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Some results related with neutrosophic hesitant fuzzy primary ideal and semiprimary ideal of ring

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Abstract

In this paper, we study neutrosophic hesitant fuzzy ideal. We introduce the notions of neutrosophic hesitant fuzzy primary ideal, neutrosophic hesitant fuzzy semiprimary ideal of a ring and discussion of some important theorems and results.

Keywords: Neutrosophic hesitant fuzzy ideal of a ring $(\mathrm{NHFI}(\mathrm{Ɣ}))$, Neutrosophic hesitant fuzzy primary ideal of a ring $(\mathrm{NHFYI}(\mathrm{Ɣ}))$, Neutrosophic hesitant fuzzy semiprimary ideal of a ring $(\mathrm{NHFSYI}(\mathrm{Ɣ}))$.

1-Introduction

The theory of fuzzy set was introduced by Zadeh [22] in 1965 as generalized of a set and studied their properties. Torra [19] in (2010) defined the notion of a hesitant fuzzy set (H.F.S) which further characterized an element by a set of membership values by decreasing the loss of information during fuzzification and defined the complement, union and intersection of H.F.Ss. Many authors gave some results of the hesitant fuzzy set as [10], [12], [13], [14], [20], [21]. Mohammed Y. Abbasi, et al. [2] in

(2018) introduced the hesitant fuzzy bi-ideal, hesitant fuzzy left (resp. right and two sided) ideal, and hesitant fuzzy ideal in Γ-semigroup and examined some of their characteristics. A. Abbas and Mohammad [1] in (2021) introduced the notions of H. F ideal of a ring. Many authors gave some results of the hesitant fuzzy ideal of a ring as [8], [9], [18]. R. R. Rasool and M. J. Mohammed [15] in (2022) introduced a new result related with hesitant intuitionistic fuzzy ideal of ring.

The neutrosophic sets (NS) are explored as an important generalization of the intuitionistic set by F. Smarandache [16] in (2005). I. Arockiarani et al. [3] in (2013) introduced the concept of fuzzy neutrosophic soft. In (2018) A .Solairaju, S .Thiruveni [17] introduced the concept of Neutrosophic fuzzy ideals of near-rings. K.Hemabala, B.S.Kumar [7] in (2022) dispensed the theory of neutrosophic multi fuzzy ideals of γ near ring. in (2023) P. A. Parveen, M. H. Begum [11] introduced the new concept of neutrosophic fuzzy (N. F.) bi-ideal of BS-algebras. Durgadevi. P. , Devarasan. E. [5] in (2023) introduced properties of neutrosophic fuzzy (N. F.) ideals in Γ Rings. Some new neutrosophic operations are explored.

In this work, we define a neutrosophic hesitant fuzzy primary ideal of a ring with some results about it. Also, we proved the pre-image of the neutrosophic hesitant fuzzy primary ideal with respect to the homomorphism between two rings. Next, we introduce the concept of neutrosophic hesitant fuzzy semiprimary ideal of a ring and demonstrate new results about it.

2-Neutrosophic hesitant fuzzy primary ideal

Definition (2.1)

If $W = (T_{h_W}, I_{h_W}, F_{h_W})$ be a non-empty neutrosophic hesitant fuzzy (NHF) subset of a ring Ɣ. Then W is said

i) A NHF left ideal (NHFLI, for short) of Ɣ if for all $k, s \in$ Ɣ:

$$1)\begin{cases} T_{h_W}(k-s) \supseteq T_{h_W}(k) \cap T_{h_W}(s). \\ I_{h_W}(k-s) \supseteq I_{h_W}(k) \cap I_{h_W}(s). \\ F_{h_W}(k-s) \subseteq F_{h_W}(k) \cup F_{h_W}(s). \end{cases}$$

$$2)\begin{cases} T_{h_W}(ks) \supseteq T_{h_W}(s). \\ I_{h_W}(ks) \supseteq I_{h_W}(s). \\ F_{h_W}(ks) \subseteq F_{h_W}(s). \end{cases}$$

ii) A NHF right ideal (NHFRI, for short) of Ɣ if for all $k, s \in$ Ɣ :

$$1)\begin{cases} T_{h_W}(k-s) \supseteq T_{h_W}(k) \cap T_{h_W}(s). \\ I_{h_W}(k-s) \supseteq I_{h_W}(k) \cap I_{h_W}(s). \\ F_{h_W}(k-s) \subseteq F_{h_W}(k) \cup F_{h_W}(s). \end{cases}$$

$$2)\begin{cases} T_{h_W}(ks) \supseteq T_{h_W}(k). \\ I_{h_W}(ks) \supseteq I_{h_W}(k). \\ F_{h_W}(ks) \subseteq F_{h_W}(k). \end{cases}$$

