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Solve of Fractional Telegraph Equation via Yang Decomposition Method

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Abstract

In this study, we introduced a novel scheme to attain approximate and closed-form solutions of fractional telegraph equations, which belong to the most consequential amplitude equations in physics. The Yang transforms (YT) and the Adomian decomposition method (ADM) is combined in the proposed method. We call it the Yang Adomian decomposition method (YDM). Some examples are given to illustrate the accuracy of the numerical results by YDM. As a result, YDM demonstrates that it is a useful and simple mathematical tool for getting approximate and exact analytical solutions to linear-nonlinear fractional telegraph equations (FTEs) of the given kind. The convergence and absolute error analysis of the series solutions is also offered.

Keywords: Yang transform; Adomian decomposition method; Telegraph equation; Caputo fractional operator.

1-Introduction

Fractional calculus, a fast-developing branch of mathematics, is the study of the integrals and derivatives of functions of any order. It has been gaining popularity among scientists working on a range of issues due to the excellent results gained when different tools from this calculus were utilized to simulate specific real-world situations. What makes this calculus interesting to learn is the diversity of fractional operators. The range of fractional operators makes it easy to choose the one that will produce the best results [1].

During recent decades, researchers have been interested in studying fractional calculus and its applications, not only in mathematics but also in many other sciences, such as physics, thermodynamics, engineering, economics, etc. Fractional calculus has many applications in the field of electrical, electrochemistry, statistics, and probability. In addition, fractional differential equations can describe many cosmological phenomena that traditional differential equations cannot describe [2,3]. Various approximation and methodologies, like the fractional Adomian decomposition method (FADM), fractional homotopy method (FHPM), fractional function decomposition method, fractional variational iteration method (FVIM), fractional reduce differential transform method (FRDTM), fractional differential transform method, fractional Laplace variational iteration method, fractional Laplace homotopy perturbation method (FLHPM), fractional Laplace decomposition method (FLDM), fractional Sumudu homotopy analysis method, fractional Sumudu variational iteration method (FVIM), fractional Sumudu decomposition method (FSDM), fractional natural decomposition method (FNDM) [4-60]. In this paper, we apply the Yang decomposition technique to find solution of fractional differential equations with the fractional operator Caputo. The order of the paper is as follows: The basic definitions for calculus and fractional integration are presented in section 2, the method used is analyzed in section 3, many examples are given that explain the effectiveness of the method proposed in section 4, and finally, the conclusion is provided in section 5.

Preliminaries

This section [61-66] goes through some FC definitions and notation that will be used during this period of work.

Definition 2.1. The fractional integral operator of order $\alpha \geq 0$ Riemann Liouville, of $\varphi(\mu) \in C_{\vartheta}, \vartheta \geq -1$ is

$$I^{\alpha}u(t) = \begin{cases} \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}u(t)d\tau, & \alpha > 0, \quad t > 0. \\ u(t), & \alpha = 0 \end{cases}$$

Properties of operator I^{α}:

1. $I^{\alpha}I^{\sigma}u(t) = I^{\alpha+\sigma}u(t)$.
2. $I^{\alpha}I^{\sigma}u(t) = I^{\sigma}I^{\alpha}u(t)$.
3. $I^{\alpha}t^{m} = \frac{\Gamma(m+1)}{\Gamma(\alpha+m+1)}t^{\alpha+m}$.

Definition 2.2. The Caputo fractional derivative of order α of $u(t)$ is

$$D^{\alpha}u(t) = I^{m-\alpha}D^{m}u(t) = \frac{1}{\Gamma(m-\alpha)}\int_0^t (t-\tau)^{m-\alpha-1}u^{(m)}(\tau)d\tau$$

For $m-1 < \alpha < m$, $m \in N$, $t > 0$ and $u \in C_{-1}^{m}$.

The properties D^{α} are:

1. $D^{\alpha}k = 0$, where k is a constant.
2. $D^{\alpha}t^{\sigma} = \frac{\Gamma(\sigma+1)}{\Gamma(\sigma-\alpha+1)} t^{\sigma-\alpha}$,
3. $D^{\alpha}D^{\sigma}u(t) = D^{\alpha+\sigma}u(t)$
4. $I^{\alpha}D^{\alpha}u(t) = u(t) - \sum_{k=0}^{m-1} u^{(k)}(0)\frac{t^k}{k!}$.

