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Comparison of the Muthuswamy-Chua circuit behavior with two different models.

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Abstract

In this paper, we have learned about the memristor in a simple way from previous studies. Here, we introduce it into a simple circuit, which is the Muthuswamy- Chua circuit, which consists of a resistor, an inductor, and a nonlinear element that we will replace with the memristor. We will know the mathematical relationships using dimensionless equations, as we use two different formulas for the memristor. To compare the two formulas and know which formula is better in producing chaos and which is better in stability, we use branching diagrams, time series, and phase diagrams. We find chaotic behavior, fractals, and also the periodic binary and tri-periodic binary of the two formulas.

Keywords: memristor, Muthuswamy-Chua, comparison, chaotic attractors.

1-Introduction

Magnetic flux and charge are the sources of a memristor, a negative-terminal nonlinear circuit element. Chua proposed a theoretical of it in 1971 [1].The zero-crossing property, which is a prominent aspect of the memory system, was thoroughly analyzed by Chua and Kang in 1976 when they extended the memristor to the memory system . Many scientists and researchers were interested in studying and using the physical memristor that Hewlett Packard Laboratories (HP labs) produced for the first time by Strukov and others in [2,3].

After the three fundamental components of an electrical circuit the resistor, capacitor, and inducer the World Chua categorized the memristor as the fourth element [1]. Applications for memristors include nonvolatile memory based on memristors, neuromorphic systems, digital design (logic gates and reconfigurable switches), analog domain, and more [4] . Chua disclosed that the origin of complexity is local action. Sustain oscillations and magnify weak signals, the nonlinear dynamical system requires local activity [1].

This work aims to study the properties of the circuit presented by Chua and Muthusamy [5], using two different memristor formulas and to find out the best formula in terms of chaos and stability in these formulas by comparing the two formulas.

This work is organized as follows: the memristor is the first portion, followed by Muthusamy-Chua, the bifurcation form, the time series, and the chaotic attraction of each bifurcation form; the conclusion is the last section.

2-Memristor

In a paper published to Leon Chua developed the memristor hypothesis [1]. To achieve conceptual symmetry with the resistor, inductor, and capacitor, Chua firmly thought that a fourth device existed. A relationship between two of the four fundamental circuit variables defines the fundamental passive circuit elements, from which this symmetry is derived. During that period, the memristor a device that would link charge and flux which are defined as time integrals of voltage and current was still speculative [2]. That team at HP Labs, led by scientist R. Stanley Williams, would not, however, disclose the development of a switching memristor until [6 ,7], some thirty-seven years later.

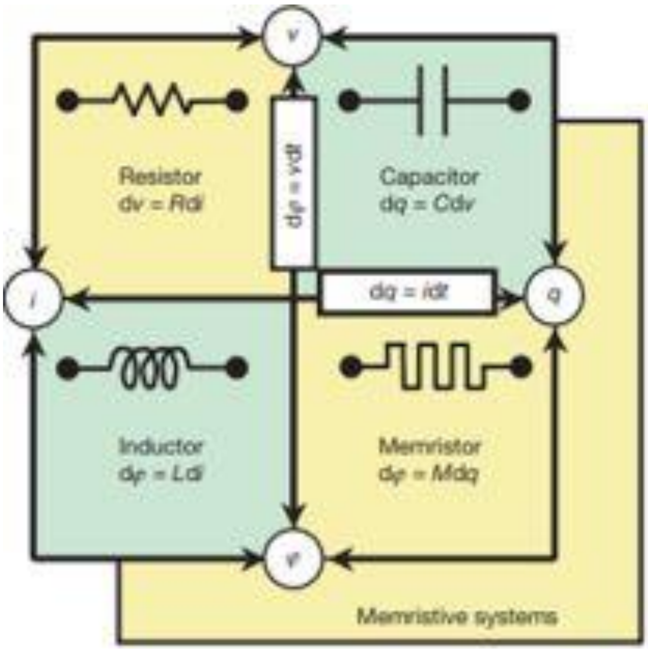

Fig.(1) The four basic double-terminal circuit components are the memristor (M), resistor (R), capacitor (C), and inductor (L) [4].

Memristor systems, a broader class of dynamical devices, includes resistors and memristors as subsets. It should be noted that nonlinear elements can be produced by R, C, L, and M being functions of the independent variable in their defining equations. A charge-controlled memristor, for instance, has a single-valued function M(q) that defines it where the "voltage momentum," also known as "flux," and the "current momentum," also known as "charge," are, in accordance with the terminology presented in [8] :

$$\varphi(t) = \int_{-\infty}^{t} v(\tau)d\tau \rightarrow v = \varphi'(t)$$
$$q(t) = \int_{-\infty}^{t} i(\tau)d\tau \rightarrow i = q'(t) \qquad (1)$$

There are four basic variables in circuit theory: charge (q), magnetic flux (φ), voltage (v), and current (i). There are six possible pairings of these four elements. Them the well-known relations express two [4]:

$$v = R\,i$$
$$\varphi = L\,i \qquad (2)$$
$$q = C\,v$$

correspond to the axiomatic definitions of a resistor (R), an inductor (L) and a capacitor (C) respectively. There is only one undefined combination remaining from the six conceivable combinations involving the charge q, the magnetic flux φ, the voltage v, and the current i. This combination links φ and q. Chua has linked the "missing" memristors to this precise connection, where the slope (dφ / dq) is defined as the (incremental) memristance M (q).