iii) A NHF ideal (NHFI, for short) of Ɣ if for all $k, s \in Ɣ$:

$$1)\begin{cases} T_{h_W}(k-s) \supseteq T_{h_W}(k) \cap T_{h_W}(s). \\ I_{h_W}(k-s) \supseteq I_{h_W}(k) \cap I_{h_W}(s). \\ F_{h_W}(k-s) \subseteq F_{h_W}(k) \cup F_{h_W}(s). \end{cases}$$

$$2)\begin{cases} T_{h_W}(ks) \supseteq T_{h_W}(k) \cup T_{h_W}(s). \\ I_{h_W}(ks) \supseteq I_{h_W}(k) \cup I_{h_W}(s). \\ F_{h_W}(ks) \subseteq F_{h_W}(k) \cap F_{h_W}(s). \end{cases}$$

Definition (2.2)[4]

A proper ideal I of a ring Ɣ is said to be primary ideal of Ɣ if, $\forall\, k, s \in Ɣ$ such that $ks \in I$, then either $k \in I$ or $s^n \in I$, for some $n \in N$.

Definition (2.3)

A neutrosophic hesitant fuzzy ideal $W = \langle T_{h_W}, I_{h_W}, F_{h_W} \rangle$ of a ring Ɣ is called to be neutrosophic hesitant fuzzy primary ideal of a ring Ɣ (in short, NHFƔI) if for any $k, s \in Ɣ$, then

$T_{h_W}(ks) = T_{h_W}(k),\ I_{h_W}(ks) = I_{h_W}(k),\ F_{h_W}(ks) = F_{h_W}(k).$

or

$T_{h_W}(ks) \subseteq T_{h_W}(s^n),\ I_{h_W}(ks) \subseteq I_{h_W}(s^n),\ F_{h_W}(ks) \supseteq F_{h_W}(s^n).$

NHFƔI(Ɣ) represents the collection of all NHFƔIs in Ɣ.

Example(2.4)

Let $(Z, +, .)$ be a ring , and $W = \langle T_{h_W}, I_{h_W}, F_{h_W} \rangle$ be a N.H.F subset of Z as follows:

$$T_{h_W}(k) = \begin{cases} (0.1,0.4] & \text{if k is even} \\ (0.1,0.3] & \text{if k is odd} \end{cases}$$

$$I_{h_W}(k) = \begin{cases} [0.2,0.7) & \text{if k is even} \\ (0.2,0.5) & \text{if k is odd} \end{cases}$$

$$F_{h_W}(k) = \begin{cases} [0.3,0.5) & \text{if k is even} \\ (0.2,0.6) & \text{if k is odd} \end{cases}$$

Then, we can easily prove that $W \in$ NHFƔI(Z).

Theorem (2.5)

Let Ɣ be a ring and $V, W \in$ NHFƔI(Ɣ), where $V = \langle T_{h_V}, I_{h_V}, F_{h_V} \rangle$, $W = \langle T_{h_W}, I_{h_W}, F_{h_W} \rangle$. Then $V \cap W \in$

NHFYI(Ɣ).

Proof

Suppose that $V, W \in$ NHFYI(Ɣ), and $a, b \in$ Ɣ

Clearly, $V \cap W \in$ NHFI(Ɣ),

$V \cap W = \langle T_{h_V}, I_{h_V}, F_{h_V} \rangle \cap \langle T_{h_W}, I_{h_W}, F_{h_W} \rangle = \langle T_{h_V} \cap T_{h_W}, I_{h_V} \cap I_{h_W}, F_{h_V} \cup F_{h_W} \rangle$

$$\left(T_{h_V} \cap T_{h_W}\right)(ab) = \cup_{\left[y_1 \in T_{h_V}(ab)\right],\left[y_2 \in T_{h_W}(ab)\right]} min\{y_1, y_2\}$$

$$= \cup_{\left[y_1 \in T_{h_V}(a)\right],\left[y_2 \in T_{h_W}(a)\right]} min\{y_1, y_2\} = \left(T_{h_V} \cap T_{h_W}\right)(a).$$

Then $\left(T_{h_V} \cap T_{h_W}\right)(ab) = \left(T_{h_V} \cap T_{h_W}\right)(a)$ (1)

Similarly,

$\left(I_{h_V} \cap I_{h_W}\right)(ab) = \left(I_{h_V} \cap I_{h_W}\right)(a)$ (2)

$\left(F_{h_V} \cup F_{h_W}\right)(ab) = \left(F_{h_V} \cup F_{h_W}\right)(a)$ (3)

From (1), (2) and (3), we get $V \cap W \in$ NHFYI(Ɣ).