Definition 2.3. The MLF with $\alpha > 0$ is

$$E_{\alpha}(z) = \sum_{m=0}^{\infty}\frac{z^{\alpha}}{\Gamma(m\alpha+1)}$$

3. Yang transform

Definition 2.4. The Yang transform of the function is

$$Y\{u(t)\} = \int_{0}^{\infty} e^{-\frac{t}{v}} u(t)dt\,, \;\; t > 0,$$

with v representing the transform variable.

Few properties of YT is stated as.

The YT $Y[f(t)]$ *of the caputo fractional derivative as defined by*

1. $Y\left[{}_{0}^{c}D_{x}^{\alpha} f(t)\right] = Y\left[\frac{f(t)}{v^{\alpha}}\right] - \sum_{k=0}^{n-1} \frac{U^{(k)}(0^{+})}{v^{(\alpha-k-1)}}$

Where $\mathrm{n} - 1 < \alpha < n$

2. $f(v) = a \;\; \rightarrow \;\; Y[f(v)] = av \qquad , \; a \;\; is\ aconstant$

3. $f(v) = v^{\alpha} \;\; \rightarrow \;\; Y[f(v)] = v^{\alpha+1} \;\; \Gamma(\alpha + 1)$

4. If $f(v) = v^{\alpha} \;\; then\ Y^{-1}[f(v)] = \frac{v^{\alpha-1}}{\Gamma(\alpha-1)}$

3. Formulation of Yang decomposition method for fractional Telegraph Equation

We now consider the following and hence illustrate the basic

$${}_{0}^{c}D_{x}^{\alpha}U(x,t) = A(x,t)\partial_{t}^{2}U(x,t) + B(x,t)\partial_{t}U(x,t) + C(x,t)U(x,t) + U^{r}(x,t) + g(x,t), \qquad (1)$$

with the initial condition $U(0,t)\ and\ U_x(o,t)$, $0 < x < a$, $0 < \alpha \leq 2$ and $A(x,t)$, $B(x,t)$, $C(x,t)$

are continues functions and $U^r(x,t)$ is nonlinear function.

Applying the YT to both sides of (1), we have

$$Y\left[\frac{U(x,t)}{v^\alpha}\right] - \sum_{k=0}^{n-1} \frac{U^{(k)}(0^+)}{v^{(\alpha-k-1)}} = Y\begin{bmatrix} A(x,t)\partial_t^2 U(x,t) + B(x,t)\partial_t U(x,t) \\ +C(x,t)U(x,t) + U^r(x,t) + g(x,t)\end{bmatrix}, \qquad (2)$$

Or

$$Y[U(x,t)] = v^\alpha \sum_{k=0}^{n-1} \frac{U^{(k)}(0^+)}{v^{(\alpha-k-1)}} + v^\alpha\, Y[g(x,t)] + v^\alpha\, Y[A(x,t)\partial_t^2 U(x,t) + B(x,t)\partial_t U(x,t) + C(x,t)U(x,t) + U^r(x,t)]. \qquad (3)$$

Hence, applying the inverse YT to the both sides of (3) , we conclude that.

$$Y^{-1}[U(x,t)] = Y^{-1}\left[v^\alpha \sum_{k=0}^{n-1} \frac{U^{(k)}(0^+)}{v^{(\alpha-k-1)}} + v^\alpha\, Y[g(x,t)] + v^\alpha\, Y[A(x,t)\partial_t^2 U(x,t) + B(x,t)\partial_t U(x,t) + C(x,t)U(x,t) + U^r(x,t)]\right]. \qquad (4)$$

$$U(x,t) = Y^{-1}\left[v^{\alpha}\sum_{k=0}^{n-1}\frac{U^{(k)}(0^{+})}{v^{(\alpha-k-1)}} + v^{\alpha}\, Y[g(x,t)] + v^{\alpha}\, Y[A(x,t)\partial_t^2 U(x,t) + B(x,t)\partial_t U(x,t) + C(x,t)U(x,t) + U^{r}(x,t)]\right]. \quad (5)$$

So that

$U(x,t) = \mu(x,t) + Y^{-1}\left[v^{\alpha}\, Y[A(x,t)\partial_t^2 U(x,t) + B(x,t)\partial_t U(x,t) + C(x,t)U(x,t) + U^{r}(x,t)]\right]$, (5)

where

$$\mu(x,t) = Y^{-1}\left[v^{\alpha}\sum_{k=0}^{n-1}\frac{U^{(k)}(0^{+})}{v^{(\alpha-k-1)}} + v^{\alpha}\, Y[g(x,t)]\right]. \quad (6)$$

