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Yang Adomian Decomposition Method for Solving PDEs

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Abstract

The Yang Adomian decomposition technique (YADM) is an excellent analytical tool employed in this study to solve the partial differential equations (PDEs). The result of the suggested approach is stated as a series of Adomian components that converges to the precise solution of the problem. To show the applicability of the recommended technique, examples are presented.

Keywords: Partial differential equations; Yang transform; Adomian decomposition method.

1. Introduction

Partial differential equations (PDEs) also occupy a large sector of pure mathematical research, in which the usual questions are, broadly speaking, on the identification of general qualitative features of solutions of various partial differential equations, such as existence, uniqueness, regularity and stability. Among the many open questions are the existence and smoothness of solutions to the Navier–Stokes equations, named as one of the Millennium Prize Problems in 2000. PDEs are ubiquitous in mathematically oriented scientific fields, such as physics and engineering. For instance, they are foundational in the modern scientific understanding of sound, heat, diffusion, electrostatics, electrodynamics, thermodynamics, fluid dynamics, elasticity, general relativity, and quantum mechanics (Schrödinger equation, Pauli equation etc.). They also arise from many purely mathematical considerations, such as differential geometry and the calculus of variations; among other notable applications, they are the fundamental tool in the proof of the Poincaré conjecture from geometric topology. It is to be noted that several methods are usually used in solving PDEs [1]. The newly developed Adomian decomposition method and the related improvements of the modified technique and the noise terms phenomena will be effectively used. The Adomian decomposition method was formally proved to provide the solution in terms of a rapid convergent infinite series that may yield the exact solution in many cases. Moreover, the other traditional methods, that are usually used in solve PDEs and fractional PDEs [2-74].

Our goal is to demonstrate the YADM, which is a coupling technique of YT and ADM, and to utilize it to solve the PDEs. The remainder of this work is divided into the following sections. In section 2, the definition of Yang transform and its properties are provided. Section 3 implements the YADM analysis. Section 4 demonstrates how YADM may be used. The conclusion of this work is found in Section 5.

2. Yang Transform

Definition 2.1 [75]. The Yang transform of the function is

$$Y\{u(t)\} = \int_0^\infty e^{-\frac{t}{v}} u(t)dt\,, \quad t > 0,$$

with v representing the transform variable.

Few properties of YT is stated as.

1. $Y\{1\} = v.$
2. $Y\{t\} = v^2.$
3. $Y\{u^{(n)}(t)\} = \frac{Y\{u(t)\}}{v^n} - \sum_{k=0}^{n-1} \frac{u^{(k)}(0)}{v^{n-k-1}}\,, n = 1,2,3, \ldots$

3.Analysis of the Yang Adomian decomposition method

The YADM is explored in this section for the solution of nonhomogeneous fractional nonlinear PDEs

$$L_t^{(n)} u\,(x,t) + R\,u(x,t) + N\,u(x,t) = g(x,t\,), \quad t > 0, \tag{1}$$

where R and N are linear and nonlinear operators, respectively, with the initial conditions

$$u^{(k)}(x,0) = c_k\,, k = 0,1, \ldots., n-1 \tag{2}$$

Taking Yang transform (YT) to Eq. (1), we obtain

$$Y\left\{L_t^{(m)} u\,(x,t)\right\} = Y\,\{\,g\,(x\,,t) - R\,u\,(x\,,t) + N\,u\,(x\,,t)\},$$

or

$$\frac{Y\{u\,(x,t)\}}{v^n} - \frac{\sum_{k=0}^{n-1} u^{(k)}(x,0)}{v^{n-k-1}} = Y\,\{g\,(x\,,t) - R\,u\,(x\,,t) - N\,u\,(x\,,t)\}.$$

This equivalent

$$Y\{\, u(x,t)\} = vu(x,0) + v^2 u'(x,0) + \cdots + v^n u^{(n-1)}(x,0) + \ v^n\, Y\{g(x,t)\}$$
$$- v^n\, Y\{R\big(u(x,t)\big) + N\big(u(x,t)\big)\}. \tag{3}$$

