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Disconnected Multi-Effect Domination for Several Graphs Constructed by Corona Operation

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Abstract:

In this paper, disconnected multi-effect domination is discussed, a new domination model in graphs is introduced. Let $G = (V, E)$ is a finite, simple, nontrivial, and undirected graph without isolated vertex. A dominating subset $D \subseteq V$ is a disconnected multi-effect dominating set in G if for every vertex $v \in D$, $|N(v) \cap (V - D)| \geq 2$ and $G[D]$ is disconnected subgraph. The minimum cardinality over all disconnected multi-effect dominating sets in G is the disconnected multi-effect domination number of G denoted by $\gamma_{dm}(G)$. In this work some graphs generated by corona operation will be studied.

Keywords: Disconnected multi-effect domination, Disconnected multi-effect dominating set, Corona

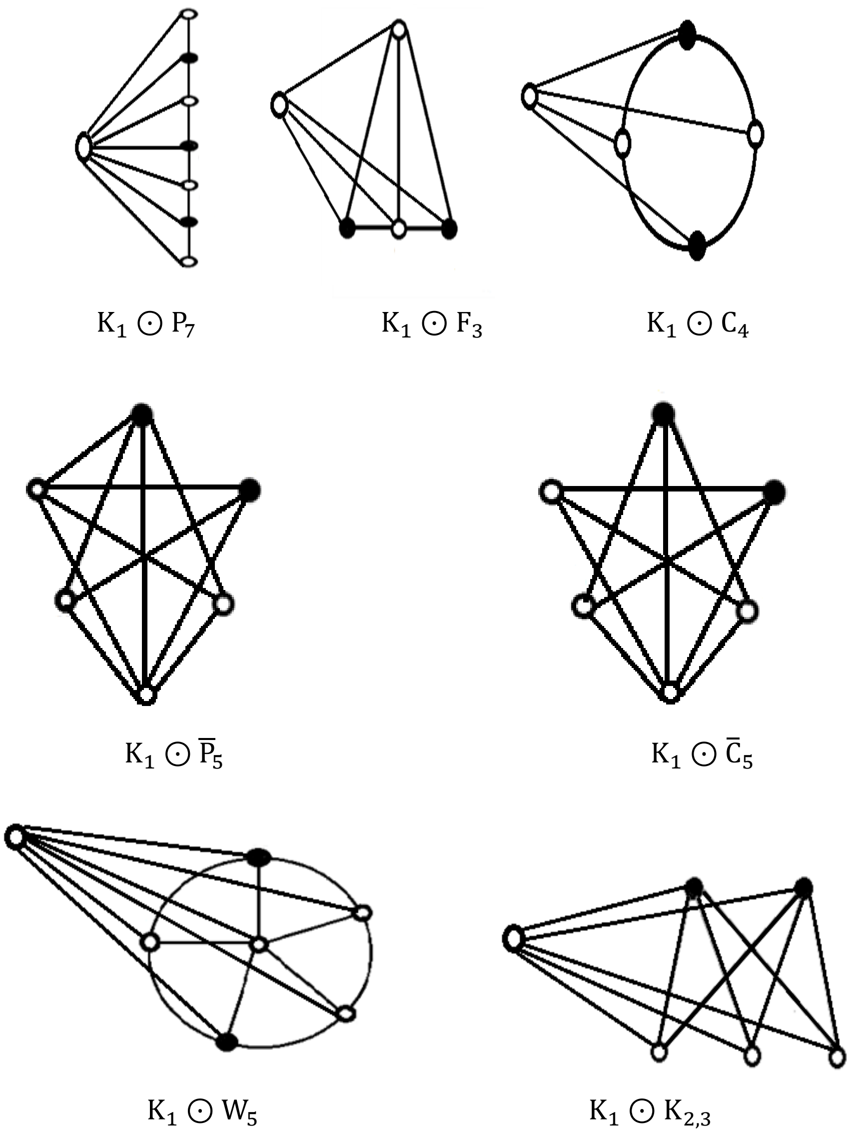
1-Introduction

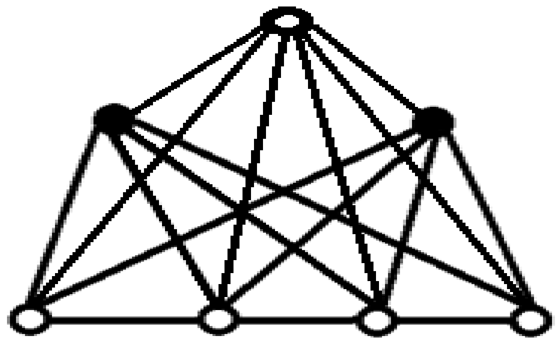
Assume that $G = (V, E)$ be a graph has no isolated vertices with size $m = |E|$ and order $n = |V|$. $G[D]$ is the subgraph of G induced by the vertices in set D and the edges incident between them. (The corona $G_1 \odot G_2$ of two graphs G_1 (with n_1 vertices and m_1) and G_2 (with $n_2$vertices and m_2 edges) is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2, and then joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2). See [12, 15, 23, 25] for a list of terms specifically associated with graph theory. One of the fastest growing areas in graph theory is the study of domination problems, a comprehensive examination of the basics of domination is presented see [13, 17]. An dominating set D is considered a minimal dominating set if it does not contain any proper dominating subset The domination number $\gamma(G)$ is the cardinality of the minimum dominating set. Domination has many applications and can take different forms. Many kinds of domination have surfaced according to their intended purpose. Numerous researchers such [1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 