or

$$\left(T_{h_V} \cap T_{h_W}\right)(ab) = \cup_{\left[y_1 \in T_{h_V}(ab)\right],\left[y_2 \in T_{h_W}(ab)\right]} min\{y_1, y_2\}$$

$$\subseteq \cup_{\left[y_1 \in T_{h_V}(b^n)\right],\left[y_2 \in T_{h_W}(b^n)\right]} min\{y_1, y_2\} = \left(T_{h_V} \cap T_{h_W}\right)(b^n), \text{ for some } n \in N.$$

Then $\left(T_{h_V} \cap T_{h_W}\right)(ab) \subseteq \left(T_{h_V} \cap T_{h_W}\right)(b^n)$ (4)

Similarly,

$\left(I_{h_V} \cap I_{h_W}\right)(ab) \subseteq \left(I_{h_V} \cap I_{h_W}\right)(b^n)$ (5)

$\left(F_{h_V} \cup F_{h_W}\right)(ab) \supseteq \left(F_{h_V} \cup F_{h_W}\right)(b^n)$ (6)

From (4), (5) and (6), we get $V \cap W \in$ NHFYI(Ɣ).

Proposition (2.6)

Let $\{W_i, i \in I\}$ be a family of a NHFYI(Ɣ), then $(\cap_{i \in I} W_i)$ is a NHFYI(Ɣ).

Proof

Similarly to prove theorem (2.5).

Theorem (2.7)

Let Ɣ be a ring and $V, W \in$ NHFYI(Ɣ), where $V = \langle T_{h_V}, I_{h_V}, F_{h_V} \rangle$, $W = \langle T_{h_W}, I_{h_W}, F_{h_W} \rangle$. Then $V \cup W \in$ NHFYI(Ɣ).

Proof

Suppose that $V, W \in NHFYI(Ɣ)$, and $a, b \in Ɣ$.

Clearly, $V \cup W \in NHFI(Ɣ)$,

$V \cup W = \langle T_{h_V}, I_{h_V}, F_{h_V} \rangle \cup \langle T_{h_W}, I_{h_W}, F_{h_W} \rangle = \langle T_{h_V} \cup T_{h_W}, I_{h_V} \cup I_{h_W}, F_{h_V} \cap F_{h_W} \rangle$

$$\left(I_{h_V} \cup I_{h_W}\right)(ab) = \cup_{\left[y_1 \in I_{h_V}(ab)\right],\left[y_2 \in I_{h_W}(ab)\right]} max\{y_1, y_2\}$$

$$= \cup_{\left[y_1 \in I_{h_V}(a)\right],\left[y_2 \in I_{h_W}(a)\right]} max\{y_1, y_2\} = \left(I_{h_V} \cup I_{h_W}\right)(a)$$

Then $\left(I_{h_V} \cup I_{h_W}\right)(ab) = \left(I_{h_V} \cup I_{h_W}\right)(a)$ (7)

Similarly,

$\left(T_{h_V} \cup T_{h_W}\right)(ab) = \left(T_{h_V} \cup T_{h_W}\right)(a)$ (8)

$\left(F_{h_V} \cap F_{h_W}\right)(ab) = \left(F_{h_V} \cap F_{h_W}\right)(a)$ (9)

From (7), (8) and (9), we get $V \cup W \in NHFYI(Ɣ)$.

or

$$\left(I_{h_V} \cup I_{h_W}\right)(ab) = \cup_{\left[y_1 \in I_{h_V}(ab)\right],\left[y_2 \in I_{h_W}(ab)\right]} max\{y_1, y_2\}$$

$$\subseteq \cup_{\left[y_1 \in I_{h_V}(b^n)\right],\left[y_2 \in I_{h_W}(b^n)\right]} max\{y_1, y_2\} = \left(I_{h_V} \cup I_{h_W}\right)(b^n), \text{ for some } n \in N.$$

Then $\left(I_{h_V} \cup I_{h_W}\right)(ab) \subseteq \left(I_{h_V} \cup I_{h_W}\right)(b^n)$ (10)

Similarly,

$\left(T_{h_V} \cup T_{h_W}\right)(ab) \subseteq \left(T_{h_V} \cup T_{h_W}\right)(b^n)$ (11)

$\left(F_{h_V} \cap F_{h_W}\right)(ab) \supseteq \left(F_{h_V} \cap F_{h_W}\right)(b^n)$ (12)

From (10), (11) and (12), we get $V \cup W \in NHFYI(Ɣ)$.

Proposition (2.8)

Let $\{W_i , i \in I\}$ be a family of a NHFYI(Ɣ), then $(\cup_{i \in I} W_i)$ is a NHFYI(Ɣ).

Proof

Similarly to prove theorem (2.7).

