Journal of Education for Pure Science- University of Thi-Qar
Vol14, No. 2 (2024)
Website: jceps.utq.edu.iq Email: jceps@eps.utq.edu.iq

DOI: https://doi.org/10.32792/jeps.v14i2.434

Solving Biological Population Model by Using FADM within Atangana-Baleanu fractional derivative

Ghazusan Ali Abdul Hussein[1], Djelloul Ziane[2],

[1]Department of Mathematics, College of Education for Pure Science, University of Thi-Qar, Nasiriyah, Iraq.

[2]Laboratory of mathematics and its applications (LAMAP), University of Oran1, Oran, Algeria.

[*] Corresponding email: ghazwanali84@gmail.com

Received 10/4/2024 Accepted 12/5/2024, Published 1/6/2024

Abstract

In this paper, we are concerned with finding approximate solutions to biological population model by using fractional Adomian decomposition method (FADM). The presented method is considered in the Atangana-Baleanu fractional derivative operator (ABFDO). By using an initial value, the explicit solution of the equation has been presented in the closed form and then its numerical solution has been represented graphically. The present method performs extremely well in terms of efficiency and simplicity.

Keywords: Fractional biological population model; Atangana-Baleanu fractional operator; Adomian decomposition method.

1. Introduction

Fractional calculus has attracted significant interests in the field of engineering and applied sciences in the last few years. The elementary knowledge of fractional calculus can be found in [1,2]. Fractional differential equations contain derivatives of any complex or real order, being considered as general form of differential equations. The comprehensive applications in real world problems are described by fractional partial differential equation and it is found to be an effective tool in interpretation and modeling of numerous problems appear in physics and applied mathematics [3-5].

Recently, a great effort has been expended to develop the exact and approximate behavior of fractional partial differential equations (PDEs). In this effort several enthusiastic methods have been applied for the solution of fractional PDEs such as homotopy analysis method, expansion methods, homotopy analysis transform method, fractional difference method, operational method, variational iteration method, homotopy perturbation method, direct approach, Lie symmetry analysis, differential transform method, reproducing kernel method, extended differential transform, method, mesh less methods, Sumudu variational iteration method, Sumudu decomposition method, Laplace homotopy perturbation method, and Laplace variational iteration method and another methods [5-82].

2. Preliminaries

Definition 1. Suppose that the function $\mathcal{U} \in H^1(\mathcal{E}_1, \mathcal{E}_2), \mathcal{E}_1 > \mathcal{E}_2$, then the Atangana-Baleanu operator in Caputo sense of $\mathcal{U}$ at $0 < a < 1$ is,

$$ {}^{\mathcal{A}ß}\mathcal{D}_{\mathcal{T}}^{a}\mathcal{U}(\mathcal{T}) = \frac{ß(a)}{1-a}\int_0^t \mathrm{E}_a\left(-\frac{a(\mathcal{T}-\delta)^a}{1-a}\right)\mathcal{U}'(\delta)\mathrm{d}\delta\,,\ \ \mathcal{T} \geq 0, \tag{2.1} $$

where $ß(a)$ is a function such that $ß(0) = ß(1) = 1$ and $\mathcal{U}'(\delta)$ is the derivative of $\mathcal{U}$.

Definition 2. The Atangana-Baleanu fractional integral (ABFI) of order α defined as follows

$$ {}^{\mathcal{A}ß}_{a}I_{\mathcal{T}}^{\alpha}\,\mathcal{U}(\mathcal{T}) = \frac{1-\alpha}{M(\alpha)}\mathcal{U}(\mathcal{T}) + \frac{\alpha}{M(\alpha)}\,\frac{1}{\Gamma(\alpha)}\int_a^{\mathcal{T}}(\mathcal{T}-x)^{\alpha-1}\,\mathcal{U}(x)\,dx \tag{2.2} $$

The properties of ABFI is defined as follows**:**

1. ${}^{AB}_{a}I_{t}^{\alpha}\,{}^{\mathcal{A}ß}\mathcal{D}_{\mathcal{T}}^{a}\,\mathcal{U}(\mathcal{T}) = \mathcal{U}(\mathcal{T}) - \mathcal{U}(a)$.

2. ${}^{AB}_{a}I_{\mathcal{T}}^{\alpha}\,c = \frac{c}{M(\alpha)}\left(1-\alpha+\frac{\mathcal{T}^{\alpha}}{\Gamma(\alpha)}\right)$.

3. ${}^{AB}_{a}I_{\mathcal{T}}^{\alpha}\,\mathcal{T}^{k} = \frac{\mathcal{T}^{k}}{M(\alpha)}\left(1-\alpha+\frac{\alpha\,\Gamma(k+1)\,\mathcal{T}^{\alpha}}{\Gamma(\alpha+k+1)}\right)$.