$$M(q)=\frac{d\varphi}{dq}\rightarrow d\varphi=M(q)dq \qquad (3)$$

Taking into account Eq.(1):

$$v=M(q)i \qquad (4)$$

The value of the memristor, which represents the second degree in this research, is in two different formulas that we apply in the circuit, where the first formula is [9];

$$\begin{aligned} V_M &= \beta(z^2-1)i_M \\ M(z) &= \beta(z^2-1) \end{aligned} \qquad (5(a))$$

The second formula for the memristor that we use has been used on the modified Chua circuit, which is as follows [10]

$$\begin{aligned} V &= (-\alpha+3\beta z^2)\mathrm{i} \\ W(z) &= -\alpha+3\beta z^2 \end{aligned} \qquad (5(b))$$

Due to the presence of α in the new memristor formula, we will change it to another symbol which is (α=ά), so that it does not collide with α in the equation of the internal state variable of the memristor.

z denotes the internal state variable of the resistor, and (ά, β) are related to the memristor. Where this formula was used on the Chua circuit Eq.(5(b)), but here we will use it on the simplest circuit, which is the Muthusamy circuit When introducing the new value to the Muthusamy equation and studying the branch behavior, the time series, and the chaotic attractor.

2-Muthuswamy- Chua circuit

Muthuswamy and Chua introduced the most basic electronic circuit that generates chaotic attractors [5]. This circuit comprises a linear passive inductor, a linear passive capacitor, and a nonlinear active memristor [11], and its behavior can be characterized by a differential system[12,13,14] .

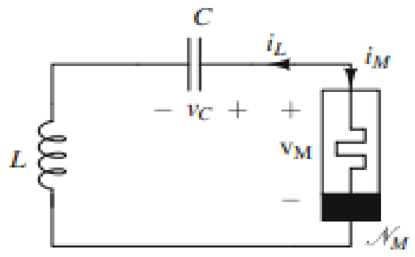

Fig (2) The Muthuswamy -Chua circuit [12].

A mathematical description of Fig. (1) can be obtained by applying Kirchhoff's law to electrical circuits [5]:

$$\begin{aligned} C\frac{dV_C}{dt} &= i_L \\ L\frac{di_L}{dt} &= M(z)i_M - V_C \\ \frac{dz}{dt} &= i_M - \alpha z - i_M z \end{aligned} \qquad (6)$$

The following mathematical model describes the circuit's behavior called Muthuswamy equations with memristor [10] .

$$\begin{aligned} x' &= ay \\ y' &= -b(M(z)y + x) \\ z' &= -y - \alpha z + yz \end{aligned} \qquad (7(a))$$

$$\begin{aligned} x' &= ay \\ y' &= -b(W(z)y + x) \\ z' &= -y - \alpha z + yz \end{aligned} \qquad (7(b))$$

($x = V_C$) where the voltages across the capacitor, ($i_L = y$) the current through the inductor, (z) the voltages across the memristor, ($a = 1/C$), ($b = 1/L$) represent fixed parameters and are included in the Eq. (7, (a & b)). To observe the chaotic evolutions of constant parameter change, a bifurcation plot between the voltages across the capacitor and the parameter α was used to study the resulting behavior and compare the two formulations. (M&W). When we substitute the values of (a = 0.83, f = 1.34, and e = 1.4), we see that Fig. (3)

When (b=0.3) is in Fig. (3, A. 1), it has been observed that it starts off regularly, then it transitions to a chaotic zone with dynamics strewn throughout the region encompassed between 0.1 and 1, before returning to periodicity. On the other hand, we see that Fig. (3, B.1) starts with several periods with near amplitudes, and a sizable region is contained within the range of 0.1 to 1. In addition to being double periodic, it also exhibits periodic multiplicity with varying amplitudes and, at the conclusion, periodic multiplicity with amplitudes larger than at the start.

At (b = 0.43) we discover that in Fig. (3, A.2) it begins periodically, and in the region between 0.1 and 1 the chaos decreases and appears quasi-periodic, but the chaos ends at 0.8, after which it becomes a periodic region and takes the Fig. (3, B.2) and begins to multiply periodically. The region contains between 0.1 and 1.1 a periodic quasi-binary, along with a small part of a periodic quadrilateral. The chaos begins from 0.55 to 1.1, and ends with a periodicity.

Regarding (b = 0.76), we observe that the periodicity amplitude has grown in Fig. (3,A.3), but that the chaos has greatly decreased in the region between 0.1 and 1. It has also been divided into two portions, with the periodicity duo dividing them. The chaos terminates at 0. 6. The periodicity then starts, but as before, it is shown in Fig. (3,B.3), where it starts with the periodicity multiplicity, continues to the zone of chaos that we may limit from 0.1 to 1, terminates at 0.8, where the region of chaos is dynamically rich, and then the periodicity starts.

In Fig. (3,A.4), we observe a chaotic zone that nearly vanished as it was limited between 0 and 0.1, at which point a stable periodicity forms if (b = 3.3). With regard to Fig. (3, B.4), we observe that periodicity has started, chaos has also considerably lessened, and that chaos is now confined to the range of 0.3 and 0.5 before steady periodicity follows.

