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Double Stage Shrunken Technique For Estimate Shape Parameter of The Burr XII Distribution by Katti's Region

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Abstract

The current research is concerned with double stage shrunken technique in order to estimate the shape parameter (Burr XII) distribution, when previous knowledge regarding the value shape is available as the original estimate. The formulas for the Relative efficiency, mean squared error, and bias ratio are derived, For the aforementioned expressions, numerical results and conclusions were shown.

Keywords: Burr XII distribution, Katti's Region, maximum likelihood estimator, shrinkage technique , Relative efficiency.

Introduction

The BurrXII distribution was first developed by Burr (1942), and its wide range of applications including acceptance sampling plans, failure time modeling, and reliability attracted the attention of researchers. The reader can locate applications from Burr (1942) and Ali and Jaheen (2002)[4] as well as G. Srinivasa Rao and colleagues (2015)[6]. The density function of the Burr-XII distribution with two parameters is as follows:

$$f(x;\alpha,\beta) = \alpha\beta \frac{x^{\beta-1}}{(1+x^{\beta})^{\alpha+1}} \qquad x,\alpha,\beta > 0 \qquad \dots\dots\dots(1)$$

and corresponding cumulative distribution function:

$$F(x;\alpha,\beta) = 1 - \left(1 + x^{\beta}\right)^{-\alpha}$$

Many paper proposed Shrunken Technique; Abbas Najim and .et al (2017)[8] A single-stage shrinkage estimator to calculate the form parameter of the Burr X distribution; Al Hemyari and et. Al (2019)[11] applying a two-stage shrinkage testimator with type II censored data to test the reliability function of the exponential failure model; Jebur et. Al (2021)[3] Examine two stress-strength models (series and bounded) in generalized inverse Rayleigh distribution-based systems reliability. Bayesian approaches under informative and non-informative assumptions are applied to produce some estimates of shrinkage estimators, Using Monte Carlo simulations based on the Mean squared Error criterion, the proposed techniques are compared.; Abd Ali and et. al (2022)[5] proposed two stage shrinkun Bayesian estimator for the shape parameter of pareto distribution; In this research, we calculate the double stage shrinkage estimator (Katti's estimator) for the shape parameter of the Burr XII distribution and compare it with the maximum likelihood estimator.

1. **Maximum Likelihood Estimation**

The MLE of α defined by:

$$L(\alpha,\beta)=\prod_{i=1}^{n} f(x_i;\alpha,\beta)$$

$$=\prod_{i=1}^{n}\alpha\beta\frac{x^{\beta-1}}{(1+x^{\beta})^{\alpha+1}}$$

$$=(\alpha\beta)^n\frac{(\prod_{i=1}^{n}x_i)^{\beta-1}}{[\prod_{i=1}^{n}(1+x_i{}^{\beta})]^{\alpha+1}}$$

$$\frac{\partial LnL(\alpha,\beta)}{\partial\alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}Ln(1+x_i{}^{\beta})$$

Then :

$$\hat{\alpha}=\frac{n}{\sum_{i=0}^{n}Ln(1+x_i{}^{\beta})} \quad \dots\dots\dots(2)$$

Since $x\sim$Burr distribution XII then $\ln(1+x^{\beta})$ has Exponential distribution,

then $\frac{1}{\hat{\alpha}}=\sum_{i=1}^{n}\ln(1+x_i{}^{\beta})$ has Gamma distribution G(n, nα),

consequently the distribution of $\hat{\alpha}$ is inverted gamma distribution has the

following P.d.f:

$$f(\hat{\alpha},n)=\frac{(n\alpha)^n\left(\frac{1}{\hat{\alpha}}\right)^{(n+1)}\exp\left(\frac{-n\alpha}{\hat{\alpha}}\right)}{\Gamma(n)} \quad ;\hat{\alpha}>0 \quad \dots\dots\dots(3)$$

2. Shrinkage Estimation

In most cases, the sample unit is very expensive and challenging to obtain or high data acquisition cost, so it is better to use inexpensive estimates. In addition, the researchers proposed a two-stage pooling estimation procedure. Katti [7] is the most outstanding researcher he proposed a A technique to calculate the mean of a normal distribution given the variance and the mean's initial value (μ_0), Shah(1964)[9] using the Katti[7] research method on the variance of a normal distribution in cases where the mean is unknown, as per Arnold and Al-Bayyati (1970)[1][2]) Attempt to allocate a constant k to both μ_0 and

μ_1. such thata A number of k around 0 indicates a strong belief that θ_0 is in close proximity to the genuine mean θ, while a value around 1 indicates that the double stage shrinkage estimator mostly relies on the sample, and they utilized a double stage estimate of the mean by utilizing prior information; where they linked the style of Katti [7] and (Thompson(1968)[10]) to suggest double stage shrinkage estimator.