Now, suppose that

$$U(x,t) = \sum_{n=0}^{\infty} U_n(x,t), \quad (7)$$

$$U^{r}(x,t) = \sum_{n=0}^{\infty} A_n(x,t).$$

Substituting series (7) in (5), we have

$$\sum_{n=0}^{\infty} U_n(x,t) = \mu(x,t) + Y^{-1}\left[v^{\alpha}\, Y[A(x,t)\partial_t^2 U_n(x,t) + B(x,t)\partial_t U_n(x,t) + C(x,t)U_n(x,t) + A_n(x,t)]\right]. \quad (8)$$

For the recursive iteration system, by the computing of both side of (8) , we get the components of the approximation as the of the following respectively .

$$U_0(x,t) = \mu(x,t).$$

$$U_1(x,t) = \left[Y^{-1}\left[v^{\alpha}\, Y[A(x,t)\partial_t^2 U_0(x,t) + B(x,t)\partial_t U_0(x,t) + C(x,t)U_0(x,t) + A_0(x,t)]\right]\right] \quad (9)$$

$$U_2(x,t) = \left[Y^{-1}\left[v^{\alpha}\, Y\begin{bmatrix}A(x,t)\partial_t^2 U_1(x,t) + B(x,t)\partial_t U_1(x,t) + \\ C(x,t)U_1(x,t) + A_1(x,t)\end{bmatrix}\right]\right] \quad (10)$$

$$U_3(x,t) = \left[Y^{-1}\left[v^{\alpha}\, Y[A(x,t)\partial_t^2 U_2(x,t) + B(x,t)\partial_t U_2(x,t) + C(x,t)U_2(x,t) + A_2(x,t)]\right]\right]. \quad (11)$$

$$U_{n+1}(x,t) = \left[Y^{-1}\left[v^{\alpha}\, Y[A(x,t)\partial_t^2 U_n(x,t) + B(x,t)\partial_t U_n(x,t) + C(x,t)U_n(x,t) + A_n(x,t)]\right]\right]. \quad (12)$$

5. Illustrative Examples

Example 5.1. Consider the one-dimensional space FTE

$$ {}_{0}^{c}D_{x}^{\alpha}U(x,t) = D_t^2U(x,t) + 4D_tU(x,t) + 4U(x,t), \quad 0 < x < 1, \quad 0 < \alpha \leq 2, \tag{13}$$

with the initial and boundary conditions

$$U(0,t) = 1 + e^{-2t},$$

$$U_x(0,t) = 2,$$

$$U(x,0) = 1 + e^{2t}$$

$$U_t(x,0) = -2 .$$

Applying the YT on the both side of (13), we have

$$Y[{}_{0}^{c}D_{x}^{\alpha}U(x,t)] - Y[D_t^2U(x,t) + 4D_tU(x,t) + 4U(x,t)] = 0 \tag{14}$$

or

$$\frac{Y[U(x,t)]}{v^{\alpha}} - \sum_{k=0}^{m-1}\frac{U^{(k)}}{v^{\alpha-k-1}} = Y[D_t^2U(x,t) + 4D_tU(x,t) + 4U(x,t)] \tag{15}$$

$$\frac{Y[U(x,t)]}{v^{\alpha}} - \frac{U_{(0)}^{(0)}}{v^{\alpha-0-1}} - \frac{U_0^{(1)}}{v^{\alpha-1-1}} = Y[D_t^2U(x,t) + 4D_tU(x,t) + 4U(x,t)]$$

$$Y[U(x,t)] = v^{\alpha}\left[\frac{1\_e^{-2t}}{v^{\alpha-1}} + \frac{2}{v^{\alpha-2}}\right] + v^{\alpha}Y[D_t^2U(x,t) + 4D_tU(x,t) + 4U(x,t)]$$

$$Y[U(x,t)] = v + ve^{-2t} + 2v^{\alpha} + v^{\alpha}Y[D_t^2U(x,t) + 4D_tU(x,t) + 4U(x,t)]. \tag{16}$$

Applying the invers YT to the both side of (16), we get

$U(x,t) = Y^{-1}[v + ve^{-2t} + 2v^{\alpha}] + Y^{-1}\big[[v^{\alpha}Y[D_t^2 U(x,t) + 4D_t U(x,t) + 4U(x,t)]\big]$ (17).

$$U(x,t) = e^{-2t} + 2x + 1 + Y^{-1}\big[[v^{\alpha}Y[D_t^2 U(x,t) + 4D_t U(x,t) + 4U(x,t)]\big]$$