3. Analysis of FADM

Let us consider the following fractional partial differential equation:

$$ {}^{AB}_{a}D_{\mathcal{T}}^{\alpha}\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T}) + R\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T}) + N\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T}) = g\,(\mathcal{X},\mathcal{Y},\mathcal{T}),\ \ 0 < \alpha < 1,\ \mathcal{T} > 0, \tag{3.1} $$

with respect to the initial condition

$$\mathcal{U}(\mathcal{X},\mathcal{Y},0)=f(\mathcal{X},\mathcal{Y}), \tag{3.2}$$

where ${}^{AB}_{a}D^{\alpha}_{\mathcal{T}}\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})$ Atangana-Baleanu fractional operator.

The technique is based on using the operator ${}^{AB}_{a}I^{\alpha}_{\mathcal{T}}$, and operator's inverse ${}^{AB}_{a}D^{\alpha}_{\mathcal{T}}$ on both sides of Eq. (3.1) to have

$${}^{AB}_{a}I^{\alpha}_{\mathcal{T}}\{{}^{AB}_{a}D^{\alpha}_{\mathcal{T}}\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})\}={}^{AB}_{a}I^{\alpha}_{\mathcal{T}}[g(\mathcal{X},\mathcal{Y},\mathcal{T})]-{}^{AB}_{a}I^{\alpha}_{\mathcal{T}}[R\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})+N\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})],$$

or equivalent

$$\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})=\mathcal{U}(\mathcal{X},\mathcal{Y},0)+{}^{AB}_{a}I^{\alpha}_{\mathcal{T}}[g(\mathcal{X},\mathcal{Y},\mathcal{T})]-{}^{AB}_{a}I^{\alpha}_{\mathcal{T}}[R\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})+N\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})] \tag{3.4}$$

The infinite series shown here reflects the ADM solution of $\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})$as

$$\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})=\sum_{n=0}^{\infty}\mathcal{U}_n(\mathcal{X},\mathcal{Y},\mathcal{T}), \tag{3.5}$$

The problem's nonlinear term may be written as an Adomian polynomial as follows:

$$N\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})=\sum_{n=0}^{\infty}A_n, \tag{3.6}$$

where

$$A_n=\frac{1}{n!}\left[\frac{\partial^n}{\partial\omega^n}N\left(\sum_{i=0}^{n}\omega_i u^i\right)\right]_{\omega=0}.$$

We get Eq. (3.4) by combining Eq. (3.5) and Eq. (3.6)

$$\sum_{n=0}^{\infty}\mathcal{U}_n(\mathcal{X},\mathcal{Y},\mathcal{T})=f(\mathcal{X},\mathcal{Y})+{}^{AB}_{a}I^{\alpha}_{t}[g(\mathcal{X},\mathcal{Y},\mathcal{T})]-{}^{AB}_{a}I^{\alpha}_{\mathcal{T}}\left[R\left(\sum_{n=0}^{\infty}\mathcal{U}_n\right)+\sum_{n=0}^{\infty}A_n\right]. \tag{3.7}$$

We present recursive relations as a continuation of the ADM,

$$\begin{aligned}\mathcal{U}_0(\mathcal{X},\mathcal{Y},\mathcal{T})&=f(\mathcal{X},\mathcal{Y})+{}^{AB}_{a}I^{\alpha}_{\mathcal{T}}[g(\mathcal{X},\mathcal{Y},\mathcal{T})],\\ \mathcal{U}_{n+1}(\mathcal{X},\mathcal{Y},\mathcal{T})&=-{}^{AB}_{a}I^{\alpha}_{\mathcal{T}}\left[R\left(\mathcal{U}_n(\mathcal{X},\mathcal{Y},\mathcal{T})\right)+A_n\right],\ n\geq 0.\end{aligned} \tag{3.8}$$

Consequently, the solution for the series is

$$\mathcal{U}_n(\mathcal{X},\mathcal{Y},\mathcal{T})=\mathcal{U}_0(\mathcal{X},\mathcal{Y},\mathcal{T})+\mathcal{U}_1(\mathcal{X},\mathcal{Y},\mathcal{T})+\mathcal{U}_2(\mathcal{X},\mathcal{Y},\mathcal{T})+\cdots$$

4. Applications

Example 1. Consider the following population model with the Atangana-Baleanu operator

$${}^{\mathcal{AB}}\mathcal{D}^{\alpha}_{\mathcal{T}}\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})=\frac{\partial^2\,\mathcal{U}^2}{\partial\mathcal{X}^2}+\frac{\partial^2\,\mathcal{U}^2}{\partial\mathcal{Y}^2}+\mathcal{U}\quad,\quad 0<a\leq 1, \tag{4.1}$$

subject to the initial condition $\mathcal{U}(\mathcal{X},0)=\sqrt{\mathcal{X}\mathcal{Y}}$.