Definition (2.9)

Let X be a non-empty set and W be a NHF set in X, then the $(\delta, \vartheta, \gamma)$-level set defined by

$W_{H_{(\delta,\vartheta,\gamma)}} = \left\{x \in X : T_{h_W} \supseteq \delta , I_{h_W} \supseteq \vartheta , F_{h_W} \subseteq \gamma\right\}$ where $\delta, \vartheta, \gamma \subseteq [0,1]$ with $\delta^+ + \vartheta^+ + \gamma^- \leq 3$.

Theorem (2.10)

Let $A = \langle T_{h_A}, I_{h_A}, F_{h_A} \rangle$ be NHFYI of a ring Ɣ, then $A_{H_{(\delta,\vartheta,\gamma)}}$ is primary ideal of Ɣ if $T_{h_A}(0) \supseteq \delta$, $I_{h_A}(0) \supseteq \vartheta$, $F_{h_A}(0) \subseteq \gamma$.

Proof

Let $T_{h_A}(0) \supseteq \delta$, $I_{h_A}(0) \supseteq \vartheta$, $F_{h_A}(0) \subseteq \gamma$, then $A_{H_{(\delta,\vartheta,\gamma)}} \neq \emptyset$

Let $a, b \in$ Ɣ, $ab \in A_{H_{(\delta,\vartheta,\gamma)}}$.

Since $ab \in A_{H_{(\delta,\vartheta,\gamma)}}$ implies $T_{h_A}(ab) \supseteq \delta$, $I_{h_A}(ab) \supseteq \vartheta$, $F_{h_A}(ab) \subseteq \gamma$.

Since A be NHFYI of a ring Ɣ, then

$T_{h_A}(ab) = T_{h_A}(a) \supseteq \delta$, $I_{h_A}(ab) = I_{h_A}(a) \supseteq \vartheta$ and $F_{h_A}(ab) = F_{h_A}(a) \subseteq \gamma$.

Implies $T_{h_A}(a) \supseteq \delta$, $I_{h_A}(a) \supseteq \vartheta$ and $F_{h_A}(a) \subseteq \gamma$ then $a \in A_{H_{(\delta,\vartheta,\gamma)}}$.

If

$T_{h_A}(b^n) \supseteq T_{h_A}(ab) \supseteq \delta$, $I_{h_A}(b^n) \supseteq I_{h_A}(ab) \supseteq \vartheta$ and $F_{h_A}(b^n) \subseteq F_{h_A}(ab) \subseteq \gamma$.

Implies $T_{h_A}(b^n) \supseteq \delta$, $I_{h_A}(b^n) \supseteq \vartheta$ and $F_{h_A}(b^n) \subseteq \gamma$ then $b^n \in A_{H_{(\delta,\vartheta,\gamma)}}$, for some $n \in N$.

Then $\forall\, a, b \in$ Ɣ, $ab \in A_{H_{(\delta,\vartheta,\gamma)}}$ imply $a \in A_{H_{(\delta,\vartheta,\gamma)}}$ or $b^n \in A_{H_{(\delta,\vartheta,\gamma)}}$.

Hence $A_{H_{(\delta,\vartheta,\gamma)}}$ is primary ideal of Ɣ.

Theorem (2.11)

Let A be a constant then A ∈ NHFYI(Ɣ).

Proof

Let $A = \{\langle k, T_{h_A}(k), I_{h_A}(k), F_{h_A}(k) \rangle : k \in$ Ɣ $\}$, where $T_{h_A}(k) = \delta$, $I_{h_A}(k) = \vartheta$, $F_{h_A}(k) = \gamma$, $\delta, \vartheta, \gamma \subseteq [0,1]$ and let $a, b \in$ Ɣ, then $ab \in$ Ɣ.

Clearly, A ∈ NHFI(Ɣ)

$T_{h_A}(ab) = \delta = T_{h_A}(a)$, then $T_{h_A}(ab) = T_{h_A}(a)$;

$I_{h_A}(ab) = \vartheta = I_{h_A}(a)$, then $I_{h_A}(ab) = I_{h_A}(a)$;

$F_{h_A}(ab) = \gamma = F_{h_A}(a)$, then $F_{h_A}(ab) = F_{h_A}(a)$.

Then A ∈ NHFYI(Ɣ).

or

Since b ∈ Ɣ, then $b^n \in$ Ɣ, for some $n \in N$, imply $T_{h_A}(b^n) = \delta$, $I_{h_A}(b^n) = \vartheta$, $F_{h_A}(b^n) = \gamma$;

$T_{h_A}(ab) = \delta = T_{h_A}(b^n)$, then $T_{h_A}(ab) \subseteq T_{h_A}(b^n)$;

$I_{h_A}(ab) = \vartheta = I_{h_A}(b^n)$, then $I_{h_A}(ab) \subseteq I_{h_A}(b^n)$;

$F_{h_A}(ab) = \gamma = F_{h_A}(b^n)$, then $F_{h_A}(ab) \supseteq F_{h_A}(b^n)$.