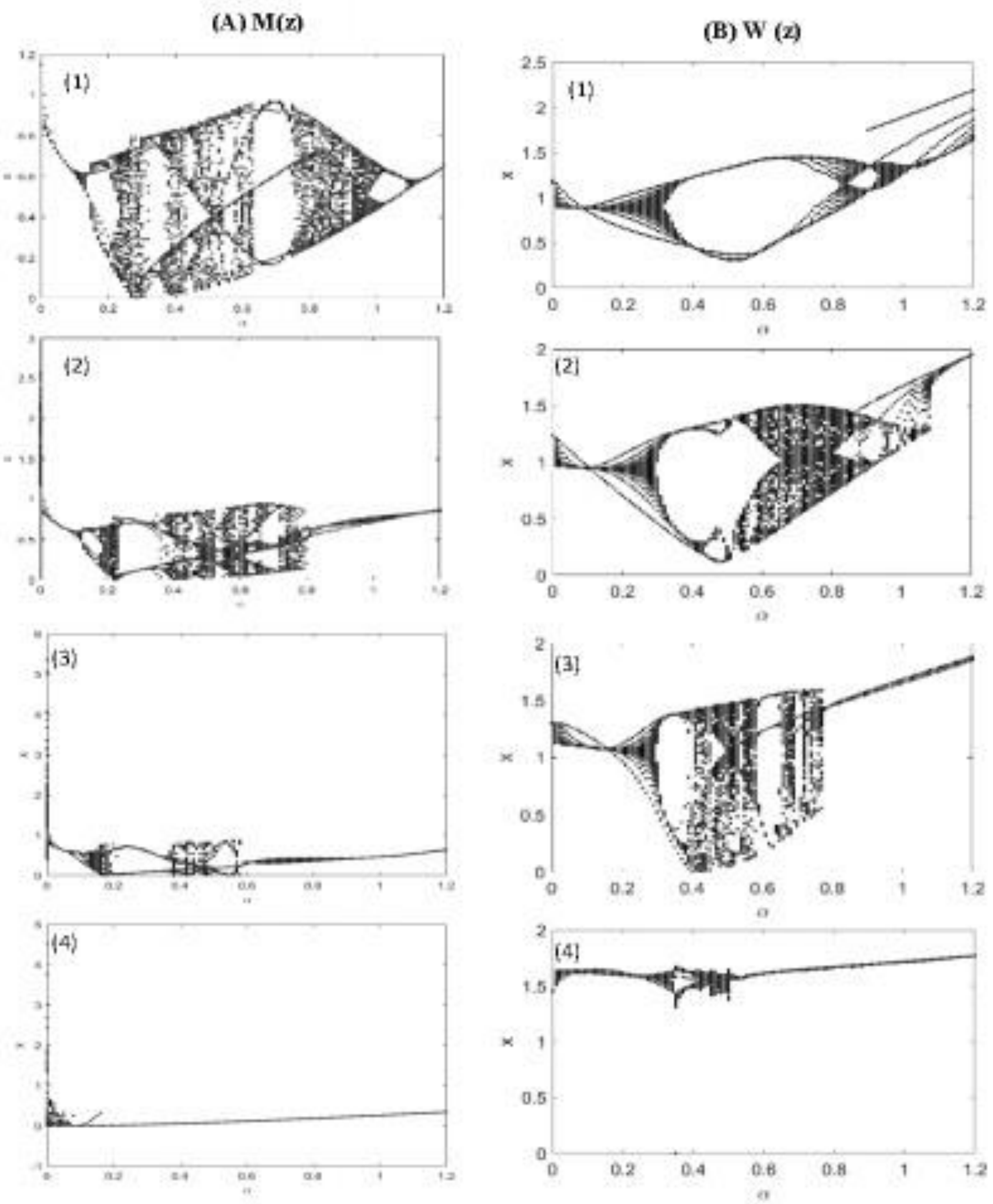

Fig. (3) Bifurcation diagram as a function of the control parameter α (1) b=0.3 (2) b=0.43 (3) b=0.76 (4) b=3.3.

The time series and the chaotic attractor diagrams, which appear in Fig. (3,(A,B),1) at alpha = 0.4 and b = 0.3, and the comparison of the two formulas for the memory resistance M show that it is in a stable double periodic state in Fig. (4,B) when the time series representing the voltages across the capacitor in Fig. (4,A) is in a state of chaos.

The current flowing through the inductor in Fig. (4, A) is chaotic, whereas in Fig. (4, B) it is double per. Similarly, the voltage across the memristor in Fig. (4, A) is chaotic, whereas in Fig. (4, B) it is double periodic.