Using the operator ${}^{AB}_{a}I^{\alpha}_{\mathcal{T}}$, on both sides of Eq. (4.1), we have

$${}^{AB}I^{\alpha}_{\mathcal{T}}\left[{}^{\mathcal{AB}}\mathcal{D}^{\alpha}_{\mathcal{T}}\,\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T})=\frac{\partial^2\,\mathcal{U}^2}{\partial\mathcal{X}^2}+\frac{\partial^2\,\mathcal{U}^2}{\partial\mathcal{Y}^2}+\mathcal{U}\right], \tag{4.2}$$

or

$$\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T}) = \sqrt{\mathcal{X}\mathcal{Y}} + {}^{AB}I_{\mathcal{T}}^{\alpha}\left[\frac{\partial^2\mathcal{U}^2}{\partial\mathcal{X}^2} + \frac{\partial^2\mathcal{U}^2}{\partial\mathcal{Y}^2} + \mathcal{U}\right]. \tag{34}$$

Suppose that

$$\mathcal{U}(\mathcal{X},\mathcal{Y},\mathcal{T}) = \sum_{n=0}^{\infty}\mathcal{U}_n(\mathcal{X},\mathcal{Y},\mathcal{T}), \qquad \mathcal{U}^2 = \sum_{n=0}^{\infty}\mathcal{H}_n$$

$$\mathcal{H}_0 = \mathcal{U}_0^2,$$
$$\mathcal{H}_1 = 2\mathcal{U}_0\mathcal{U}_1,$$
$$\mathcal{H}_2 = 2\mathcal{U}_0\mathcal{U}_1 + \mathcal{U}_1^2.$$
$$\vdots$$

Thus,

$$\sum_{n=0}^{\infty}\mathcal{U}_n = \sqrt{\mathcal{X}\mathcal{Y}} + {}^{AB}I_{\mathcal{T}}^{\alpha}\left[\frac{\partial^2}{\partial\mathcal{X}^2}\sum_{n=0}^{\infty}\mathcal{H}_n + \frac{\partial^2}{\partial\mathcal{Y}^2}\sum_{n=0}^{\infty}\mathcal{H}_n - \sum_{n=0}^{\infty}\mathcal{U}_n\right]. \tag{35}$$

By comparing both sides of the Eq.(35), the following result is obtained,

$$\mathcal{U}_0(\mathcal{X},\mathcal{T}) = \mathcal{U}(\mathcal{X},0),$$
$$\mathcal{U}_1(\mathcal{X},\mathcal{T}) = {}^{AB}I_{\mathcal{T}}^{\alpha}\left[\frac{\partial^2}{\partial\mathcal{X}^2}\mathcal{H}_0 + \frac{\partial^2}{\partial\mathcal{Y}^2}\mathcal{H}_0 - \mathcal{U}_0\right],$$
$$\mathcal{U}_2(\mathcal{X},\mathcal{T}) = {}^{AB}I_{\mathcal{T}}^{\alpha}\left[\frac{\partial^2}{\partial\mathcal{X}^2}\mathcal{H}_1 + \frac{\partial^2}{\partial\mathcal{Y}^2}\mathcal{H}_1 - \mathcal{U}_1\right],$$
$$\mathcal{U}_3(\mathcal{X},\mathcal{T}) = {}^{AB}I_{\mathcal{T}}^{\alpha}\left[\frac{\partial^2}{\partial\mathcal{X}^2}\mathcal{H}_2 + \frac{\partial^2}{\partial\mathcal{Y}^2}\mathcal{H}_2 - \mathcal{U}_2\right].$$
$$\vdots$$

By the above algorithms,

$$\mathcal{U}_0(\mathcal{X},\mathcal{T}) = \sqrt{\mathcal{X}\mathcal{Y}},$$
$$\mathcal{U}_1(\mathcal{X},\mathcal{T}) = -a\sqrt{xy} + \sqrt{xy} + \frac{\sqrt{xy}t^a}{\Gamma(a)},$$
$$\mathcal{U}_2(\mathcal{X},\mathcal{T}) = a^2\sqrt{xy} - 2a\sqrt{xy} + \frac{\sqrt{xy}\sqrt{\pi}a(t^a)^2}{(2^a)^2\Gamma(a)\Gamma(1/2+a)} + \sqrt{xy} + 2\frac{\sqrt{xy}t^a}{\Gamma(a)} - 2\frac{\sqrt{xy}at^a}{\Gamma(a)},$$

$$\mathcal{U}_3(\mathcal{X},\mathcal{T}) = \frac{2}{3}\frac{\sqrt{xy}\pi(t^a)^3a^2\sqrt{3}}{\Gamma(a)\Gamma\left(a+\frac{1}{3}\right)\Gamma\left(a+\frac{2}{3}\right)27^a} - 3\frac{\sqrt{xy}\sqrt{\pi}(t^a)^2a^2}{4^a\Gamma(a)\Gamma\left(\frac{1}{2}+a\right)} - \sqrt{xy}a^3$$
$$+3\frac{\sqrt{xy}\sqrt{\pi}(t^a)^2a}{4^a\Gamma(a)\Gamma(1/2+a)} + 3a^2\sqrt{xy} + 3\frac{\sqrt{xy}t^aa^2}{\Gamma(a)} - 3a\sqrt{xy} - 6\frac{\sqrt{xy}t^a}{\Gamma(a)} + \sqrt{xy} + 3\frac{\sqrt{xy}t^a}{\Gamma(a)},$$

and so on.