Now shrunken estimator of $\tilde{\alpha}$ determination as below:

$$\tilde{\alpha} = \begin{cases} k\hat{\alpha} + (1-k)\alpha_0 & if\ \alpha \epsilon C \\ \frac{n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2}{n_1+n_2} & other\ wise \end{cases} \quad \ldots\ldots\ldots (4)$$

The weighted shrinkage constant (k) and the suitable test region (Represented by C) are obviously important factors in determining the estimator.

Where k shrinkage factor (0<k<1) and C is the katti's regions where the $\alpha_0 = \alpha$, Explained in the next section.

3. Choices the regions C

(Katti(1962)[7]) proposed a two-stage estimation scheme the mean θ when the variance $\delta^2$is known and when a prior estimate is supplied in θ_0 the estimator that Katti considers will be denoted by:

$$\tilde{\theta} = \begin{cases} \bar{X}_1, & \hat{\theta}_1 \in R \\ \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}, & \text{otherwise} \end{cases}$$

Therefore, our estimate was obtained based on the above hypothesis:

$$MSE(\tilde{\alpha}/\alpha, C) = \int_C (k(\hat{\alpha}_1 - \alpha_0) + \alpha_0 - \alpha)^2 f(\hat{\alpha}_1) d(\hat{\alpha}_1) + \int_0^\infty \int_0^\infty \left[\frac{n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2}{n_1+n_2} - \alpha\right]^2 f(\hat{\alpha}_1) d(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d(\hat{\alpha}_2) - \int_0^\infty \int_c \left[\frac{n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2}{n_1+n_2} - \alpha\right]^2 f(\hat{\alpha}_1) d(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d(\hat{\alpha}_2)$$

When $\alpha=\alpha_0$,then

$$\frac{\partial MSE(\tilde{\alpha}/\alpha, C)}{\partial C} = \left(k^2 - \frac{n_1^2}{(n_1+n_2)}\right)(\hat{\alpha}_1 - \alpha_0)^2 - \frac{n_2^2}{(n_1+n_2)^2} MSE(\hat{\alpha}_2/\alpha_0) - \frac{2n_1n_2}{(n_1+n_2)^2} Bias(\hat{\alpha}_2/\alpha_0)(\hat{\alpha}_1 - \alpha_0) = 0$$

Simple calculations lead to:

$$C = \{\alpha_0(1-Q), \alpha_0(1+Q)\} \qquad \ldots\ldots\ldots(5)$$

$$where\ Q = \frac{\sqrt{b} + n_1n_2\sqrt{n_2-2}}{(N^2k^2 - n^2{}_1)(n_2-1)(\sqrt{n_2-2})}$$

$$and \quad b = n^2{}_2(N^2k^2 - n_1{}^2)(n_2-1)(n_2{}^2 - 2n_2(n_2-2) + (n_2-1)(n_2-2) + n_1n_2(n_2-2)$$

And the shrinkage constant should be $k \geq \frac{n_1}{N}$,

Therefore the equation was studied in

$$k = \frac{n_1}{N} + 10^{-q} \qquad , and\ q = 1,2,3\ and\ N = n_1 + n_2$$

4. The Mean Square Error (MSE) and BIAS

The Mean Square Error was calculated for estimator $\tilde{\alpha}$

Where $MSE(\tilde{\alpha}) = E(\tilde{\alpha} - \alpha)^2$

$$MSE(\tilde{\alpha}) = \int_0^\infty \int_c [k\hat{\alpha}_1 + (1-k)\hat{\alpha}_0 - \alpha]^2 \, f(\hat{\alpha}_1) d\,(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d\,(\hat{\alpha}_2) + \int_0^\infty \int_{\bar{c}} \left[\frac{n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2}{n_1+n_2} - \alpha\right]^2 f(\hat{\alpha}_1) d\,(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d\,(\hat{\alpha}_2)$$

$$= \int_0^\infty \int_c [k(\hat{\alpha}_1 - \alpha_0) - (\alpha - \alpha_0)]^2 \, f(\hat{\alpha}_1) d\,(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d\,(\hat{\alpha}_2) + \int_0^\infty \int_0^\infty \left[\frac{n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2}{n_1+n_2} - \alpha\right]^2 f(\hat{\alpha}_1) d\,(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d\,(\hat{\alpha}_2) - \int_0^\infty \int_c \left[\frac{n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2}{n_1+n_2} - \alpha\right]^2 f(\hat{\alpha}_1) d\,(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d\,(\hat{\alpha}_2).$$