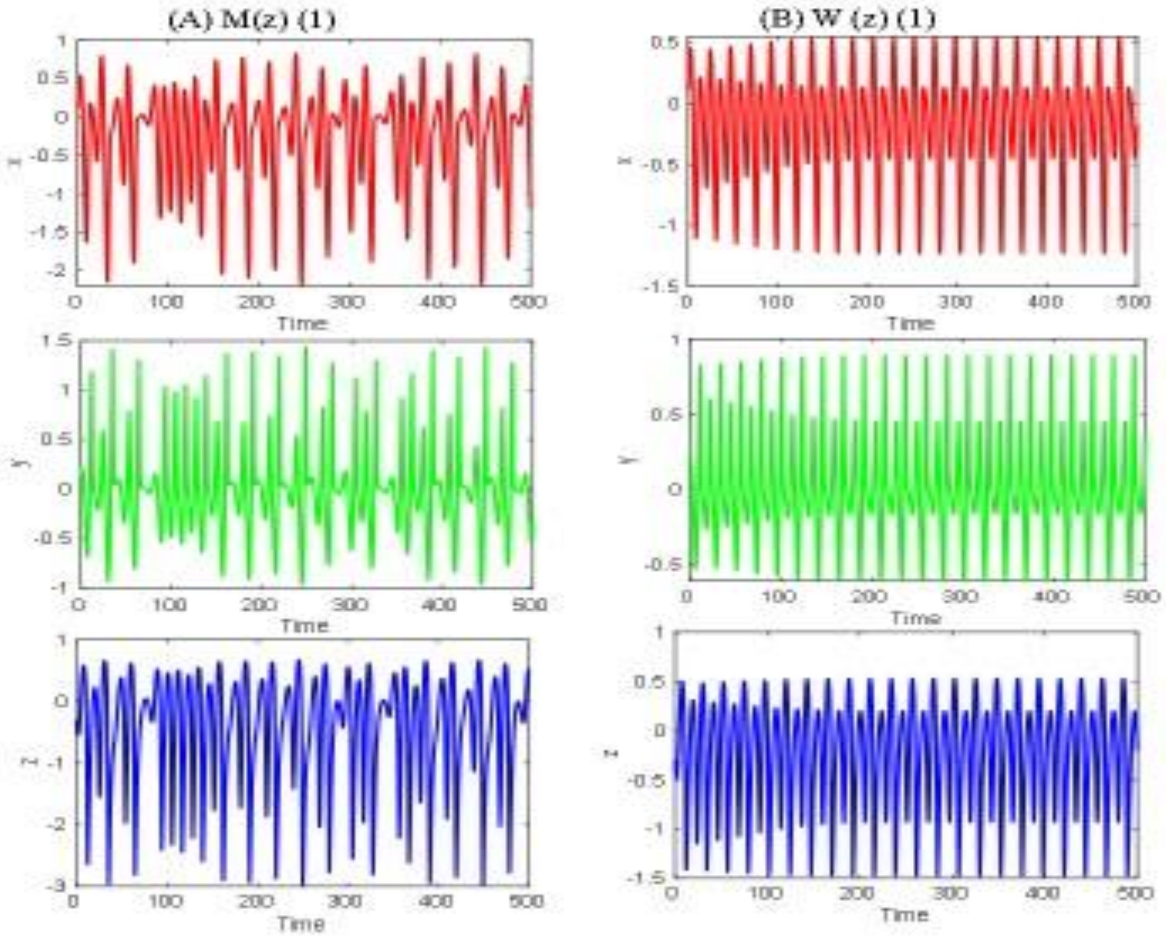

Fig. (4) Time series for Figure (3,1 (A & B)) when (b=0.3, a=0.83, β=1.34, ά=1.4 and α=0.4).

Furthermore, the chaotic attractor in Fig. (3, (A, B),1) between the current flowing through the inductor and the voltage across the capacitor in the chaotic attractor case (right side), is at Fig. (5, A). Periodic multiplicity occurs on the left. Regarding Fig. (5, B), this is the situation where a periodic binary with varying amplitudes occurs. The voltages between the memristor and the capacitor are depicted in Fig. (5, A). It also shows up on the right side when there is chaotic attraction. It is in a small-amplitude multiple periodic conditions on the left, and a double periodic state in Fig. (5, B). What we notice in the phase diagrams is that the first formula (M(z)) is close to the results found in the work [5].

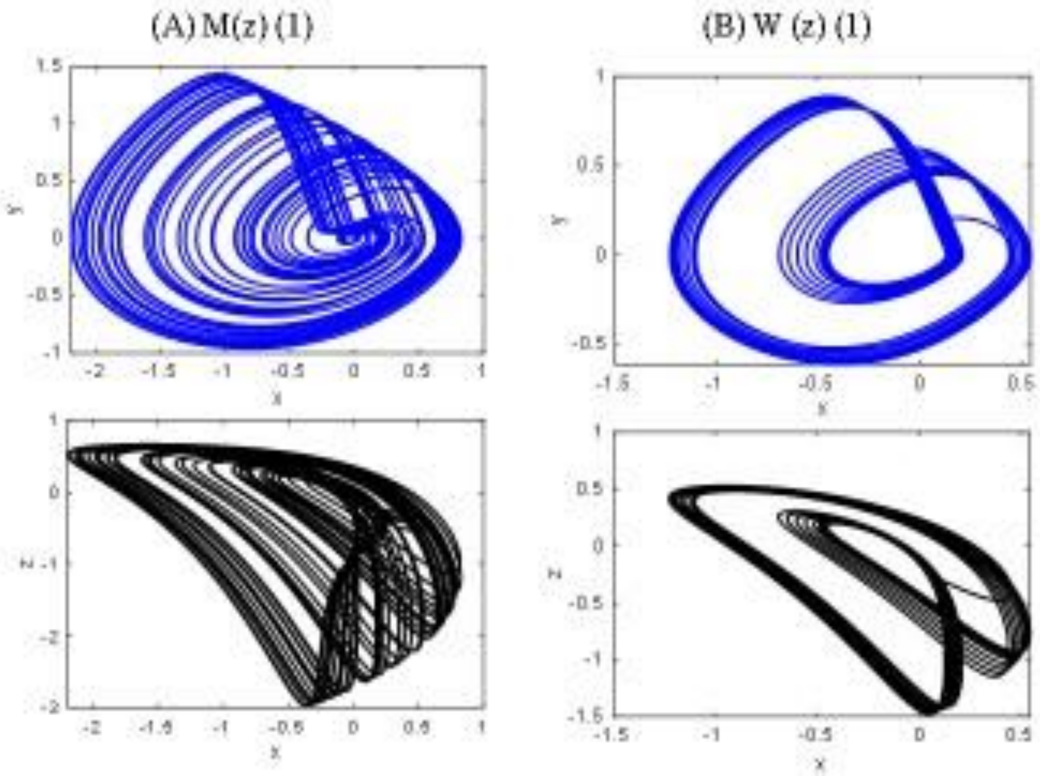

Fig .(5) The plots of chaotic attractor of charge-controlled memristor based on the simplest chaotic circuit.