Therefore, the series solution $\mathcal{U}(\mathcal{X},\mathcal{T})$ of Eq.(3.1) is given by

$$\mathcal{U}(\mathcal{X},\mathcal{T}) = -\sqrt{xy}a^3 + 4\sqrt{xy} - 6a\sqrt{xy} + 6\frac{\sqrt{xy}t^a}{\Gamma(a)} + 4a^2\sqrt{xy} + \frac{\sqrt{xy}\sqrt{\pi}(t^a)^2 a}{(2^a)^2\Gamma(a)\Gamma\left(\frac{1}{2}+a\right)}$$

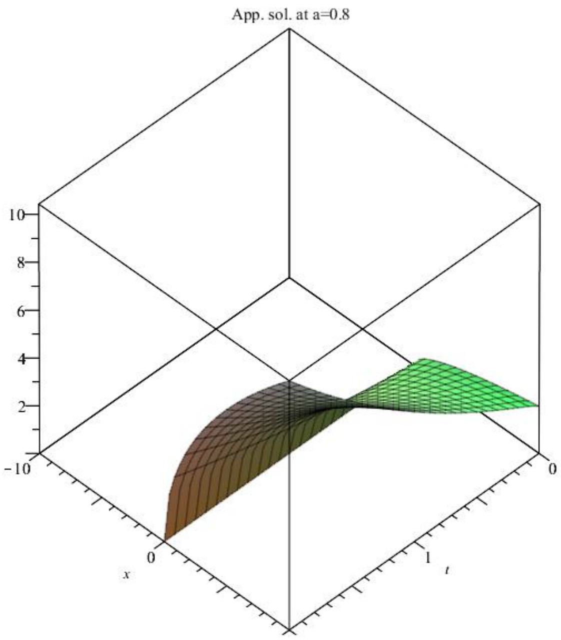
$$-8\frac{\sqrt{xy}at^a}{\Gamma(a)} + 3\frac{\sqrt{xy}t^a a^2}{\Gamma(a)} + \frac{2}{3}\frac{\sqrt{xy}\pi(t^a)^3 a^2\sqrt{3}}{\Gamma(a)\Gamma\left(a+\frac{1}{3}\right)\Gamma\left(a+\frac{2}{3}\right)27^a} - 3\frac{\sqrt{xy}\sqrt{\pi}(t^a)^2 a^2}{4^a\Gamma(a)\Gamma\left(\frac{1}{2}+a\right)}$$

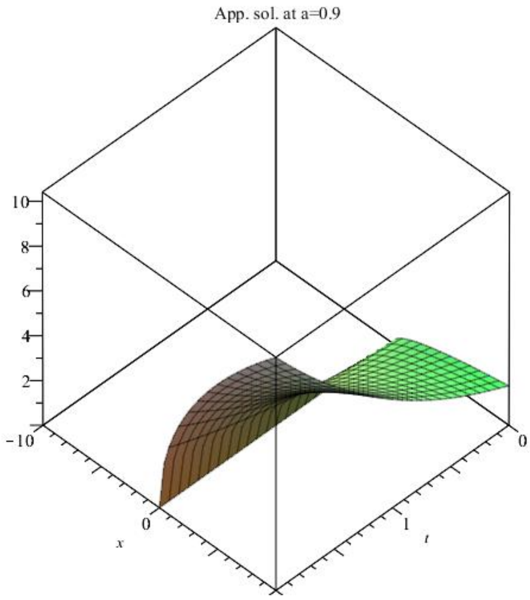
$$+3\frac{\sqrt{xy}\sqrt{\pi}(t^a)^2 a}{4^a\Gamma(a)\Gamma(1/2+a)} + \cdots \qquad (36)$$

If we put $a \to 1$ in Eq.(36) , we get the approximate

$$\mathcal{U}(\mathcal{X},\mathcal{T}) = \frac{1}{2}\sqrt{xy}t^2 + \frac{1}{6}\sqrt{xy}t^3 + \sqrt{xy} + \sqrt{xy}t + \cdots.$$