$$MSE(\tilde{\alpha}) = \alpha^2 \Big[k^2 \left[\frac{n_1^2\alpha^2}{(n_1-1)(n_1-2)\Gamma(n_1-2)} \int_{c^*} x^{n_1-3}\exp(-x)\,dx - \frac{2n_1\alpha^2\gamma}{(n_1-1)\Gamma(n_1-1)} \int_{c^*} x^{n_1-2}\exp(-x)\,dx + \frac{\alpha^2\gamma^2}{\Gamma(n_1)} \int_{c^*} x^{n_1-1}\exp(-x)\,dx\right] - 2k(\alpha - \alpha\gamma)\left[\frac{n_1\alpha}{(n_1-1)\Gamma(n_1-1)} \int_{c^*} x^{n_1-2}\exp(-x)\,dx - \frac{\alpha\gamma}{\Gamma(n_1)} \int_{c^*} x^{n_1-1}\exp(-x)\,dx\right] +$$
$$\frac{(\alpha-\alpha\gamma)^2}{\Gamma(n_1)} \int_{c^*} x^{n_1-1}\exp(-x)\,dx + \frac{n_1^4\alpha^2}{(n_1+n_2)^2\Gamma(n_1)} \int_0^\infty x^{n_1-3}\exp(-x)\,dx - \frac{2n_1^3\alpha^2}{(n_1+n_2)^2\Gamma(n_1)} \int_0^\infty x^{n_1-2}\exp(-x)\,dx + \frac{n_1^2\alpha^2}{(n_1+n_2)^2} + \frac{n_2^2\alpha^2}{(n_1+n_2)^2} +$$
$$\frac{2n_1n_2\alpha^2}{(n_1+n_2)^2}\left[\left(\frac{n_1}{\Gamma(n_1)} \int_0^\infty x^{n_1-2}\exp(-x)\,dx - 1\right)\left(\frac{n_2}{\Gamma(n_2)} \int_0^\infty y^{n_2-2}\exp(-y)\,dy - 1\right)\right] + \frac{n_2^4\alpha^2}{(n_1+n_2)^2\Gamma(n_2)} \int_0^\infty y^{n_2-3}\exp(-y)\,dy - \frac{2n_2^3\alpha^2}{(n_1+n_2)^2\Gamma(n_2)} \int_0^\infty y^{n_2-2}\exp(-y)\,dy -$$
$$\frac{n_1^3\alpha^2}{(n_1+n_2)^2(n_1-1)(n_1-2)\Gamma(n_1-2)} \int_{C^*} x^{n_1-3}\exp(-x)\,dx + \frac{2n_1^3\alpha^2}{(n_1+n_2)^2(n_1-1)\Gamma(n_1-1)} \int_{C^*} x^{n_1-2}\exp(-x)\,dx - \frac{n_1^2\alpha^2}{(n_1+n_2)^2\Gamma(n_1)} \int_{C^*} x^{n_1-1}\exp(-x)\,dx -$$
$$\frac{2n_1n_2\alpha^2}{(n_1+n_2)^2}\left[\left(\frac{n_1}{(n_1-1)\Gamma(n_1-1)} \int_{C^*} x^{n_1-2}\exp(-x)\,dx - \frac{1}{\Gamma(n_1)} \int_{C^*} x^{n_1-1}\exp(-x)\,dx\right)\left(\frac{n_2}{\Gamma(n_2)} \int_0^\infty y^{n_2-2}\exp(-y)\,dy - 1\right)\right] -$$
$$\frac{n_1^2\alpha^2}{(n_1+n_2)^2\Gamma(n_1)} \int_{C^*} x^{n_1-1}\exp(-x)\,dx - \frac{n_2^2\alpha^2}{(n_1+n_2)^2}\left[\left(\frac{1}{\Gamma(n_1)} \int_{C^*} x^{n_1-1}\exp(-x)\,dx\right)\left(\frac{n_2^2}{\Gamma(n_2)} \int_0^\infty y^{n_2-3}\exp(-y)\,dy\right)\right] +$$
$$\frac{2n_2^2\alpha^2}{(n_1+n_2)^2}\left[\left(\frac{1}{\Gamma(n_1)} \int_{C^*} x^{n_1-1}\exp(-x)\,dx\right)\left(\frac{n_2^2}{\Gamma(n_2)} \int_0^\infty y^{n_2-2}\exp(-y)\,dy\right)\right]\Big] \quad \ldots\ldots\ldots(6)$$

