1-\alpha+\frac{\alpha\Gamma(\,\alpha+1)\,t^{\alpha}}{\Gamma(2\alpha+1)}\right)\right.$$

$$\left.+\frac{t^{2\alpha}}{\left(\Gamma(\alpha)\right)^2}\left(1-\alpha+\frac{\alpha\Gamma(2\alpha+1)t^{\alpha}}{\Gamma(3\alpha+1)}\right)\right)$$

If □□□□1, then the closed form solution of (4.1) is

$$u(x,y,t)=\frac{-0.5x+y+Z-2.25xt}{1-2.25t^2}$$

$$v(x,y,t)=\frac{x-0.5y+Z-2.25yt}{1-2.25t^2}$$

$$w(x,y,t)=\frac{x+y-0.5Z-2.25zt}{1-2.25t^2}$$

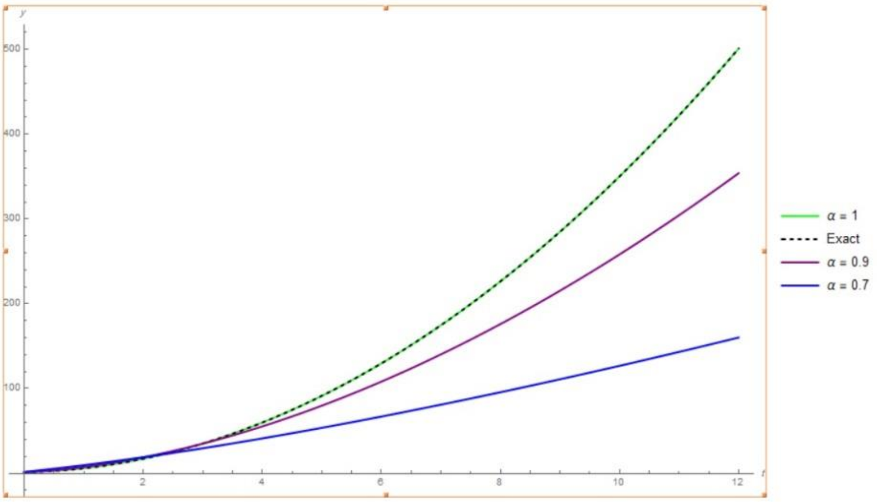

Figure 1. Plots of the exact and approximate solutions $u(x, y, z, t)$ for different values of α

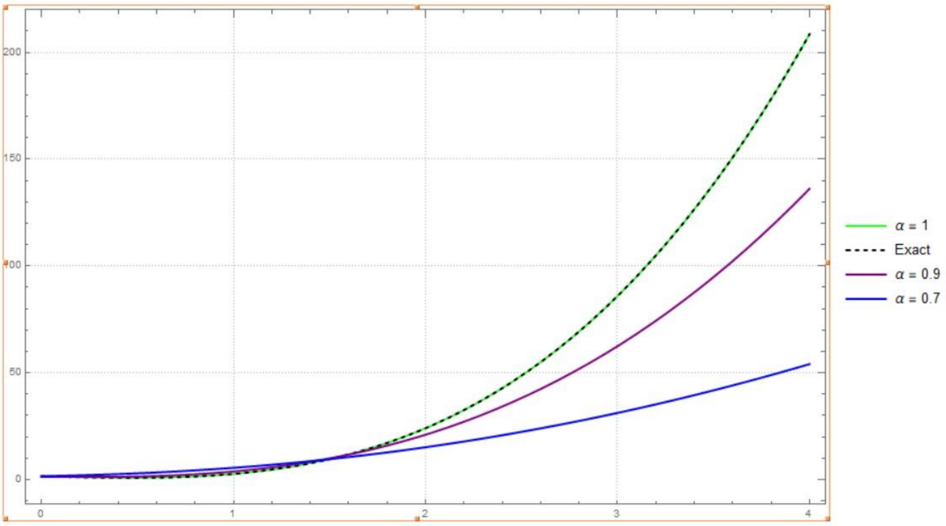

Figure 2. Plots of the exact and approximate solutions $v(x, y, z, t)$ for different values of α

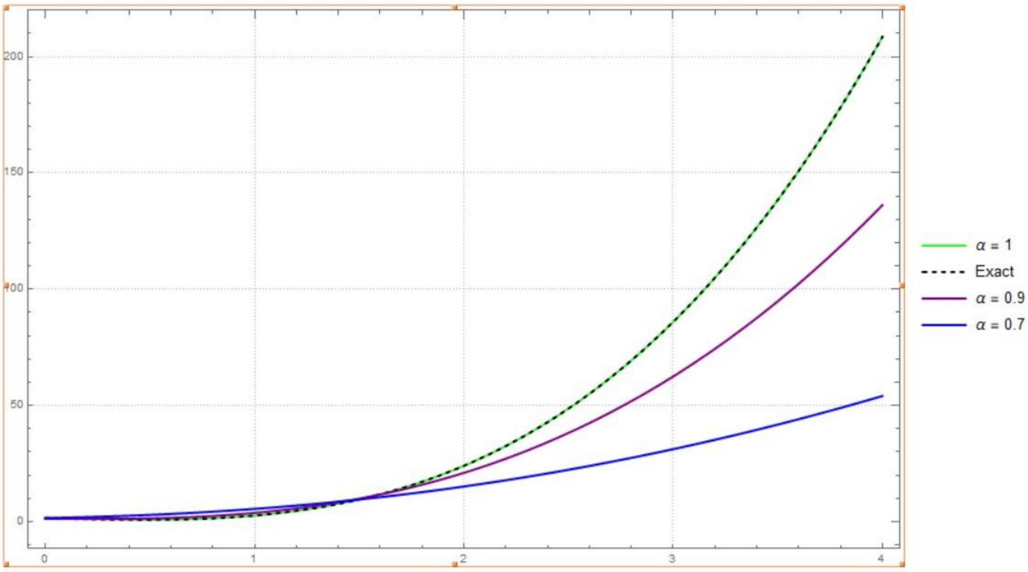

Figure 3. Plots of the exact and approximate solutions $w(x, y, z, t)$ for different values of α

5. Conclusions

We used VIT with ABFO to evaluate the fractional-order three-dimensional Navier–Stokes equations in this paper. The VIT result closely resembles the precise solution to the provided issues. The convergence of the fractional-order answers to integer-order solutions was confirmed by a graphical examination of the results. Furthermore, the proposed method is clear, simple, and low-cost to implement; it may be extended to solve additional fractional-order partial differential equations.

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