$$= \sqrt{xy}\, e^{\mathcal{T}}.$$





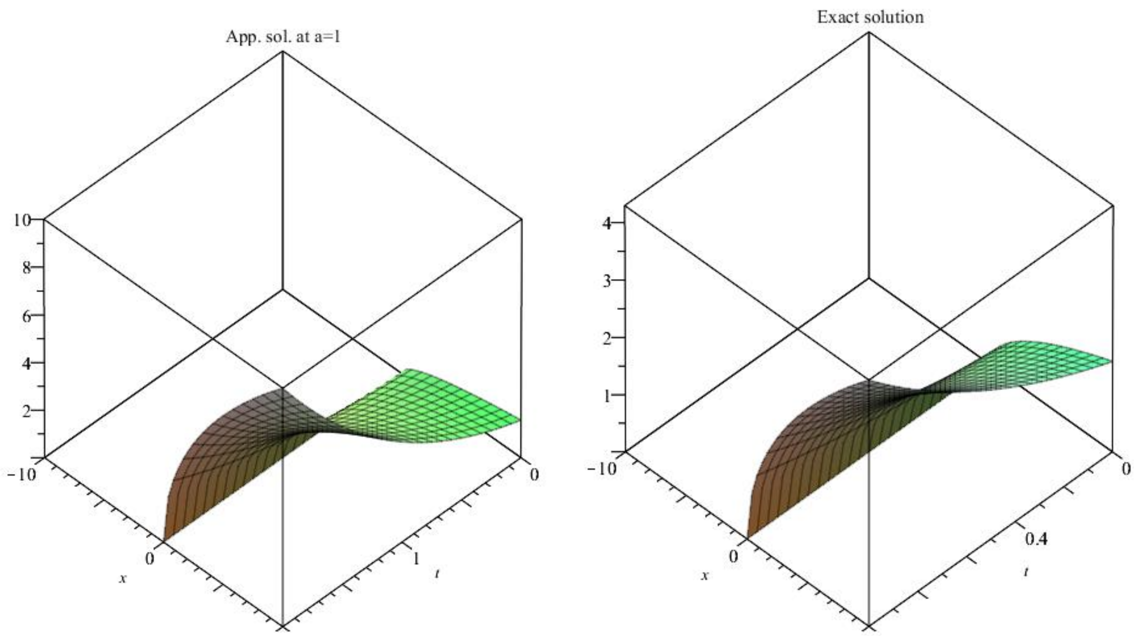


Figure 1. The graph of approximate and exact solutions to Eq.(3.1).

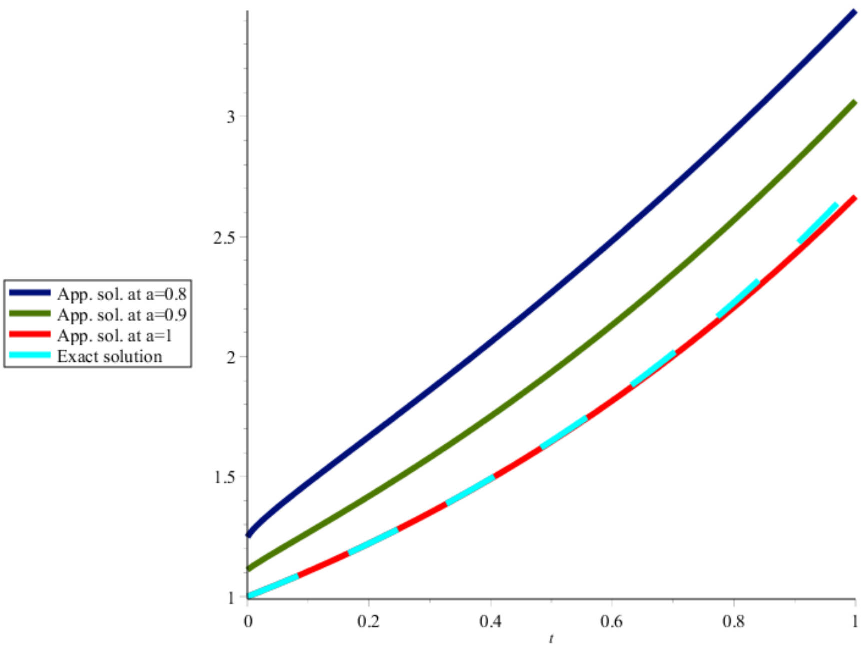


Figure 2. Two-dimensional graph with approximate and exact solutions to Eq.(3.1).

5. CONCLUSIONS

The biological population model with Atangana-Baleanu fractional derivative has been studied. The FADM was used to successfully achieve analytical approximation solutions. The solutions were found to be infinite power series that could be expressed in closed form. When $\alpha = 1$, the results of FADM are in great agreement with the precise solution, as seen in the example.

REFERENCES

[1]R. Hilfer, *Applications of fractional calculus in physics. Singapore*, Word Scientific Company, (2000).

[2]A. A. Kilbas, H. M. Srivastava, J. T. Juan, *Theory and applications of fractional differential equations*, North- Holland, Jan Van Mill (2006).

[3]I. Petras, *Fractional-order nonlinear systems: modeling, analysis and simulation*, Beijing, Higher Education Press, (2011).

[4]I. Podlubny, *Fractional differential equations*, San Diego, Academic Press (1999).

[5]H. K. Jassim, On Local Bifurcations and Chaos of a Three-Dimensional Nonlinear System,. *Journal of college of Education for Pure Science*, 3(2) (2013), 150–158.

[6]H. Jafari, et al., Local Fractional Series Expansion Method for Solving Laplace and Schrodinger Equations on Cantor Sets within Local Fractional Operators, *International Journal of Mathematics and Computer Research*, 2(11)(2014), 736-744.

[7]H. Jafari, et al., Local Fractional Adomian Decomposition Method for Solving Two Dimensional Heat conduction Equations within Local Fractional Operators, *Journal of Advance in Mathematics*, 9 (4)(2014), 2574-2582.

[8]H. Jafari, et al., Local Fractional Laplace Variational Iteration Method for Solving Nonlinear Partial Differential Equations on Cantor Sets within Local Fractional Operators, *Journal of Zankoy Sulaimani-Part A*, 16(4)(2014), 49-57.

[9]H. Jafari, et al., Numerical Solutions of Telegraph and Laplace Equations on Cantor Sets Using Local Fractional Laplace Decomposition Method , *International Journal of Advances in Applied Mathematics and Mechanics*, 2(3) (2015), 144-151.