Then $A \in NHFYI(Ɣ)$.

Proposition (2.12)

1) Let $A^{\emptyset}$ is a neutrosophic hesitant fuzzy empty set, then $A^{\emptyset} \in NHFYI(Ɣ)$.

2) Let $A^{[0,1]}$ is a neutrosophic hesitant fuzzy complete set, then $A^{[0,1]} \in NHFYI(Ɣ)$.

Proof

Similarly to prove theorem (2.11).

Theorem (2.13)

Let $g: M \to M^*$ be an onto homomorphism of a rings

1) If $A \in NHFYI(M^*)$, then $g^{-1}(A) \in NHFYI(M)$.

2) If $A \in NHFYI(M)$, then $g(A) \in NHFYI(M^*)$.

Proof (1).

Suppose $g: M \to M^*$ be a homomorphism from a ring M into a ring M^*.

Let $A \in NHFYI(M^*)$, and $a, b \in M$.

Clearly, $g^{-1}(A) \in NHFI(M)$,

$$g^{-1}(T_{h_A})(ab) = T_{h_A}(g(ab)) = T_{h_A}(g(a)g(b)) = T_{h_A}(g(a)) = g^{-1}(T_{h_A})(a).$$

Then $g^{-1}(T_{h_A})(ab) = g^{-1}(T_{h_A})(a)$ (13)

Similarly,

$$g^{-1}(I_{h_A})(ab) = g^{-1}(I_{h_A})(a) \quad (14)$$

$$g^{-1}(F_{h_A})(ab) = g^{-1}(F_{h_A})(a) \quad (15)$$

Thus, from (13), (14) and (15), we get $g^{-1}(A) \in NHFYI(M)$.

or

$$g^{-1}(T_{h_A})(ab) = T_{h_A}(g(ab)) = T_{h_A}(g(a)g(b))$$
$$\subseteq T_{h_A}(g(b))^n$$
$$= T_{h_A}(g(b^n)) = g^{-1}(T_{h_A})(b^n), \text{ for some } n \in N.$$

Then $g^{-1}(T_{h_A})(ab) \subseteq g^{-1}(T_{h_A})(b^n)$ (16)

Similarly,

$$g^{-1}(I_{h_A})(ab) \subseteq g^{-1}(I_{h_A})(b^n) \quad (17)$$

$$g^{-1}(F_{h_A})(ab) \supseteq g^{-1}(F_{h_A})(b^n) \quad (18)$$

Thus, from (16), (17) and (18), we get $g^{-1}(A) \in NHFYI(M)$.

Proof (2)

Similarly to proof (1).

3-Neutrosophic hesitant fuzzy semiprimary ideal

Definition (3.1)[6]

Let I be an ideal of a ring Ɣ then I is called to be semiprimary ideal of Ɣ if, $\forall$ k, s $\in$ Ɣ, ks $\in$ I, implies that either a power of **k** or a power of **s** belongs to I.

Definition (3.2)

A neutrosophic hesitant fuzzy ideal $A = \langle T_{h_A}, I_{h_A}, F_{h_A} \rangle$ of a ring Ɣ is called neutrosophic hesitant fuzzy semiprimary ideal of a ring Ɣ (in short, NHFSYI) if for any k, s $\in$ Ɣ,

Either $T_{h_A}(ks) \subseteq T_{h_A}(k^n)$, $I_{h_A}(ks) \subseteq I_{h_A}(k^n)$, $F_{h_A}(ks) \supseteq F_{h_A}(k^n)$.

or $T_{h_A}(ks) \subseteq T_{h_A}(s^m)$, $I_{h_A}(ks) \subseteq I_{h_A}(s^m)$, $F_{h_A}(ks) \supseteq F_{h_A}(s^m)$, for some n, m $\in$ N.

NHFSYI(Ɣ) represents the collection of all NHFSYIs in Ɣ.

Example (3.3)

Let (Z, +, .) be a ring , and $V = \langle T_{h_V}, I_{h_V}, F_{h_V} \rangle$ be a N.H.F subset of Z as follows:

$$T_{h_V}(k) = \begin{cases} (0.1,0.4] & \text{if k is even} \\ (0.1,0.3] & \text{if k is odd} \end{cases}$$

$$I_{h_V}(k) = \begin{cases} [0.2,0.7) & \text{if k is even} \\ (0.2,0.5) & \text{if k is odd} \end{cases}$$

$$F_{h_V}(k) = \begin{cases} [0.3,0.5) & \text{if k is even} \\ (0.2,0.6) & \text{if k is odd} \end{cases}$$

Then, we can easily prove that V $\in$ NHFSYI(Z).