Where $x = \frac{n_1}{\hat{\alpha}_1}$, $and\ y = \frac{n_2}{\hat{\alpha}_2}$,

C^* refers to supplemented region of, C and

$$C^* = \left\{\frac{n_1}{\gamma(1+Q)}, \frac{n_1}{\max(0,\gamma(1-Q))}\right\}$$

And the BIAS is define by:

$$Bias(\tilde{\alpha}) = E(\tilde{\alpha}_D) - \alpha$$

Then

$$Bias(\tilde{\alpha}) = \int_0^\infty \int_C [k\hat{\alpha}_1 + (1-k)\hat{\alpha}_0]\ f(\hat{\alpha}_1) d\,(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d\,(\hat{\alpha}_2) + \int_0^\infty \int_{\bar{C}} [\frac{n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2}{n_1+n_2}] f(\hat{\alpha}_1) d\,(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d\,(\hat{\alpha}_2)$$

$$Bias(\tilde{\alpha}) = \int_0^\infty \int_C [k\hat{\alpha}_1 + (1-k)\hat{\alpha}_0]\ f(\hat{\alpha}_1) d\,(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d\,(\hat{\alpha}_2) + \int_0^\infty \int_0^\infty [\frac{n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2}{n_1+n_2}] f(\hat{\alpha}_1) d\,(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d\,(\hat{\alpha}_2) - \int_0^\infty \int_C [\frac{n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2}{n_1+n_2}] f(\hat{\alpha}_1) d\,(\hat{\alpha}_1) f(\hat{\alpha}_2)\, d\,(\hat{\alpha}_2)$$

$$Bias(\tilde{\alpha}) = \frac{kn_1\alpha}{\Gamma(n_1)} \int_{C^*} x^{n_1-2} \exp(-x)\, dx + \frac{(1-k)\alpha_0}{\Gamma(n_1)} \int_{C^*} x^{n_1-1} \exp(-x)\, dx + \frac{n_1^2\,\alpha\Gamma(n_1-1)}{(n_1+n_2)\Gamma(n_1)} + \frac{n_2^2\,\alpha\Gamma(n_2-1)}{(n_1+n_2)\Gamma(n_2)} - \frac{n_1^2\,\alpha}{(n_1+n_2)\Gamma(n_1)} \int_{C^*} x^{n_1-2} \exp(-x)\, dx - \frac{n_2^2\,\alpha\Gamma(n_2-1)}{(n_1+n_2)\Gamma(n_1)\Gamma(n_2)} \int_{C^*} x^{n_1-1} \exp(-x)\, dx - \alpha\,. \quad \ldots\ldots\ldots(7)$$

5. The Relative Efficiency (R.Eff) and Bias Ratio BI.R:

Now we calculate Relative Efficiency defined by:

$$R.Eff(\tilde{\alpha}) = \frac{MSE(\hat{\alpha})}{MSE(\tilde{\alpha})} \qquad \dots\dots\dots (8)$$

The $MSE(\tilde{\alpha})$ is defined in equation 6, and $MSE(\hat{\alpha})$ the mean square error of estimation $(\hat{\alpha})$defined by:

$$MSE(\hat{\alpha}) = \alpha^2 \left\{ \frac{n_1^4}{(n_1+n_2)^2(n_1-1)(n_1-2)} - \frac{2n_1^3}{(n_1+n_2)^2(n_1-1)} - \frac{2n_2^3}{(n_1+n_2)^2(n_2-1)} + \frac{n_1^2}{(n_1+n_2)^2} + \frac{n_2^2}{(n_1+n_2)^2} + \frac{2n_1\,n_2}{(n_1+n_2)^2(n_1-1)(n_2-1)} + \frac{n_2^4}{(n_1+n_2)^2(n_2-1)(n_2-2)} \right\}$$

And the Bias Ratio defined by:

$$BI.R(\tilde{\alpha}) \quad = \frac{Bias(\tilde{\alpha})}{\alpha} \qquad \dots\dots\dots (9)$$

To determine how effective the suggested estimator is and how much biased it is, one must depends on: n_1,n_2, γ, q, We have considered a few constant values: n_1=6,12,18; n_2=6,12,18; γ =0.25:0.25:1.75; q= 1,2,3;