[10] H. Jafari, et al., A Coupling Method of Local Fractional Variational Iteration Method and Yang-Laplace Transform for Solving Laplace Equation on Cantor Sets, *International Journal of pure and Applied Sciences and Technology*, 26(1) (2015), 24-33.

[11] H. K. Jassim, Local Fractional Laplace Decomposition Method for Nonhomogeneous Heat Equations Arising in Fractal Heat Flow with Local Fractional Derivative, *International Journal of Advances in Applied Mathematics and Mechanics*, 2(4)(2015), 1-7.

[12] H. K. Jassim, Homotopy Perturbation Algorithm Using Laplace Transform for Newell-Whitehead-Segel Equation, *International Journal of Advances in Applied Mathematics and Mechanics*, 2(4) (2015) 8-12.

[13] H. Jafari, et al., Application of the Local fractional Adomian Decomposition and Series Expansion Methods for Solving Telegraph Equation on Cantor Sets Involving Local Fractional Derivative Operators, *Journal of Zankoy Sulaimani-Part A*, vol. 17, no. 2, pp. 15-22 , 2015.

[14] H. K. Jassim, Analytical Solutions for System of Fractional Partial Differential Equations by Homotopy Perturbation Transform Method, *International Journal of Advances in Applied Mathematics and Mechanics*, vol.3, no. 1, pp. 36-40, 2015.

[15] H. K. Jassim, Analytical Approximate Solution for Inhomogeneous Wave Equation on Cantor Sets by Local Fractional Variational Iteration Method, *International Journal of Advances in Applied Mathematics and Mechanics*, vol.3, no. 1, pp. 57-61, 2015.

[16] H. K. Jassim, Local Fractional Variational Iteration Transform Method to Solve partial differential equations arising in mathematical physics, *International Journal of Advances in Applied Mathematics and Mechanics*, vol.3, no. 1, pp. 71-76, 2015.

[17] H. Jafari, et al., Local Fractional Variational Iteration Method for Nonlinear Partial Differential Equations within Local Fractional Operators, *Applications and Applied Mathematics,* vol. 10, no. 2, pp. 1055-1065, 2015

[18] H. K. Jassim, The Approximate Solutions of Helmholtz and Coupled Helmholtz Equations on Cantor Sets within Local Fractional Operator, *Journal of Zankoy Sulaimani-Part A*, vol. 17, no. 4, pp. 19-25, 2015.

[19] H. K. Jassim, The Approximate Solutions of Fredholm Integral Equations on Cantor Sets within Local Fractional Operators, *Sahand Communications in Mathematical Analysis*, Vol. 16, No. 1, 13-20, 2016.

[20] H. Jafari, et al., Approximate Solution for Nonlinear Gas Dynamic and Coupled KdV Equations Involving Local Fractional Operator, *Journal of Zankoy Sulaimani-Part A*, vol. 18, no.1, pp.127-132, 2016

[21] H. Jafari, et al., A new approach for solving a system of local fractional partial differential equations, *Applications and Applied Mathematics*, Vol. 11, No. 1, pp.162-173, 2016.

[22] H. K. Kadhim, et al., Application of Local Fractional Variational Iteration Method for Solving Fredholm Integral Equations Involving Local Fractional Operators, *Journal of University of Thi-Qar,* Vol. 11, No. 1, pp. 12-18, 2016.

[23] H. Jafari, et al., Application of Local Fractional Variational Iteration Method to Solve System of Coupled Partial Differential Equations Involving Local Fractional Operator, *Applied Mathematics & Information Sciences Letters*, Vol. 5, No. 2, pp. 1-6, 2017.

[24] H. K. Jassim, The Analytical Solutions for Volterra Integro-Differential Equations Involving Local fractional Operators by Yang-Laplace Transform, *Sahand Communications in Mathematical Analysis*, Vol. 6 No. 1 (2017), 69-76.

[25] H. K. Jassim, Some Dynamical Properties of Rössler System, *Journal of University of Thi-Qar*, 3(1) (2017), 69-76.

[26] H. K. Jassim, A Coupling Method of Regularization and Adomian Decomposition for Solving a Class of the Fredholm Integral Equations within Local Fractional Operators, 2(3) (2017), 95-99.

[27] H. K. Jassim, On Approximate Methods for Fractal Vehicular Traffic Flow, *TWMS Journal of Applied and Engineering Mathematics*, 7(1)(2017), 58-65.

[28] H. K. Jassim, A Novel Approach for Solving Volterra Integral Equations Involving Local Fractional Operator, *Applications and Applied Mathematics*, 12(1) (2017), 496 – 505.

[29] H. K. Jassim, Extending Application of Adomian Decomposition Method for Solving a Class of Volterra Integro-Differential Equations within Local Fractional Integral Operators, *Journal of college of Education for Pure Science*, 7(1) (2017), 19-29.