14, 18, 19, 22, 24], worked on the domination of vertices and presented different definitions and characteristics in their works and some researchers [20, 21] worked on discrete topoiogical graph. In this paper, Disconnected multi-effect domination is a novel model of domination in graphs that is presented. The cardinality of the minimum disconnected multi-effect dominating set of the graph G is called the disconnected multi-effect domination number of the G, denoted by $\gamma_{dm}(G)$. [8] some studies have applied the corona operation to various graphs.

2- Disconnected multi-effect domination for some graphs generated by corona operation

The study of the disconnected multi-effect domination in this section will focus on some graphs using the corona operation.

Proposition 2.1. Assume that G be any graph with order m has $\gamma_{dm} -$ set, then $\gamma_{dm}(K_1 \odot G) = \gamma_{dm}(G)$.
Proof. Since there exists one vertex in K_1 is adjacent with all vertices of G and G has $\gamma_{dm} -$ set. Then, each vertex of D of G dominates two or more vertices of $V - D$ and G[D] is disconnected graph such that the vertex u of K_1 belongs to $V - D$. Hence $\gamma_{dm}(K \odot G) = \gamma_{dm}(G)$ and $\gamma_{dm} -$ set. For example, see Fig 1.



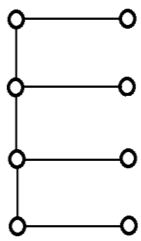


$K_1 \odot K_{2,4}$

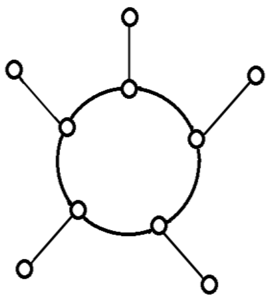
Figure 1. $\boldsymbol{\gamma_{dm}(K_1 \odot G)}$**.**

Proposition 2.2. Assume that G connected graph with order n, then $G \odot K_1$ has no disconnected multi-effect domination.

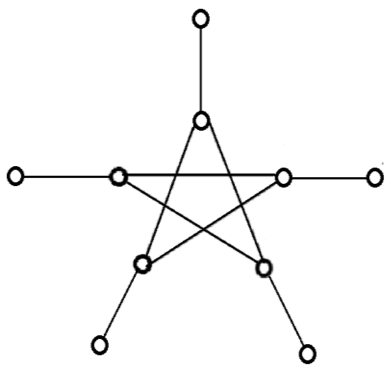
Proof. Since each vertex of G is adjacent with one vertex of K_1. If $D = V(G)$, then each vertex of D dominates one vertex in all copies of K_1, and $G[D]$ is connected graph. If D consist of all vertices of all copies of K_1, then each vertex in K_1 dominates one vertex in G. If $D \subseteq V(G)$, then there exists at least one vertex of all copies K_1 don't dominates by any vertex of G. In all above cases are contraction of our definition. After that, $G \odot K_1$ has no disconnected multi-effect domination. For example, see Fig 2.