Results

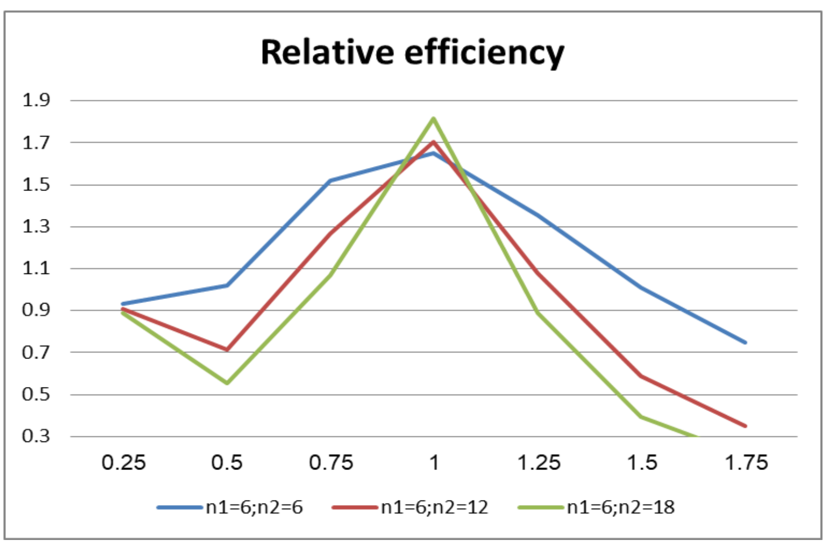

Figure (1) Relative efficiency when q=1

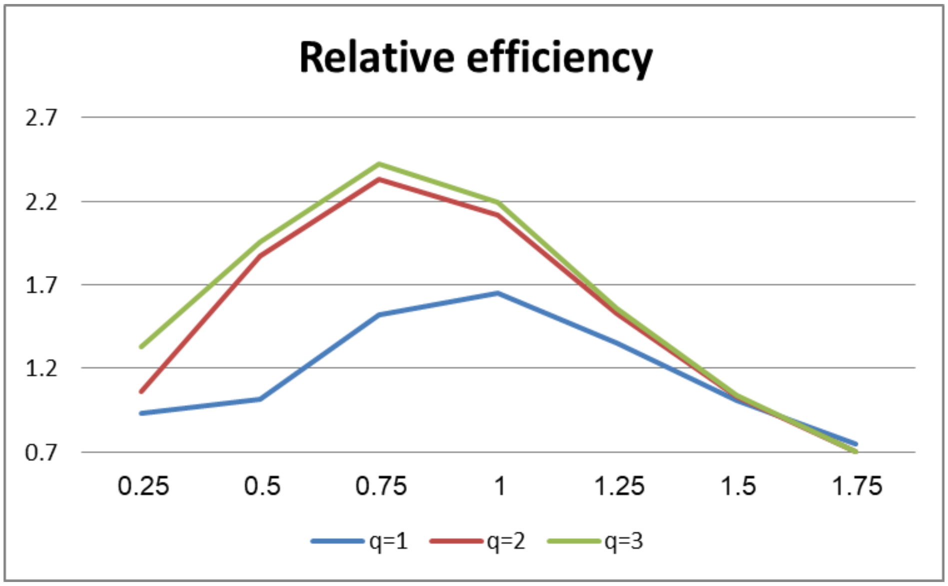

Figure (2) Relative efficiency when n1=n2=6

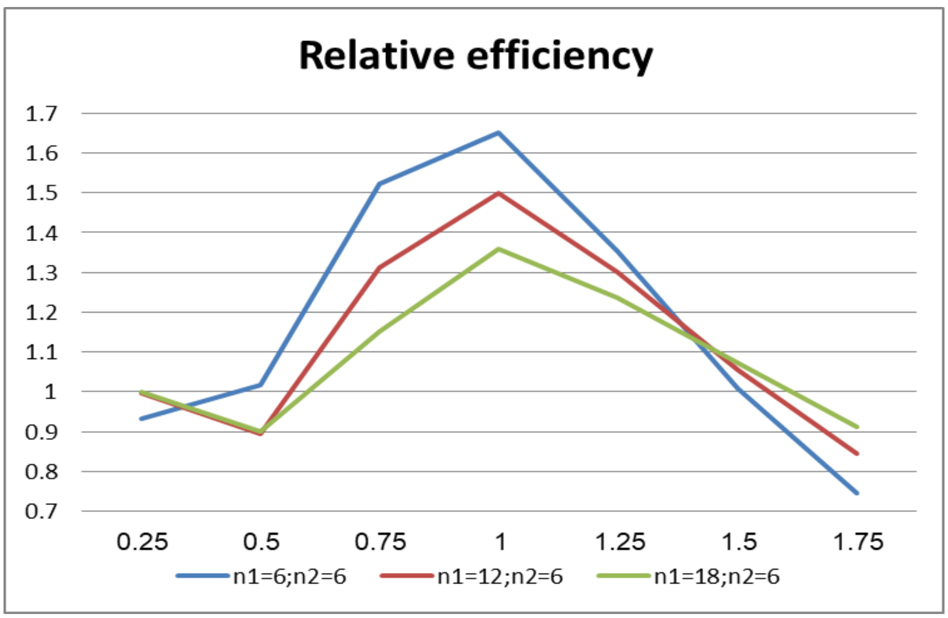

Figure (3) Relative efficiency when q=1

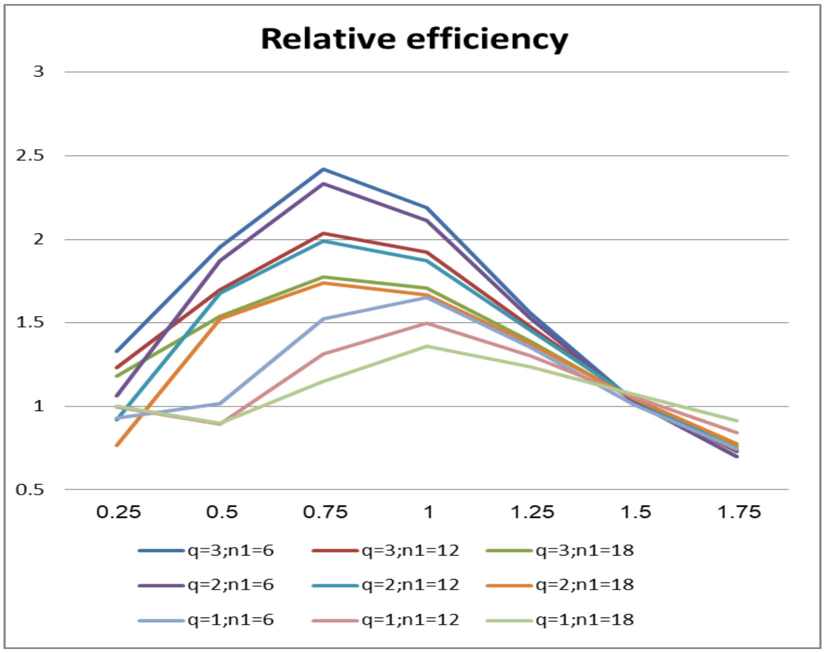

Figure (4) Relative efficiency when n2=6

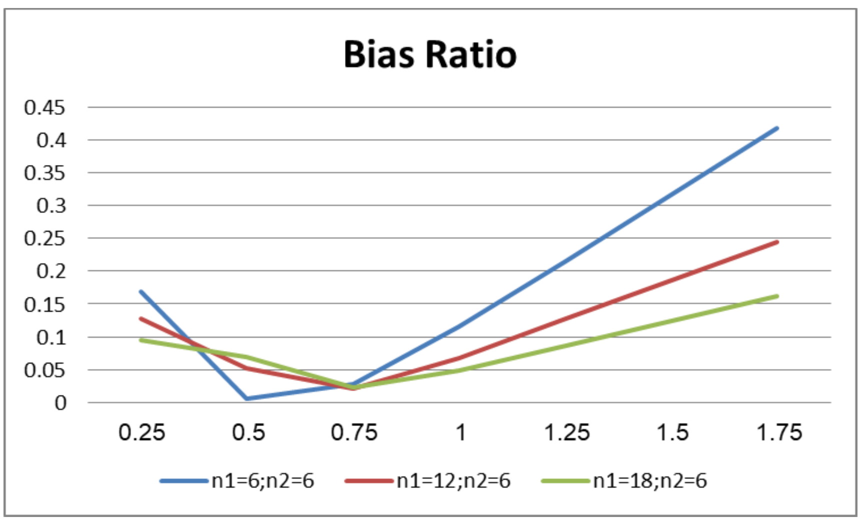

Figure (5) Bias Ratio when q=1

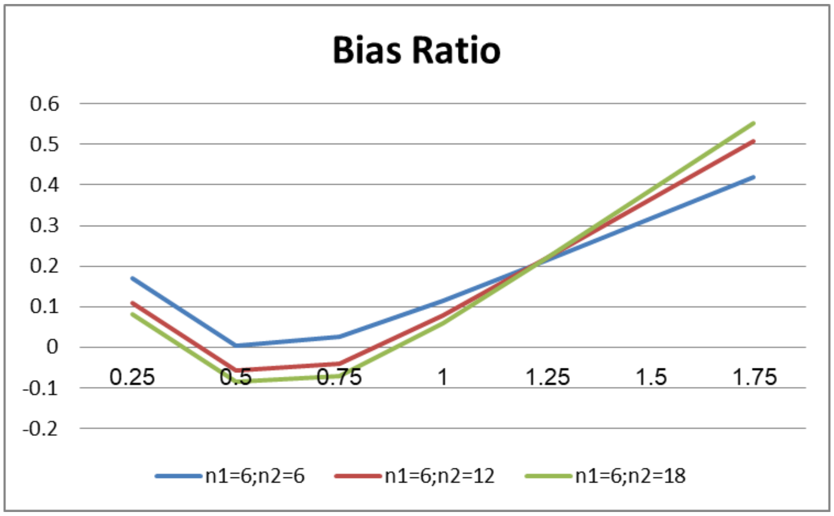

Figure (6) Bias Ratio when q=1

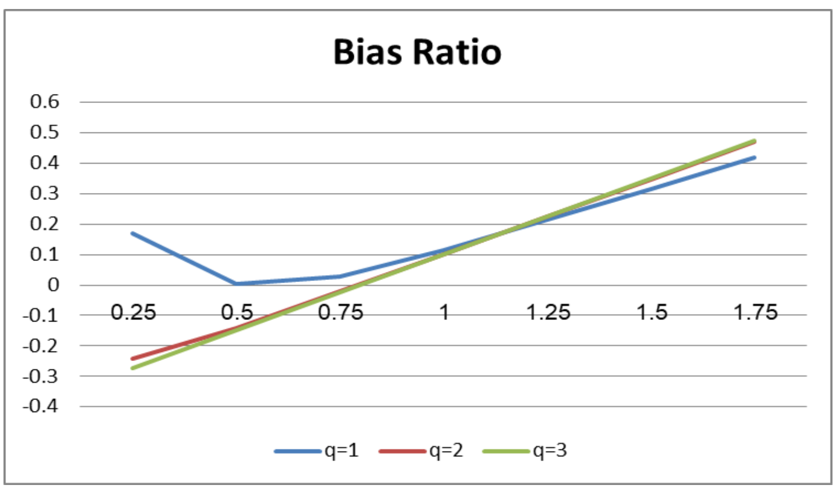

Figure (7) Bias Ratio when n1=n2=6

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(Table 1 : Relative Efficiency)

q	n1	n2	γ = 0.25	0.5	0.75	1	1.25	1.5	1.75
1	6	6	0.9318951	1.0176247	1.5216603	1.6522996	1.3540469	1.0092413	0.7465487
		12	0.9062284	0.714232	1.2666768	1.7023891	1.0794425	0.5857071	0.3475268