When we take the time series in Fig. (3, (A,B), 2) with the same values as above, we find that Fig. (6, A) represents the voltage across the capacitor in the quasi-tricyclic case, while Fig. (6, B) finds it in the double periodic case, and similarly for the current across the inductor in Figs. (6, A) and (6, B) quasi-tricyclic and double periodic, respectively. This is also seen in the voltage across the memristor in Figs.(6,A) and (6,B), which are quasi-tri-periodic and double periodic, respectively.

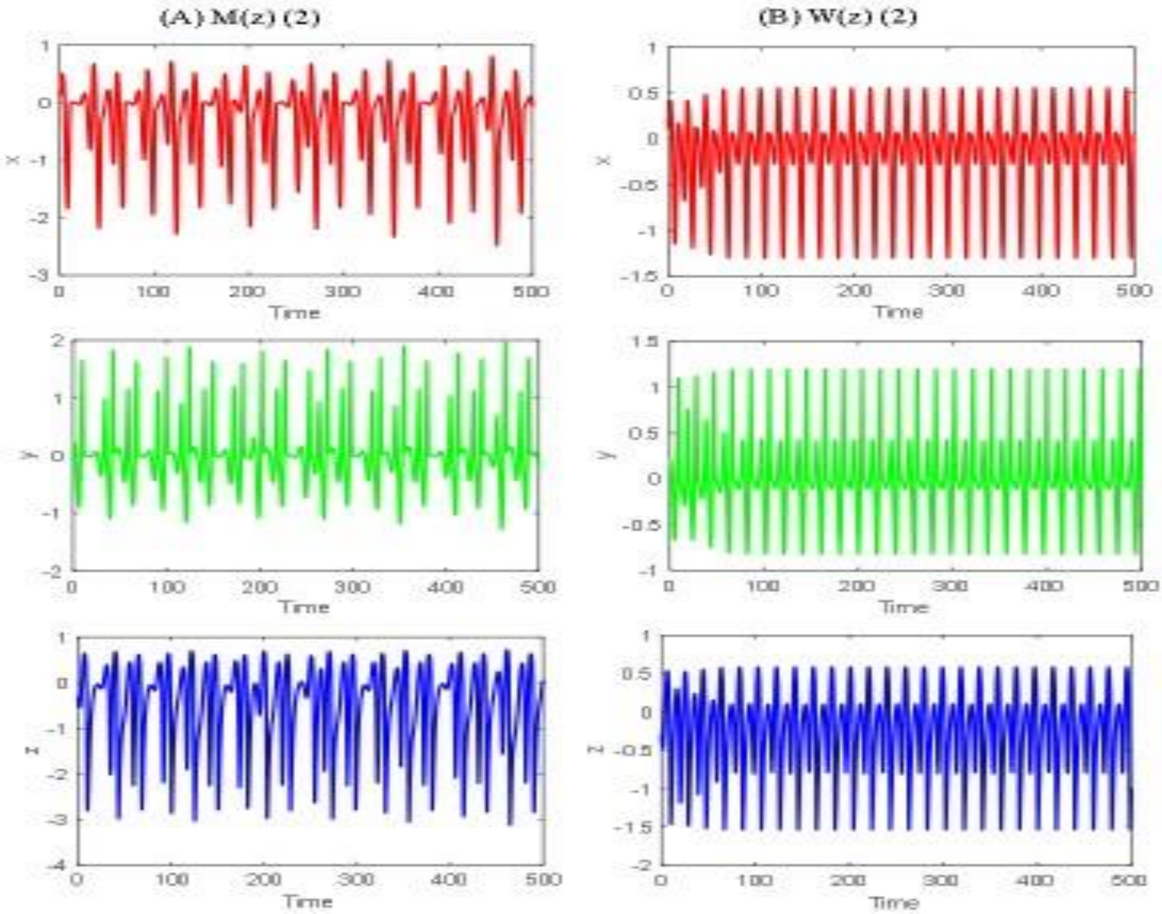

Fig. (6) Time series for Figure (3, 2 (A & B)) when (b=0.3, a=0.83, β=1.34, ά =1.4 and α=0.4).

The chaotic attraction of the time series is visible in Fig. (3, (A, B),2) when Fig. (7, A) is in a quasi-periodic triplet with varying amplitudes. We discover that the chaotic attraction is present in the right part, whereas the left part is periodic multiplicity. Regarding Fig. (7, B), it is in a state of a periodic doublet, but the beginning of the emergence of chaotic attraction in the right section, between the voltages across the capacitor and the memristor.