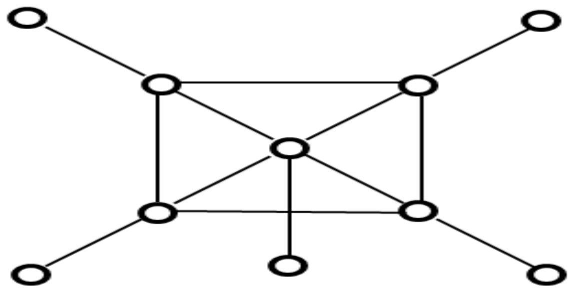
$P_4 \odot K_1$



$C_5 \odot K_1$



$\bar{C}_5 \odot K_1$



$W_4 \odot K_1$

Figure 2. $\boldsymbol{G \odot K_1}$ **has no disconnected multi-effect domination.**

Proposition 2.3. Assume that G be any graph with order m has no $\gamma_{dm}-$ set, then $K_1 \odot G$ has no disconnected multi-effect domination.

Proof. Since there exists one vertex in K_1 is adjacent with all vertices of G and since G has no $\gamma_{dm}-$ set. Then, there exists one vertex in D dominates all vertices of $V-D$, but G[D] is connected graph which is contraction. Then, $K_1 \odot G$ has no disconnected multi-effect domination. For example, see Fig 3.

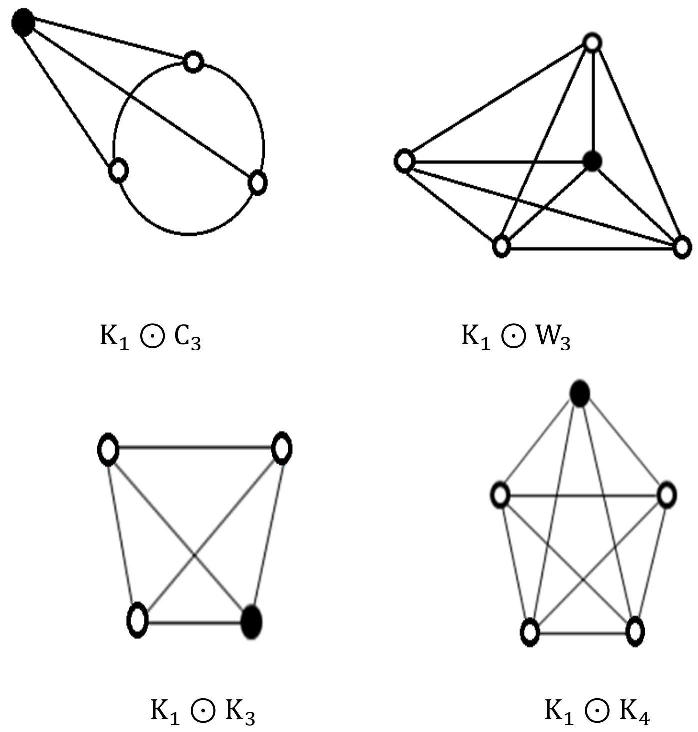


Figure 3. $K_1 \odot$ G has no disconnected multi-effect domination.

Proposition 2.4. Assume that G be any graph of order m has no $\gamma_{dm}-$ set and is not null graph and has no isolated vertex, then $\gamma_{dm}(K_1 \odot G) = \gamma(G)$.

Proof. Since G has no $\gamma_{dm}-$ set, but G has $\gamma-$ set and since there exists one vertex in K_1 is adjacent with all vertices of G and G is not null graph and has no isolated vertex, then each vertex in D of G dominates at least one vertex of G, let $v \in K_1$ belongs to $V-D$. Then every vertex in D dominates at least one vertex from D of G and the vertex v of K_1. Then, each vertex of D in G dominates two or more vertices of $V-D$. Then, each vertex of D dominates two or more vertices of $V-D$ and G[D] is disconnected graph such that the vertex v of K_1 belongs to $V-D$. Hence, $\gamma_{dm}(K_1 \odot G) = \gamma(G)$ and $\gamma_{dm}-$ set. For example, see Fig 4.

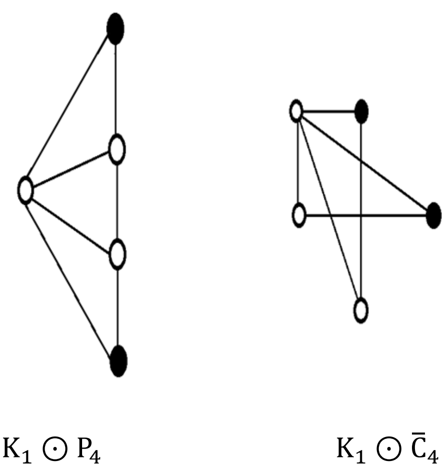


Figure 4. $\gamma_{dm}(K_1 \odot G)$.