[30] A. A. Neamah, et al., Analytical Solution of The One Dimensional Volterra Integro Differential Equations within Local Fractional Derivative, *Journal of Kufa for Mathematics and Computer*, 4 (1)(2017), 46-50

[31] H. K. Jassim, Approximate Methods for Local Fractional Integral Equations, *The Journal of Hyperstructures*, 6 (1) (2017), 40-51

[32] H. K. Jassim, Solving Poisson Equation within Local Fractional Derivative Operators, *Research in Applied Mathematics*, 1 (2017), 1-12.

[33] H. K. Jassim, An Efficient Technique for Solving Linear and Nonlinear Wave Equation within Local Fractional Operators, *The Journal of Hyperstructures*, 6(2)(2017), 136-146.

[34] H. A. Naser, et al., A New Efficient Method for solving Helmholtz and Coupled Helmholtz Equations Involving Local Fractional Operators, 6(4)(2018), 153-157.

[35] M. G. Mohammed, et al., The Approximate solutions of time-fractional Burger's and coupled time-fractional Burger's equations, *International Journal of Advances in Applied Mathematics and Mechanics*,6(4)(2019),64-70.

[36] M. G. Mohammad, et al., Symmetry Classification of First Integrals for Scalar Linearizable, *International Journal of Advances in Applied Mathematics and Mechanics*, 7(1) (2019), 20-40.

[37] M. Zayir, et al., A Fractional Variational Iteration Approach for Solving Time-Fractional Navier-Stokes Equations. *Mathematics and Computational Sciences*, 3(2) (2022), 41-47.

[38] A. T. Salman, et al., A new approximate analytical method and its convergence for time-fractional differential equations, *NeuroQuantology*, 20(6) (2022) 3670-3689.

[39] M. A. Hussein, et al., New approximate analytical technique for the solution of two dimensional fractional differential equations, *NeuroQuantology*, 20(6) (2022) 3690-3705.

[40] A. T. Salman, et al., An application of the Elzaki homotopy perturbation method for solving fractional Burger's equations, *International Journal of Nonlinear Analysis and Applications*, 13(2) (2022) 21-30.

[41] M. A. Hussein, et al., A New Numerical Solutions of Fractional Differential Equations with Atangana-Baleanu operator in Reimann sense, *International Journal of Scientific Research and Engineering Development*, 5(6)(2022) 843- 849.

[42] J. Singh, et al., An efficient computational technique for local fractional Fokker-Planck equation, *Physica A: Statistical Mechanics and its Applications*, 555(124525) (2020) 1-8.

[43] J. Vahidi, et al., Solving Laplace Equation within Local Fractional Operators by Using Local Fractional Differential Transform and Laplace Variational Iteration Methods, *Nonlinear Dynamics and Systems Theory*, 20(4) (2020) 388-396.

[44] D. Baleanu, et al.,, Exact Solution of Two-dimensional Fractional Partial Differential Equations, *Fractal Fractional*, 4(21) (2020) 1-9.

[45] M. G. Mohammed, et al., A Modification Fractional Homotopy Analysis Method for Solving Partial Differential Equations Arising in Mathematical Physics, *IOP Conf. Series: Materials Science and Engineering*, 928 (042021) (2020) 1-22.

[46] H. A. Eaued, et al., A Novel Method for the Analytical Solution of Partial Differential Equations Arising in Mathematical Physics, *IOP Conf. Series: Materials Science and Engineering*, 928 (042037) (2020) 1-16.

[47] J. Vahidi, et al., A New Technique of Reduce Differential Transform Method to Solve Local Fractional PDEs in Mathematical Physics, *International Journal of Nonlinear Analysis and Applications*, 12(1) (2021) 37-44.

[48] S. M. Kadhim, et al., How to Obtain Lie Point Symmetries of PDEs, *Journal of Mathematics and Computer science*, 22 (2021) 306-324.

[49] M. A. Shareef, et al., On approximate solutions for fractional system of differential equations with Caputo-Fabrizio fractional operator, *Journal of Mathematics and Computer science*, 23 (2021) 58-66.

[50] S. A. Khafif, et al., SVIM for solving Burger's and coupled Burger's equations of fractional order, *Progress in Fractional Differentiation and Applications*, 7(1) (2021)1-6.

[51] H. A. Kadhim, et al., Fractional Sumudu decomposition method for solving PDEs of fractional order, *Journal of Applied and Computational Mechanics*, 7(1) (2021) 302-311.

[52] H. Jafari, et al., On the approximate solutions for a system of coupled Korteweg-de Vries equations with local fractional derivative, *Fractals*, 29(5)(2021) 1-7.

[53] M. G. Mohammed, et al., Natural homotopy perturbation method for solvingnonlinear fractional gas dynamics equations, *International Journal of Nonlinear Analysis and Applications*, 12(1) (2021) 813-821.

[54] M. G. Mohammed, et al., Numerical simulation of arterial pulse propagation using autonomous models, *International Journal of Nonlinear Analysis and Applications*, 12(1) (2021) 841-849.

[55] H. K. Jassim, A new approach to find approximate solutions of Burger's and coupled Burger's equations of fractional order, *TWMS Journal of Applied and Engineering Mathematics*, 11(2) (2021) 415-423.