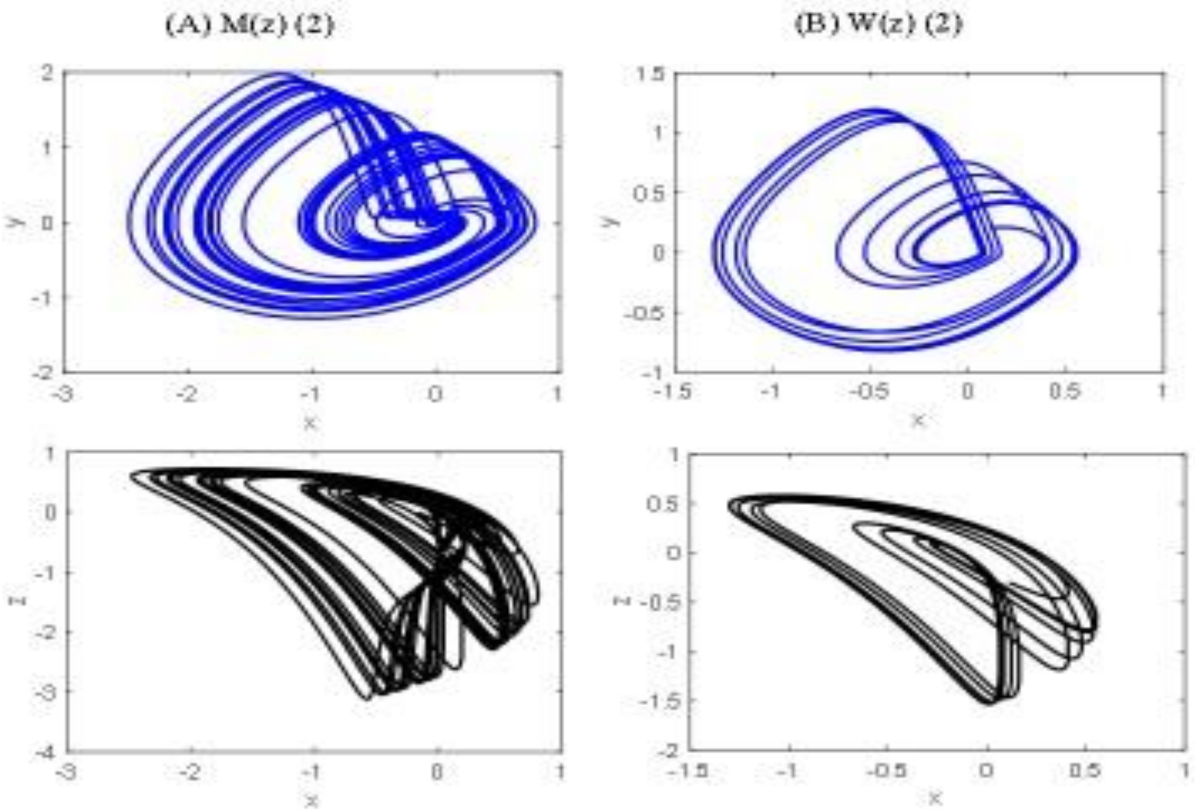

Fig. (7) The plots of chaotic attractor of charge-controlled memristor based on the simplest chaotic circuit.

The time series in Fig. (3, (A, B), 3) is the previous value. At the voltage across the capacitor in Fig. (8, A), we observe the periodic multiplicity of different amplitudes; in Fig. (8, B), it is a quasi-periodic bipolarity of varying amplitudes; however, at the current through the inductor in Fig. (8, A), the periodic multiplicity of different amplitudes; in Fig. (8, B), it is a quasi-periodic bipolarity of varying amplitudes. This arrangement is also observed at the voltage across the memristor in Figs. (8, A) and (8, B).