Applying the inverse of YT of Eq.(3), we have

$$u(x,t) = u(x,0) + tu'(x,0) + \cdots + \frac{t^n}{n!} u^{(n-1)}(x,0) + Y^{-1}\left(v^n\, Y\{g(x,t)\}\right)$$
$$- Y^{-1}\left(v^n\, Y\{R\big(u(x,t)\big) + N\big(u(x,t)\big)\}\right). \tag{4}$$

The infinite series shown here reflects the YADM solution of $u(x,t)$ as

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \tag{5}$$

The problem's nonlinear term may be written as an Adomian polynomial as follows:

$$N\, u(x,t) = \sum_{n=0}^{\infty} A_n, \tag{6}$$

where

$$A_n = \frac{1}{n!}\left[\frac{\partial^n}{\partial \lambda^n} N\left(\sum_{i=0}^{n} \lambda_i u^i\right)\right]_{\lambda=0}.$$

By adding Eq. (5) and Eq. (6) in Eq. (4), we get

$$\sum_{n=0}^{\infty} u_n = u(x,0) + tu'(x,0) + \cdots + \frac{t^n}{n!} u^{(n-1)}(x,0) + Y^{-1}\left(v^n\, Y\{g(x,t)\}\right)$$
$$- Y^{-1}\left(v^n\, Y\left\{R\left(\sum_{n=0}^{\infty} u_n\right) + \sum_{n=0}^{\infty} A_n\right\}\right). \tag{7}$$

When both sides of Eq. (7) are compared, we get:

$$u_0(x,t) = u(x,0) + tu'(x,0) + \cdots + \frac{t^n}{n!} u^{(n-1)}(x,0) + Y^{-1}\left(v^n\, Y\{g(x,t)\}\right).$$
$$u_1(x,t) = -Y^{-1}\left(v^n\, Y\{\, R(u_0) + A_0\}\right)$$
$$u_2(x,t) = -Y^{-1}\left(v^n\, Y\{\, R(u_1) + A_1\}\right)$$
$$\vdots \tag{8}$$

$$u_{n+1}(x,t) = -Y^{-1}\left(v^n\, Y\{\, R(u_n) + A_n\}\right), \qquad n = 0,1,\ldots$$

Thus, the approximate solution of Eq. (1) is:

$$u(x,t) = u_0 + u_1 + u_2 + \cdots \tag{9}$$

4. Applications of YADM

Example 4.1. Let us consider the partial differential equation

$$u_t + uu_x = x^2 + xt^2, \tag{10}$$

subject to initial condition

$$u(x,0) = 0.$$

Applying the Yang transform of Eq.(10), we get

$$Y\,\{u_t(x,t)\} + Y\,\{u\, u_x\} = Y[x^2 + xt^2]$$

$$\frac{1}{v} Y\{\, u(x,t)\} - u(x,0) = Y\{x^2 + xt^2\}$$

$$Y\{\, u(x,t)\} = v\, Y\{x\} + vY\,\{xt^2\} - vY\,\{uu_x\}$$

$$u(x,t) = Y^{-1}(vY\{x\}) + Y^{-1}(vY\{vt^2\}) - Y^{-1}(vY\{uu_x\})$$

$$u(x,t) = xt + \frac{xt^3}{3} - Y^{-1}(vY\{uu_x\})$$

Suppose that

$$u = \sum_{n=0}^{\infty} u_n\,, \quad uu_x = \sum_{n=0}^{\infty} A_n$$

Then, we have

$$u_0 = xt + + \frac{xt^3}{3},$$

$$A_0 = u_0\, u_{0x} = xt^2 + x\frac{t^4}{3} + x\frac{t^4}{3} + x\frac{t^6}{9}$$

$$\begin{aligned} u_1 &= -Y^{-1}(v\, Y\{A_0\}) \\ &= -\frac{xt^3}{3} - \frac{2xt^4}{3} - \frac{xt^7}{63} \end{aligned}$$