		18	0.888531	0.554085	1.068328	1.815967	0.890856	0.390995	0.20926
	12	6	0.998015	0.893438	1.31377	1.498809	1.30169	1.056156	0.844619
		12	0.998472	0.731499	1.125093	1.564749	1.078552	0.643549	0.404829
		18	0.998467	0.627046	0.975656	1.683938	0.907229	0.432018	0.240628
	18	6	0.999992	0.900802	1.151734	1.358038	1.234967	1.073421	0.912356
		12	0.999995	0.821669	1.012625	1.422305	1.067736	0.693543	0.452978
		18	0.999995	0.756821	0.901707	1.529665	0.921051	0.473002	0.276719
2	6	6	1.062967	1.874548	2.330767	2.114566	1.52778	1.027975	0.700542
		12	0.413162	0.935828	2.08305	2.314262	1.130089	0.545609	0.305348
		18	0.243943	0.531895	1.64175	2.619675	0.881023	0.349905	0.180236
	12	6	0.919655	1.67768	1.987965	1.871987	1.459327	1.045921	0.742537
		12	0.408566	0.937983	1.841215	2.047153	1.139575	0.584986	0.335095
		18	0.267256	0.553414	1.531195	2.309039	0.912392	0.378814	0.197457
	18	6	0.763853	1.523525	1.738532	1.666378	1.375944	1.046924	0.778017
		12	0.443319	0.937419	1.653163	1.805574	1.126243	0.622341	0.368919
		18	0.344242	0.577697	1.435798	2.012797	0.928006	0.410174	0.218434
3	6	6	1.329408	1.952926	2.418563	2.191226	1.560695	1.034562	0.697387
		12	0.479879	0.982956	2.175255	2.439884	1.152432	0.545453	0.302418
		18	0.248621	0.55351	1.70684	2.80837	0.895137	0.348544	0.178194
	12	6	1.23E+00	1.70E+00	2.04E+00	1.92E+00	1.48E+00	1.04E+00	7.32E-01
		12	4.91E-01	9.48E-01	1.87E+00	2.12E+00	1.15E+00	5.80E-01	3.29E-01
		18	2.61E-01	5.59E-01	1.55E+00	2.41E+00	9.18E-01	3.74E-01	1.94E-01
	18	6	1.18E+00	1.54E+00	1.78E+00	1.71E+00	1.39E+00	1.04E+00	7.62E-01
		12	5.20E-01	9.45E-01	1.68E+00	1.86E+00	1.13E+00	6.14E-01	3.60E-01
		18	2.82E-01	5.83E-01	1.44E+00	2.09E+00	9.30E-01	4.03E-01	2.13E-01