[56] L. K. Alzaki, et al., The approximate analytical solutions of nonlinear fractional ordinary differential equations, *International Journal of Nonlinear Analysis and Applications*, 12(2) (2021) 527-535.

[57] E. K. Amer, et al., Non-Bayesian estimation of Weibull Lindley burr XII distribution, , *International Journal of Nonlinear Analysis and Applications*, 12(2) (2021) 977-989.

[58] H. Ahmad, et al., An efficient hybrid technique for the solution of fractional-order partial differential equations, *Carpathian Mathematical Publications*, 13(3) (2021) 790-804.

[59] H. G. Taher, et al., Solving fractional PDEs by using Daftardar-Jafari method, *AIP Conference Proceedings*, 2386 (060002) (2022) 1-10.

[60] A. H. Mktof, et al., Weibull Lindley Pareto distribution, *AIP Conference Proceedings*, 2386(060015) (2022) 1-11.

[61] L. K. Alzaki, et al., Time-Fractional Differential Equations with an Approximate Solution, *Journal of the Nigerian Society of Physical Sciences*, 4 (3)(2022) 1-8.

[62] M. A. Hussein, et al., A Novel Formulation of the Fractional Derivative with the Order $\alpha \geq 0$ and without the Singular Kernel, *Mathematics*, 10 (21) (2022), 1-18.

[63] H. G. Taher, et al., Approximate analytical solutions of differential equations with Caputo-Fabrizio fractional derivative via new iterative method, AIP Conference Proceedings, 2398 (060020) (2022) 1-16.

[64] S. A. Sachit, et al., Revised fractional homotopy analysis method for solving nonlinear fractional PDEs, AIP Conference Proceedings, 2398 (060044) (2022) 1-15.

[65] S. H. Mahdi, et al., A new analytical method for solving nonlinear biological population model, AIP Conference Proceedings 2398 (060043) (2022) 1-12.

[66] M. Y. Zayir, et al., A unique approach for solving the fractional Navier–Stokes equation, Journal of Multiplicity Mathematics, 25(8-B) (2022) 2611-2616.

[67] H. Jafari, et al., Analysis of fractional Navier-Stokes equations, Heat Transfer, 52(3)(2023) 2859-2877.

[68] S. A. Sachit, Solving fractional PDEs by Elzaki homotopy analysis method, AIP Conference Proceedings, 2414 (040074) (2023) 1-12.

[69] H. Adnan, et al., The Weibull Lindley Rayleigh distribution, AIP Conference Proceedings, 2414 (040064) (2023) 1-17.

[70] S. H. Mahdi, A new technique of using Adomian decomposition method for fractional order nonlinear differential equations, AIP Conference Proceedings, 2414 (040075) (2023) 1-12.

[71] M A. Hussein, A New Approach for Solving Nonlinear Fractional Ordinary Differential Equations, Mathematics, 11(7)(2023) 1565.

[72] D. Ziane, et al., A. Al-Dmour, Application of Local Fractional Variational Iteration Transform Method to Solve Nonlinear Wave-Like Equations within Local Fractional Derivative, Progress in Fractional Differentiation and Applications, 9(2) (2023) 311–318.

[73] D. Kumar, et al.. A Computational Study of Local Fractional Helmholtz and Coupled Helmholtz Equations in Fractal Media, Lecture Notes in Networks and Systems, 2023, 666 LNNS, pp. 286–298.

[74] N. H. Mohsin, et al., A New Analytical Method for Solving Nonlinear Burger’s and Coupled Burger’s Equations, *Materials Today: Proceedings*, 80 (3)(2023) 3193-3195.

[75] M. A. Hussein, et al,, Analysis of fractional differential equations with Atangana-Baleanu fractional operator, Progress in Fractional Differentiation and Applications, 9(4)(2023) 681-686.

[76] M. Y. Zayir, et al,, Solving fractional PDEs by Using FADM within Atangana-Baleanu fractional derivative, *AIP Conference Proceedings*, 2845(060004) (2023) 1-11.

[77] M. Y. Zayir, et al., Approximate Analytical Solutions of Fractional Navier-Stokes Equation, *AIP Conference Proceedings*, 2834(080100) (2023) 1-10.

[78] M. A. Hussein, et al., An Efficient Homotopy Permutation Technique for Solving Fractional Differential Equations Using Atangana-Baleanu-Caputo operator, *AIP Conference Proceedings*, 2845 (060008) (2023) 1-8.

[79] A. T. Salman, et al., Solving Nonlinear Fractional PDEs by Elzaki Homotopy Perturbation Method, *AIP Conference Proceedings*, 2834(080101) (2023) 1-12.

[80] A. T. Salman, et al., Exact analytical solutions for fractional partial differential equations via an analytical approach, *AIP Conference Proceedings*, 2845(060007) (2023) 1-9.

[81] J. Singh, et al., Fractal dynamics and computational analysis of local fractional Poisson equations arising in electrostatics, *Communications in Theoretical Physics*, 75(12)(2023) 1-8.

[82] L. K. Alzaki, et al., Analytical Approximations for a System of Fractional Partial Differential Equations, Progr. Fract. Differ. Appl. 10(1)(2024) 81-89.