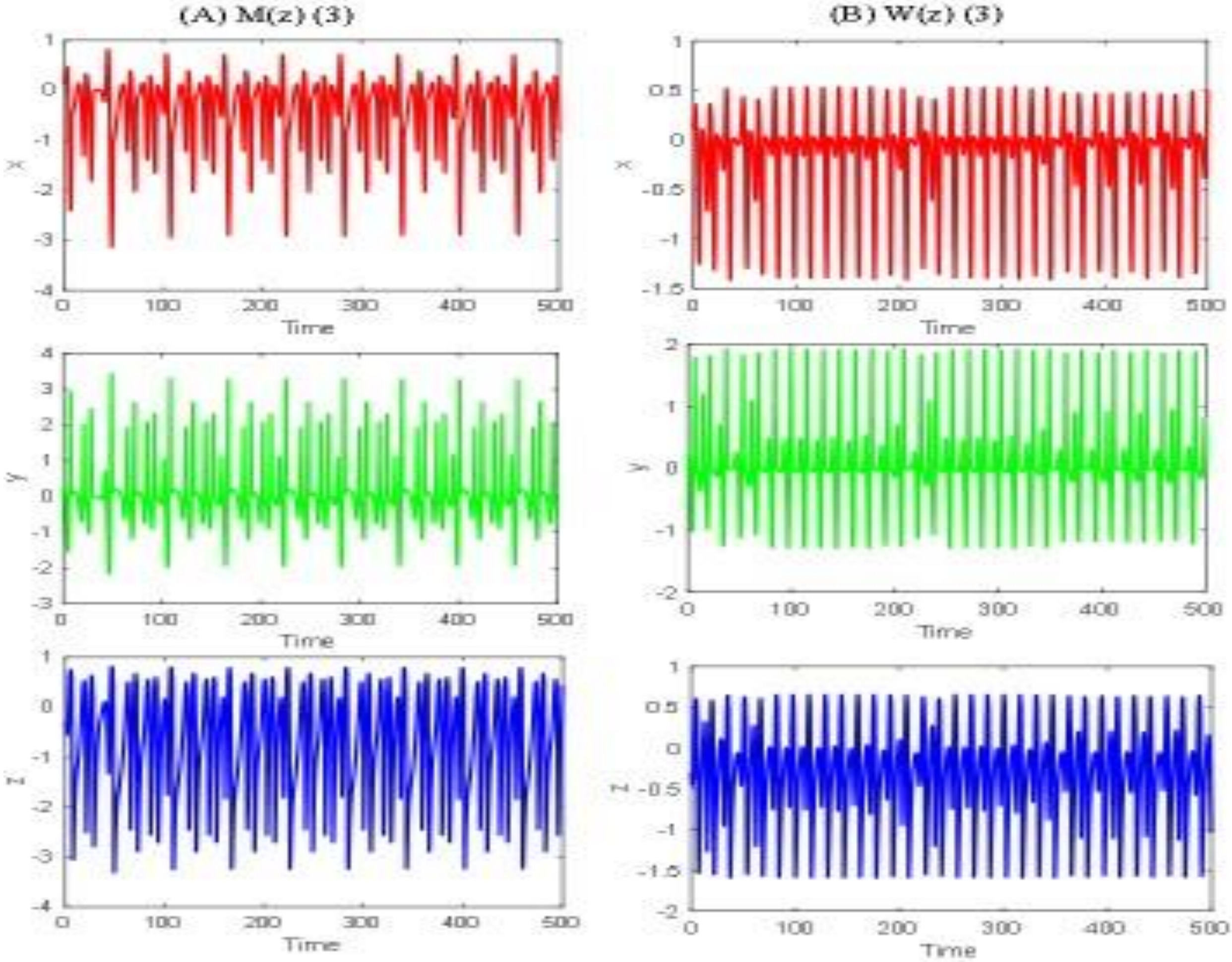

Fig. (8) Time series for Figure (3,3 (A & B)) when (b=0.3, a=0.83, β=1.34, ά =1.4 and α=0.4).

Regarding the chaotic attraction of the time series of Figs. (3 and (A, B) 3) between the current flowing through the inductor and the voltage across the capacitor, as shown in Fig. (9, A), we find that it is in a periodic state and that it appears on the right side, whereas in Fig. (9, B) Chaotic attraction emerges on the right side, while periodicity is shown on the left. And the voltages across the capacitor and the memristor, we can observe from Figs. (9, A) and (9, B) that the chaotic attraction and periodicity exhibit the same behavior between the current flowing through the inductor and the voltage across the capacitor.

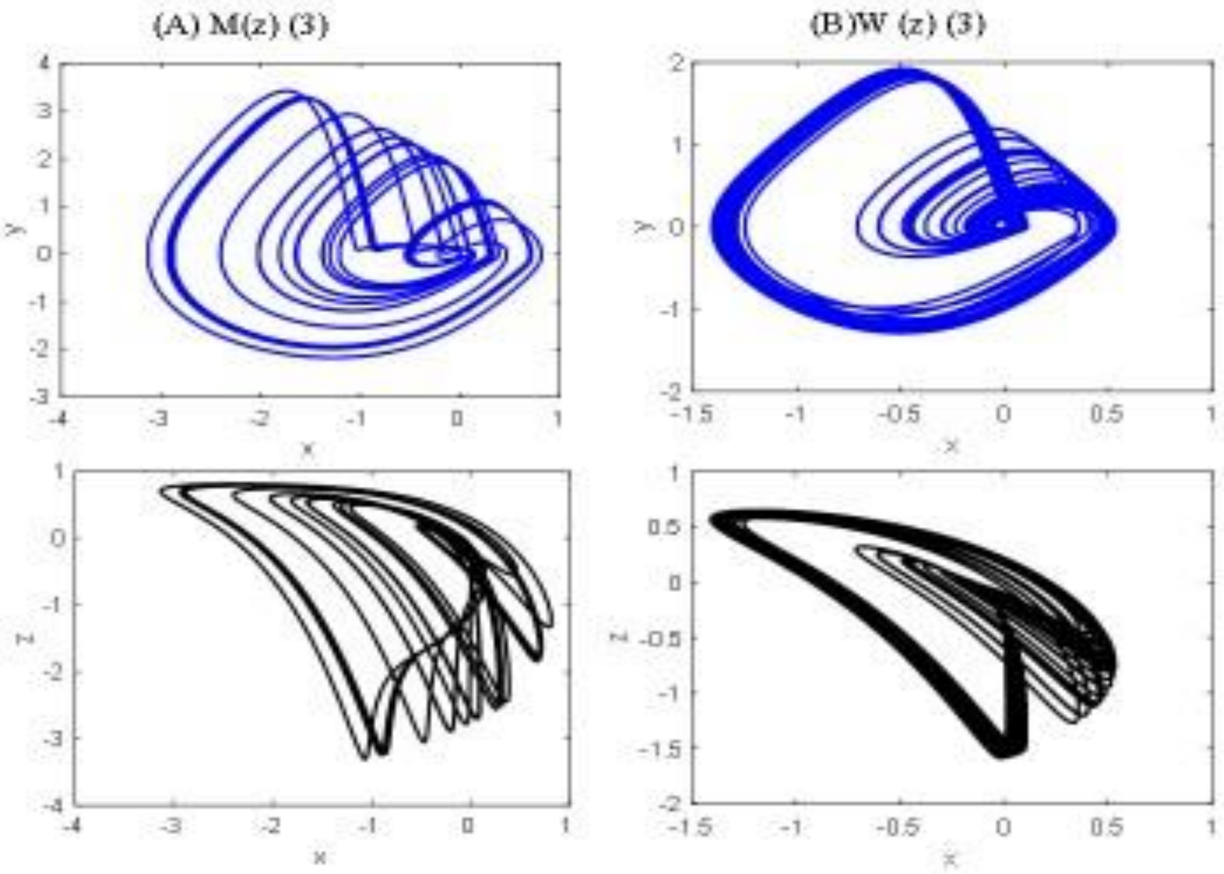

Fig. (9) The plots of chaotic attractor of charge-controlled memristor based on the simplest chaotic circuit.