Then, we have

$$U_0(x,t) = e^{-2t} + 2x + 1 \quad (18)$$

Next , when we use $U_0(x,t)$ to calculate $U_1(x,t)$

$$U_1(x,t) = Y^{-1}[v^{\alpha}Y[D_t^2 U_0(x,t) + 4D_t U_0(x,t) + 4U_0(x,t)]] \quad (19)$$

$$U_1(x,t) = Y^{-1}[v^{\alpha}Y\begin{bmatrix} D_t^2[e^{-2t} + 2x + 1\ \ ] + \\ 4D_t[e^{-2t} + 2x + 1] + 4[e^{-2t} + 2x + 1 \end{bmatrix}] \quad (20)$$

$$U_1(x,t)\ = Y^{-1}[v^{\alpha}Y[8x + 4]] \quad (21)$$

$$U_1(x,t)\ = 4Y^{-1}[2v^{2\alpha+1} + v^{2\alpha}]] \quad (22)$$

$$U_1(x,t) = 4\left[\frac{x^{\alpha}}{\Gamma(\alpha+1)} + \frac{2x^{\alpha+1}}{\Gamma(\alpha+2)}\right] \quad (23)$$

After that using $U_1(x,t)$, we get

$$U_2(x,t) = Y^{-1}[v^{\alpha}Y[D_t^2 U_1(x,t) + 4D_t U_1(x,t) + 4U_1(x,t)]] \quad (24)$$

$$U_2(x,t) = Y^{-1}\left(v^{\alpha}Y\begin{bmatrix} D_t^2(4[\frac{x^{\alpha}}{\Gamma(\alpha+1)} + \frac{2x^{\alpha+1}}{\Gamma(\alpha+2)}]) + \\ 4D_t(4[\frac{x^{\alpha}}{\Gamma(\alpha+1)} + \frac{2x^{\alpha+1}}{\Gamma(\alpha+2)})]] + 4(4[\frac{x^{\alpha}}{\Gamma(\alpha+1)}]) \end{bmatrix}\right)$$

$$U_2(x,t) = 16\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{2x^{2\alpha+1}}{\Gamma(2\alpha+2)}\right]$$

Now Use $U_2(x,t)$ colculus $U_3(x,t)$

$$U_3(x,t) = Y^{-1}[v^{\alpha}Y\begin{bmatrix} D_t^2(U_2(x,t)) \\ +4D_t(U_2(x,t))]] + 4[U_2(x,t)] \end{bmatrix}] \tag{25}$$

$$U_3(x,t) = Y^{-1}[v^{\alpha}Y\begin{bmatrix} 16[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{2x^{2\alpha+1}}{\Gamma(2\alpha+2)}]] - 4[16[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{2x^{2\alpha+1}}{\Gamma(2\alpha+2)}]]] \\ +4[16[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{2x^{2\alpha+1}}{\Gamma(2\alpha+2)}]] \end{bmatrix}$$

$$U_3(x,t) = 64[\frac{x^{3\alpha}}{\Gamma(3\alpha+1)} + \frac{2x^{3\alpha+1}}{\Gamma(3\alpha+2)}]$$

When $\alpha = 2, and\ n = 0,1,2,3, \dots\dots$ we have the solution

$$U(x,t) = e^{2x} + \left[1 + 2\mathrm{x} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \frac{(2x)^5}{5!} + \cdots. + \frac{(2x)^n}{n!}\right] \tag{26}$$

$$U(x,t) = e^{2x} + e^{-2t} \tag{27}$$

Example 5.2. Consider the following space-fractional nonlinear telegraph equation.

$$\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}} = \frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} + u^2(x,t) - e^{2x-4t} + e^{x-2t}, \tag{28}$$

$$t > 0, \qquad 0 < x < 1 \quad , \qquad 0 < \alpha \leq 1 ,$$

with the initial conditions

$$u(0,t) = 0\,, \quad u_x(0,t) = e^x,$$

$$u(x,0)=0 \quad , \quad , \quad u_t(x,0)=-2e^{x}.$$

By applying Yang transform for (17), we have

$$\mathrm{Y}\left[\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}}\right]=\mathrm{Y}\left[\frac{\partial^2 u}{\partial t^2}+2\frac{\partial u}{\partial t}+\ u^2(x,t)-e^{2x-4t}+e^{x-2t}\right] \tag{29 }$$

$$\frac{Y[u(x,t)]}{v^{2\alpha}}-\frac{u(0,t)}{v^{2\alpha-1}}-\frac{u_x(0,t)}{v^{2\alpha-2}}$$
$$=\mathrm{Y}\left[\frac{\partial^2 u}{\partial t^2}+2\frac{\partial u}{\partial t}+\ u^2(x,t)-e^{2x-4t}+e^{x-2t}\right] \tag{30}$$