Theorem (3.4)

Let Ɣ be a ring and V, W $\in$ NHFSYI(Ɣ), where $V = \langle T_{h_V}, I_{h_V}, F_{h_V} \rangle$, $W = \langle T_{h_W}, I_{h_W}, F_{h_W} \rangle$. Then $V \cap W \in$ NHFSYI(Ɣ).

Proof.

Suppose that V, W $\in$ NHFSYI(Ɣ), and a, b $\in$ Ɣ, then for some n, m $\in$ N.

Clearly, $V \cap W \in$ NHFI(Ɣ),

$V \cap W = \langle T_{h_V}, I_{h_V}, F_{h_V} \rangle \cap \langle T_{h_W}, I_{h_W}, F_{h_W} \rangle = \langle T_{h_V} \cap T_{h_W}, I_{h_V} \cap I_{h_W}, F_{h_V} \cup F_{h_W} \rangle$

$(F_{h_V} \cup F_{h_W})(ab) = \cup_{[y_1 \in F_{h_V}(ab)],[y_2 \in F_{h_W}(ab)]} max\{y_1, y_2\}$

$$\supseteq \cup_{[y_1 \in F_{h_V}(a^n)],[y_2 \in F_{h_W}(a^n)]} max\{y_1, y_2\} = (F_{h_V} \cup F_{h_W})(a^n)$$

Then $(F_{h_V} \cup F_{h_W})(ab) \supseteq (F_{h_V} \cup F_{h_W})(a^n)$ (19)

Similarly,

$(T_{h_V} \cap T_{h_W})(ab) \subseteq (T_{h_V} \cap T_{h_W})(a^n)$ (20)

$(I_{h_V} \cap I_{h_W})(ab) \subseteq (I_{h_V} \cap I_{h_W})(a^n)$ (21)

From (19), (20) and (21), we get $V \cap W \in NHFSYI(Ɣ)$.

or

$$(F_{h_V} \cup F_{h_W})(ab) = \cup_{[y_1 \in F_{h_V}(ab)],[y_2 \in F_{h_W}(ab)]} max\{y_1, y_2\}$$

$$\supseteq \cup_{[y_1 \in F_{h_V}(b^m)],[y_2 \in F_{h_W}(b^m)]} max\{y_1, y_2\} = (F_{h_V} \cup F_{h_W})(b^m)$$

Then $(F_{h_V} \cup F_{h_W})(ab) \supseteq (F_{h_V} \cup F_{h_W})(b^m)$ (22)

Similarly,

$(T_{h_V} \cap T_{h_W})(ab) \subseteq (T_{h_V} \cap T_{h_W})(b^m)$ (23)

$(I_{h_V} \cap I_{h_W})(ab) \subseteq (I_{h_V} \cap I_{h_W})(b^m)$ (24)

From (22), (23) and (24), we get $V \cap W \in NHFSYI(Ɣ)$.

Proposition (3.5)

Let $\{W_i, i \in I\}$ be a family of a NHFSYI(Ɣ), then $(\cap_{i \in I} W_i)$ is a NHFSYI(Ɣ).

Proof

Similarly to prove theorem (3.4).

Theorem (3.6)

Let Ɣ be a ring and $V, W \in NHFSYI(Ɣ)$, where $V = \langle T_{h_V}, I_{h_V}, F_{h_V} \rangle$, $W = \langle T_{h_W}, I_{h_W}, F_{h_W} \rangle$. Then $V \cup W \in NHFSYI(Ɣ)$.

Proof

Suppose that $V, W \in NHFSYI(Ɣ)$, and $a, b \in Ɣ$, then for some $n, m \in N$.

Clearly, $V \cup W \in NHFI(Ɣ)$,

$V \cup W = \langle T_{h_V}, I_{h_V}, F_{h_V} \rangle \cup \langle T_{h_W}, I_{h_W}, F_{h_W} \rangle = \langle T_{h_V} \cup T_{h_W}, I_{h_V} \cup I_{h_W}, F_{h_V} \cap F_{h_W} \rangle$

$$(T_{h_V} \cup T_{h_W})(ab) = \cup_{[y_1 \in T_{h_V}(ab)],[y_2 \in T_{h_W}(ab)]} max\{y_1, y_2\}$$

$$\subseteq \cup_{[y_1 \in T_{h_V}(a^n)],[y_2 \in T_{h_W}(a^n)]} max\{y_1, y_2\} = (T_{h_V} \cup T_{h_W})(a^n)$$

Then $(T_{h_V} \cup T_{h_W})(ab) \subseteq (T_{h_V} \cup T_{h_W})(a^n)$ (25)