(Table 2 : Bias Ratio)

q	n1	n2	γ						
			0.25	0.5	0.75	1	1.25	1.5	1.75
1	6	6	0.16919846	0.00535396	0.0269075	0.11537633	0.2154037	0.31674999	0.41783785
		12	0.11045685	-0.0559188	-0.039778	0.0787235	0.2193523	0.36330088	0.50705697
		18	0.08153605	-0.08327937	-0.0718459	0.0600461	0.2206132	0.3859295	0.55116715
	12	6	1.27E-01	0.05167632	0.02007	0.06809875	0.1269675	0.18593319	0.24453205
		12	9.08E-02	0.01476946	-0.0272651	0.0517039	0.1522845	0.25356115	0.35406056
		18	7.16E-02	-0.00628496	-0.0548731	0.04182428	0.1673422	0.29397964	0.41846268
	18	6	0.09411704	0.06908943	0.02248768	0.04889411	0.0870072	0.12492624	0.16203034
		12	0.07165749	0.04536987	-0.0135145	0.03932088	0.1151627	0.18925319	0.25016047
		18	0.05882331	0.03038353	-0.0371908	0.0329899	0.1338113	0.22988959	0.29997067
2	6	6	-0.24329688	-0.14232599	-0.0204577	0.10199986	0.2244973	0.34699805	0.46949878
		12	-0.34844284	-0.25650696	-0.0952841	0.06866263	0.232816	0.39698764	0.56115888
		18	-0.37768262	-0.31169163	-0.1325447	0.05198829	0.2369595	0.42197104	0.60698162
	12	6	-1.52E-01	-0.10000417	-0.0193163	0.06151516	0.1423485	0.22318182	0.30401515
		12	-2.19E-01	-0.19760849	-0.0761199	0.04636363	0.1688635	0.2913636	0.41386363
		18	-2.32E-01	-0.25519465	-0.1101812	0.03727261	0.1847723	0.33227261	0.47977269
	18	6	-0.0781016	-0.0751704	-0.0152937	0.04470588	0.1047059	0.16470588	0.22470588
		12	-0.09766529	-0.15821988	-0.0616131	0.03588235	0.1333823	0.23088235	0.32838235
		18	-0.09515098	-0.21275178	-0.0924863	0.03	0.1525	0.275	0.3975
3	6	6	-0.27404872	-0.14929998	-0.02455	0.1002	0.22495	0.3497	0.47445
		12	-0.43236505	-0.26596642	-0.09955	0.06686667	0.2332833	0.3997	0.56611667
		18	-0.51147642	-0.32429894	-0.1370499	0.0502	0.23745	0.4247	0.61195
	12	6	-0.18855303	-0.1054697	-0.0223864	0.06069697	0.1437803	0.22686364	0.30994697
		12	-0.32870448	-0.20395455	-0.0792045	0.04554545	0.1702955	0.29504545	0.41979545
		18	-0.41279492	-0.26304545	-0.1132955	0.03645455	0.1862045	0.33595455	0.48570455
	18	6	-0.14257353	-0.08032353	-0.0180735	0.04417647	0.1064265	0.16867647	0.23092647
		12	-0.26389706	-0.16414706	-0.0643971	0.03535294	0.1351029	0.23485294	0.33460294
		18	-0.34477939	-0.22002941	-0.0952794	0.02947059	0.1542206	0.27897059	0.40372059

Discussion and Conclusions

i. The Relative efficiency of the double stage estimation $\tilde{\alpha}$ with respected to maximum likelihood estimation $\hat{\alpha}$ has a positive relationship with value of q see Figure (2), (4) and table (1).

ii. The Relative efficiency has increasing value with respect to n_2 when γ=1 see Figure(1) and table (1).

iii. The proposed estimator $\tilde{\alpha}$ has high relative efficiency compared to the maximum likelihood estimator $\hat{\alpha}$, see table (1).

iv. When γ=1 the Relative efficiency takes high value for each n_1, n_2, q, and lessening otherwise ($\gamma \neq 1$), see Figure (1), (2),(3), and (4), and Table (1).

v. The suggested estimator $\tilde{\alpha}$ dominate $\hat{\alpha}$ especially at γ=1.

vi. The absolute value of Bias ratio of the double stage estimation $\tilde{\alpha}$ has almost small when (γ=1) for all n, q and increases else see Figures (5), (6) and (7), and table(2).

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