Arrangement and substitute the initial condition, we get

$$Y[u(x,t)]=v^{2\alpha}\left[\frac{0}{v^{2\alpha-1}}\right]+v^{2\alpha}\left[\frac{e^{x}}{v^{2\alpha-2}}\right]+v^{2\alpha}\mathrm{Y}[-e^{2x-4t}+e^{x-2t}]$$
$$+v^{2\alpha}\mathrm{Y}\left[\frac{\partial^2 u}{\partial t^2}+2\frac{\partial u}{\partial t}+\ u^2(x,t)\right] \tag{31}$$

$$Y[u(x,t)]=v^{2\alpha}e^{x}+v^{2\alpha}\mathrm{Y}[-e^{2x-4t}+e^{x-2t}]$$
$$+v^{2\alpha}\mathrm{Y}\left[\frac{\partial^2 u}{\partial t^2}+2\frac{\partial u}{\partial t}+\ u^2(x,t)\right] \tag{32}$$

Applying the invers Yang transform to both sides of equation (21) we gey

$$u(x,t)=\ Y^{-1}\left[v^{2\alpha}e^{x}+v^{2\alpha}\mathrm{Y}[-e^{2x-4t}+e^{x-2t}]\right]+Y^{-1}\left[v^{2\alpha}\mathrm{Y}\left[\frac{\partial^2 u}{\partial t^2}+2\frac{\partial u}{\partial t}+\right.\right.$$
$$\left.\left.u^2(x,t)\right]\right] \tag{33}$$

Can be write relation (22) in series as follow.

$$\sum_{n=0}^{\infty}u_{n+1}(x,t)=Y^{-1}\left[v^{2\alpha}e^{x}+v^{2\alpha}\mathrm{Y}[-e^{2x-4t}+e^{x-2t}]\right]+$$
$$Y^{-1}\ \left[v^{2\alpha}\mathrm{Y}[\partial^2 t\sum_{n=0}^{\infty}\ u_n(x,t)+2\partial t\sum_{n=0}^{\infty}\ u_n(x,t)+\right.$$
$$\left.\sum_{n=0}^{\infty}A_n(x,t)\ ]\right] \tag{34}$$

Where$A_n(x,t)$ in(34) is nonlinear term with can calculated by adomian polynomial

$$A_n(x,t) = \frac{1}{n!}\frac{\partial}{\partial \mu^n}\left[\left(\sum_{k=0}^{n} \mu U_k(x,t)\right]_{\mu=0}\right. \qquad (35)$$

Hence $u_0(x,t) = e^x$

Using $u_0(x,t)$ to get $u_1(x,t)$ and other respectively , so that

$$u_1(x,t) = \left(\frac{-2e^x t^\alpha}{\sqrt{\alpha+1}}\right), \qquad (36)$$

$$u_2(x,t) = \left(\frac{(-2)^2 e^x t^{2\alpha}}{\sqrt{2\alpha+1}}\right), \qquad (37)$$

$$u_3(x,t) = \left(\left(\frac{(-2)^3 e^x t^{3\alpha}}{\sqrt{3\alpha+1}}\right)\right) \qquad (38)$$

$$u_4(x,t) = \left(\left(\frac{(-2)^4 e^x t^{4\alpha}}{\sqrt{4\alpha+1}}\right)\right) \qquad (39)$$

Therefore, the approximate is

$$U(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots \qquad (40)$$

Then

$$U(x,t) = e^x + \left(\frac{-2e^x t^\alpha}{\sqrt{\alpha+1}}\right) + \left(\frac{(-2)^2 e^x t^{2\alpha}}{\sqrt{2\alpha+1}}\right) + \left(\frac{(-2)^3 e^x t^{3\alpha}}{\sqrt{3\alpha+1}}\right) + \left(\frac{(-2)^4 e^x t^{4\alpha}}{\sqrt{4\alpha+1}}\right) + \cdots \qquad (41)$$

substituting $\alpha = 1$, We obtain the exact solution of standard Telegraph Equation in(41) the following from

$$U(x,t) = e^{x-2t} \qquad (42).$$

6. Conclusions

In conclusion, this article investigates the use of YDM to obtain approximate analytical solutions of telegraph equations. Through a careful comparative analysis between these approximate solutions and exact solutions, supported by 2D and 3D graphs generated using the Maple platform, the analysis sheds light on the accuracy and confidence of the YDM in solving fractional differential equations.

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