Similarly,

$\left(\mathrm{I}_{h_V} \cup \mathrm{I}_{h_W}\right)(\mathrm{ab}) \subseteq \left(\mathrm{I}_{h_V} \cup \mathrm{I}_{h_W}\right)(\mathrm{a^n})$ (26)

$\left(\mathrm{F}_{h_V} \cap \mathrm{F}_{h_W}\right)(\mathrm{ab}) \supseteq \left(\mathrm{F}_{h_V} \cap \mathrm{F}_{h_W}\right)(\mathrm{a^n})$ (27)

From (25), (26) and (27), we get $\mathrm{V} \cup \mathrm{W} \in$ NHFSYI(Ɣ).

or

$$\left(\mathrm{T}_{h_V} \cup \mathrm{T}_{h_W}\right)(\mathrm{ab}) = \cup_{[y_1 \in \mathrm{T}_{h_V}(\mathrm{ab})],[y_2 \in \mathrm{T}_{h_W}(\mathrm{ab})]} max\{y_1, y_2\}$$

$$\subseteq \cup_{[y_1 \in \mathrm{T}_{h_V}(\mathrm{b^m})],[y_2 \in \mathrm{T}_{h_W}(\mathrm{b^m})]} max\{y_1, y_2\} = \left(\mathrm{T}_{h_V} \cup \mathrm{T}_{h_W}\right)(\mathrm{b^m})$$

Then $\left(\mathrm{T}_{h_V} \cup \mathrm{T}_{h_W}\right)(\mathrm{ab}) \subseteq \left(\mathrm{T}_{h_V} \cup \mathrm{T}_{h_W}\right)(\mathrm{b^m})$ (28)

Similarly,

$\left(\mathrm{I}_{h_V} \cup \mathrm{I}_{h_W}\right)(\mathrm{ab}) \subseteq \left(\mathrm{I}_{h_V} \cup \mathrm{I}_{h_W}\right)(\mathrm{b^m})$ (29)

$\left(\mathrm{F}_{h_V} \cap \mathrm{F}_{h_W}\right)(\mathrm{ab}) \supseteq \left(\mathrm{F}_{h_V} \cap \mathrm{F}_{h_W}\right)(\mathrm{b^m})$ (30)

From (28), (29) and (30), we get $\mathrm{V} \cup \mathrm{W} \in$ NHFSYI(Ɣ).

Proposition (3.7)

Let $\{\mathrm{W}_i, i \in I\}$ be a family of a NHFSYI(Ɣ), then $(\cup_{i \in I} \mathrm{W}_i)$ is a NHFSYI(Ɣ).

Proof

Similarly to prove theorem (3.6).

Theorem (3.8)

Let $\mathrm{A} = \langle \mathrm{T}_{h_A}, \mathrm{I}_{h_A}, \mathrm{F}_{h_A} \rangle$ be NHFSYI(Ɣ), then $\mathrm{A}_{\mathrm{H}_{(\delta,\vartheta,\gamma)}}$ is semiprimary ideal of Ɣ if $\mathrm{T}_{h_A}(0) \supseteq \delta$, $\mathrm{I}_{h_A}(0) \supseteq \vartheta$, $\mathrm{F}_{h_A}(0) \subseteq \gamma$.

Proof

Let $\mathrm{T}_{h_A}(0) \supseteq \delta$, $\mathrm{I}_{h_A}(0) \supseteq \vartheta$, $\mathrm{F}_{h_A}(0) \subseteq \gamma$, then $\mathrm{A}_{\mathrm{H}_{(\delta,\vartheta,\gamma)}} \neq \emptyset$

Let a, b ∈ Ɣ, $\mathrm{ab} \in \mathrm{A}_{\mathrm{H}_{(\delta,\vartheta,\gamma)}}$

Since $\mathrm{ab} \in \mathrm{A}_{\mathrm{H}_{(\delta,\vartheta,\gamma)}}$ implies $\mathrm{T}_{h_A}(\mathrm{ab}) \supseteq \delta$, $\mathrm{I}_{h_A}(\mathrm{ab}) \supseteq \vartheta$, $\mathrm{F}_{h_A}(\mathrm{ab}) \subseteq \gamma$.

Since A be NHFSYI(Ɣ), then

$\mathrm{T}_{h_A}(\mathrm{a^n}) \supseteq \mathrm{T}_{h_A}(\mathrm{ab}) \supseteq \delta$, $\mathrm{I}_{h_A}(\mathrm{a^n}) \supseteq \mathrm{I}_{h_A}(\mathrm{ab}) \supseteq \vartheta$ and $\mathrm{F}_{h_A}(\mathrm{a^n}) \subseteq \mathrm{F}_{h_A}(\mathrm{ab}) \subseteq \gamma$.