Proposition 2.5. $\gamma_{dm}(K_1 \odot S_m) = m, (m \geq 3)$.

Proof. According to the definition of the star graph is a bipartite graph of the form $K_{1,m}$. Let us table $\{ v_1, v_2, \ldots, v_m\}$ be the set of all pendent vertices of star graph. Let $D = \{ v_1, v_2, \ldots, v_m\}$, since there exists one vertex of K_1 is adjacent with all vertices of S_m, then every vertex of D is adjacent with one vertex v of S_m and one vertex u of K_1. Therefore, every vertex of D dominates exactly two vertices and $G[D]$ is disconnected graph. Thus, D is a disconnected multi-effect dominating set and $\gamma_{dm}(K_1 \odot S_m) = m$. For example, see Fig 5.

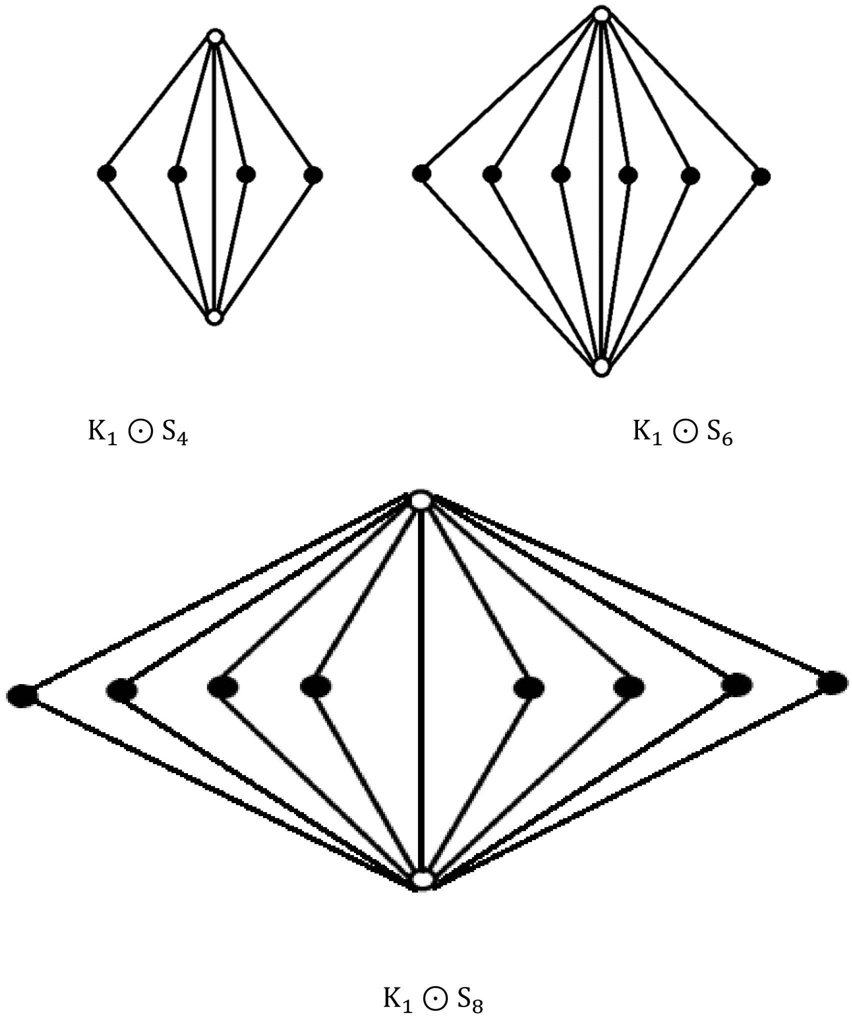


Figure 5. $\boldsymbol{\gamma_{dm}(K_1 \odot S_m)}$.

3- Conclusion

The disconnected multi-effect domination is determined when every vertex $v \in D$, $|N(v) \cap (V - D)| \geq 2$ and $G[D]$ is disconnected subgraph for some graphs that obtained from operation corona. It was concluded G connected graph with order n, then $G \odot K_1$ has no disconnected multi-effect domination. It was concluded G be any graph with order m has no γ_{dm} − set, then $K_1 \odot G$ has no disconnected multi-effect domination.

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