$$\vdots$$

Therefore, the approximate solution is

$$u(x,t) = u_0 + u_1 + u_2 + \cdots = xt$$

Example 2.4. Consider the system of partial differential equation

$u_t + wu_x + u = 1$ $\qquad\qquad u(x,0) = e^x$

$w_t + uw_x - w = 1 \qquad w(x,0)= e^{-x} \qquad (11)$

Taking YT of (11), we obtain

$$Y\{u_t(x,t)\} + Y\{wu_x(x,t)\} + Y\{u(x,t)\} = Y\{1\}$$

$$Y\{w_t(x,t)\} + Y\{uw_x(x,t)\} - Y\{w(x,t)\} = Y\{1\}$$

or

$$\frac{1}{v}Y\{u\,(x,t)\} - u(x,0) + Y\{wu_x\} + Y\{u\} = Y\{1\}$$

$$\frac{1}{v}Y\{w\,(x,t\}) - w(x,0) + Y\{uw_x(x,t)\} - Y\{w\} = Y\{1\}$$

This equivalent to

$Y\{\,u\,(x,t)\} = vt + ve^{x} - vY\{wu_x\} - vY\{u\}$

$Y\{w\,(x,t\}) = vt + ve^{-x} - v\,Y\{uw_x\} + vY\{w\}$

Applying the inverse of YT, we get

$$u(x,t) = t + e^{x} - Y^{-1}(\,vY\{wu_x\}) - Y^{-1}(v\,Y\{u\})$$

$$w(x,t) = t + e^{-x} - y^{-1}\,(v\,Y\{uw_x\}) + Y^{-1}(v\,Y\{w\})$$

Assume that

$u = \sum_{n=0}^{\infty} u_n \qquad , \qquad wu_x = \sum_{n=0}^{\infty}(A_n)$

$w = \sum_{n=0}^{\infty} w_n \qquad , \qquad uw_x = \sum_{n=0}^{\infty}(B_n)$

$u_0 = t + e^{x} \qquad A_0 = w_0\, u_0 x$

$w_0 = t + e^{-x} \qquad B_0 = u_0 w_{0x}$

$u_1 = -Y^{-1}(v\,Y(A_0)) - Y^{-1}(\,v\,Y(w_0)\,)$

$u_1 = -Y^{-1}(vY(te^{x}+1)) - Y^{-1}(v\,Y(t+e^{-x}))$

$\quad = \frac{t^2}{2!}e^{x} - t - \frac{t^2}{2!} - e^{-x}$

$w_1 = -Y^{-1}(vY\,\{u_0 w_{0x}\}) - Y^{-1}(v\,Y(w_0))$

$\quad = -Y^{-1}(vy(-te^{-x})) - Y^{-1}(vy(t+e^{-x}))$

$\quad = -\frac{t^2}{2!}e^{-x} - t + \frac{t^2}{2!} + te^{-x}$

$\vdots$

Therefore, the approximate solution is

$$u(x,t) = u_0 + u_1 + u_2 + \cdots$$

$$= t + e^{x} - \frac{t^2}{2!}e^{x} - t + \frac{t^2}{2!} - te^{x} + \cdots$$

$$= e^{x} - te^{x} - \frac{t^2}{2!}e^{x} + \cdots$$

$$w(x,t) = w_0 + w_1 + w_2 + \cdots$$

$$= t + e^{-x} - t - \frac{t^2}{2!}e^{-x} + \frac{t^2}{2!} - te^{-x} + \cdots$$

$$= e^{-x} - te^{-x} - \frac{t^2}{2!}e^{-x} + \cdots$$

5. Conclusions

The YADM was effectively used in this work to discover approximate solutions to partial differential equations. The analytical approach generates a convergence analysis that fast converges to the exact solution. The simplicity and high precision of the analytical method are clearly illustrated, for example, involves solving certain equations, like that of linear and nonlinear fractional partial differential equations, as well as an example of a nonlinear system of partial differential equations.

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