Implies $\mathrm{T}_{h_A}(\mathrm{a^n}) \supseteq \delta$, $\mathrm{I}_{h_A}(\mathrm{a^n}) \supseteq \vartheta$ and $\mathrm{F}_{h_A}(\mathrm{a^n}) \subseteq \gamma$ then $\mathrm{a^n} \in \mathrm{A}_{\mathrm{H}_{(\delta,\vartheta,\gamma)}}$, for some n ∈ N.

If

$\mathrm{T}_{h_A}(\mathrm{b^m}) \supseteq \mathrm{T}_{h_A}(\mathrm{ab}) \supseteq \delta$, $\mathrm{I}_{h_A}(\mathrm{b^m}) \supseteq \mathrm{I}_{h_A}(\mathrm{ab}) \supseteq \vartheta$ and $\mathrm{F}_{h_A}(\mathrm{b^m}) \subseteq \mathrm{F}_{h_A}(\mathrm{ab}) \subseteq \gamma$

Implies $\mathrm{T}_{h_A}(\mathrm{b^m}) \supseteq \delta$, $\mathrm{I}_{h_A}(\mathrm{b^m}) \supseteq \vartheta$ and $\mathrm{F}_{h_A}(\mathrm{b^m}) \subseteq \gamma$ then $\mathrm{b^m} \in \mathrm{A}_{\mathrm{H}_{(\delta,\vartheta,\gamma)}}$, for some m ∈ N.

Then $\forall$ a, b $\in$ Ɣ, ab $\in A_{H_{(\delta,\vartheta,\gamma)}}$ imply $a^n \in A_{H_{(\delta,\vartheta,\gamma)}}$ or $b^m \in A_{H_{(\delta,\vartheta,\gamma)}}$

Hence $A_{H_{(\delta,\vartheta,\gamma)}}$ is semiprimary ideal of Ɣ.

Theorem (3.9)

Let $g: R \to R^*$ be an onto homomorphism of a rings

1) If $A \in NHFSYI(M^*)$, then $g^{-1}(A) \in NHFSYI(M)$.

2) If $A \in NHFSYI(M)$, then $g(A) \in NHFSYI(M^*)$.

Proof (1)

Similarly to proof (2).

Proof (2)

Suppose $g: M \to M^*$ be an onto homomorphism from a ring M into a ring M^*.

Let $A \in NHFSYI(M)$, and $y, e \in M^*$, since g onto homomorphism, then there exist $a, b \in M$ such that $g(a) = y \Longrightarrow a = g^{-1}(y)$, $g(b) = e \Longrightarrow b = g^{-1}(e)$, for some $n, m \in N$.

$$\left([0,1] - g\left([0,1] - F_{h_A}\right)\right)(ye) = \cap_{a=g^{-1}(y),\, b=g^{-1}(e)} F_{h_A}(ab)$$

$$\supseteq \cap_{a^n=g^{-1}(y^n)} F_{h_A}(a^n) = \left([0,1] - g\left([0,1] - F_{h_A}\right)\right)(y^n)$$

Hence

$$\left([0,1] - g\left([0,1] - F_{h_A}\right)\right)(ye) \supseteq \left([0,1] - g\left([0,1] - F_{h_A}\right)\right)(y^n) \quad (31)$$

Similarly,

$$g\left(T_{h_A}\right)(ye) \subseteq g\left(T_{h_A}\right)(y^n) \quad (32)$$

$$g\left(I_{h_A}\right)(ye) \subseteq g\left(I_{h_A}\right)(y^n) \quad (33)$$

Thus from (31), (32) and (33) we get $g(A) \in NHFSYI(M)$.

or

$$\left([0,1] - g\left([0,1] - F_{h_A}\right)\right)(ye) = \cap_{a=g^{-1}(y),\, b=g^{-1}(e)} F_{h_A}(ab)$$

$$\supseteq \cap_{b^m=g^{-1}(e^m)} F_{h_A}(b^m) = \left([0,1] - g\left([0,1] - F_{h_A}\right)\right)(e^m)$$

Hence

$$\left([0,1] - g\left([0,1] - F_{h_A}\right)\right)(ye) \supseteq \left([0,1] - g\left([0,1] - F_{h_A}\right)\right)(e^m) \quad (34)$$

Similarly,

$$g\left(T_{h_A}\right)(ye) \subseteq g\left(T_{h_A}\right)(e^m) \quad (35)$$

$$g\left(I_{h_A}\right)(ye) \subseteq g\left(I_{h_A}\right)(e^m) \quad (36)$$

Thus from (34), (35) and (36) we get $g(A) \in NHFSYI(M)$.

6-Conclusion

In this paper we introduced the concept of neutrosophic hesitant fuzzy primary ideal and also introduced neutrosophic hesitant fuzzy semiprimary ideal of a ring. Then prove several theorems pertaining to this sets illustrate with example.

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