For the time series in Fig. (3(A,B)4) for the same values, we notice that the voltage across the capacitor in Fig. (10,A) is periodic, while in Fig. (10,B) it is bi-periodic with small amplitudes between cycles; in addition, the current across the inductor in Fig. (10,A) has a small pulse giving it a bi-periodic appearance, and shows small cycles, but in Fig. (10,B) it is also bi-periodic with small amplitudes between cycles; and finally, the voltage across the memristor in Fig. (10,A) is periodic, while in Fig. (10,A) it is bi-periodic with small amplitudes between cycles.

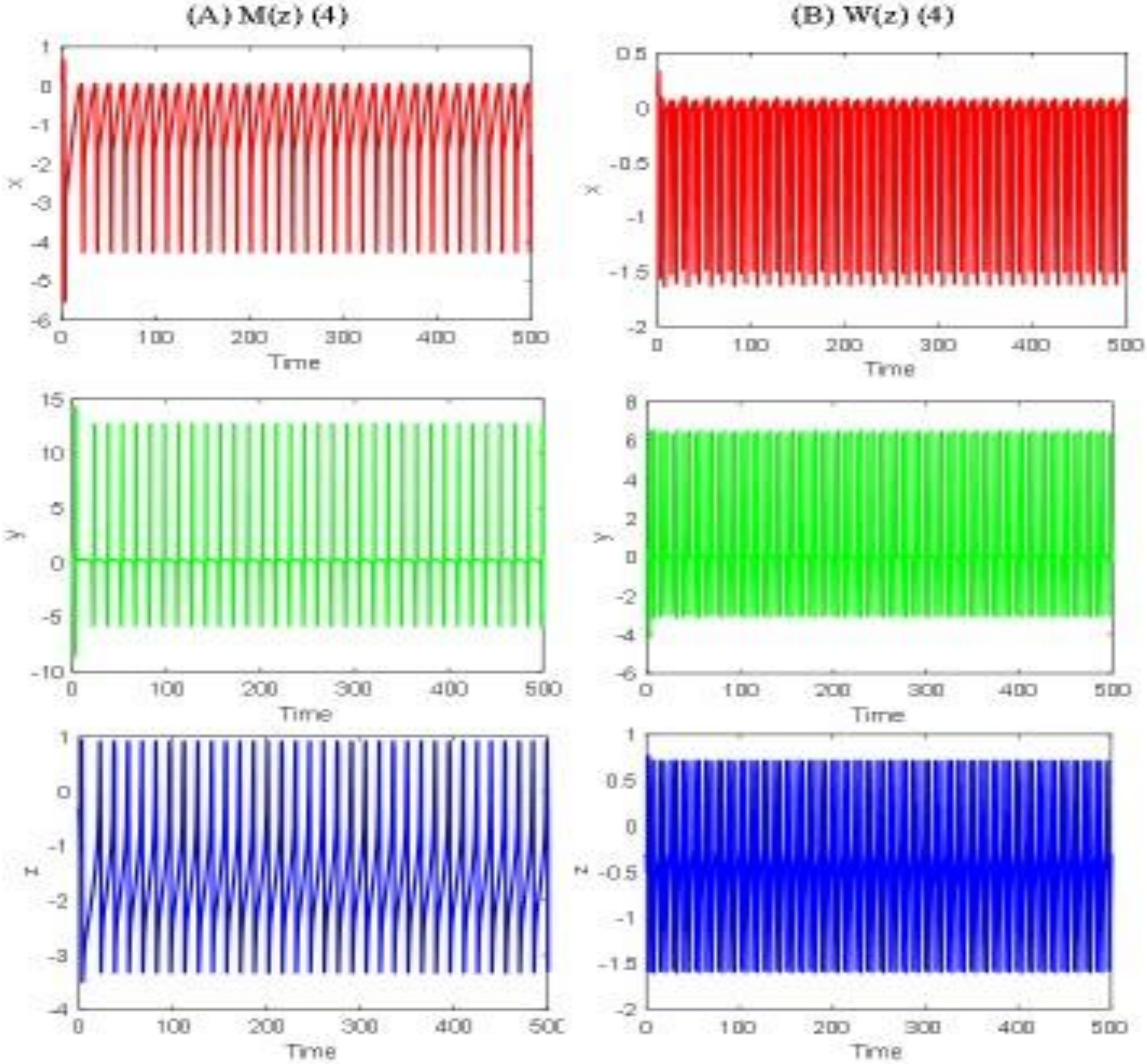

Fig. (10) Time series for Figure (3,1 (A & B)) when (b=0.3, a=0.83, β=1.34, ά=1.4 and α=0.4).

Regarding the chaotic attraction to Fig. (3(A, B) 4), there is double periodicity in Fig. (11, A) and double periodicity with comparable amplitude between the turns in Fig. (11, B). This attraction is caused by the current passing through the inductor and the voltage across the capacitor. When it comes to the voltages across the capacitor and the current across the inductor in Figs. (11, A) and (11, B), they are similar to those between the capacitor and the memristor.

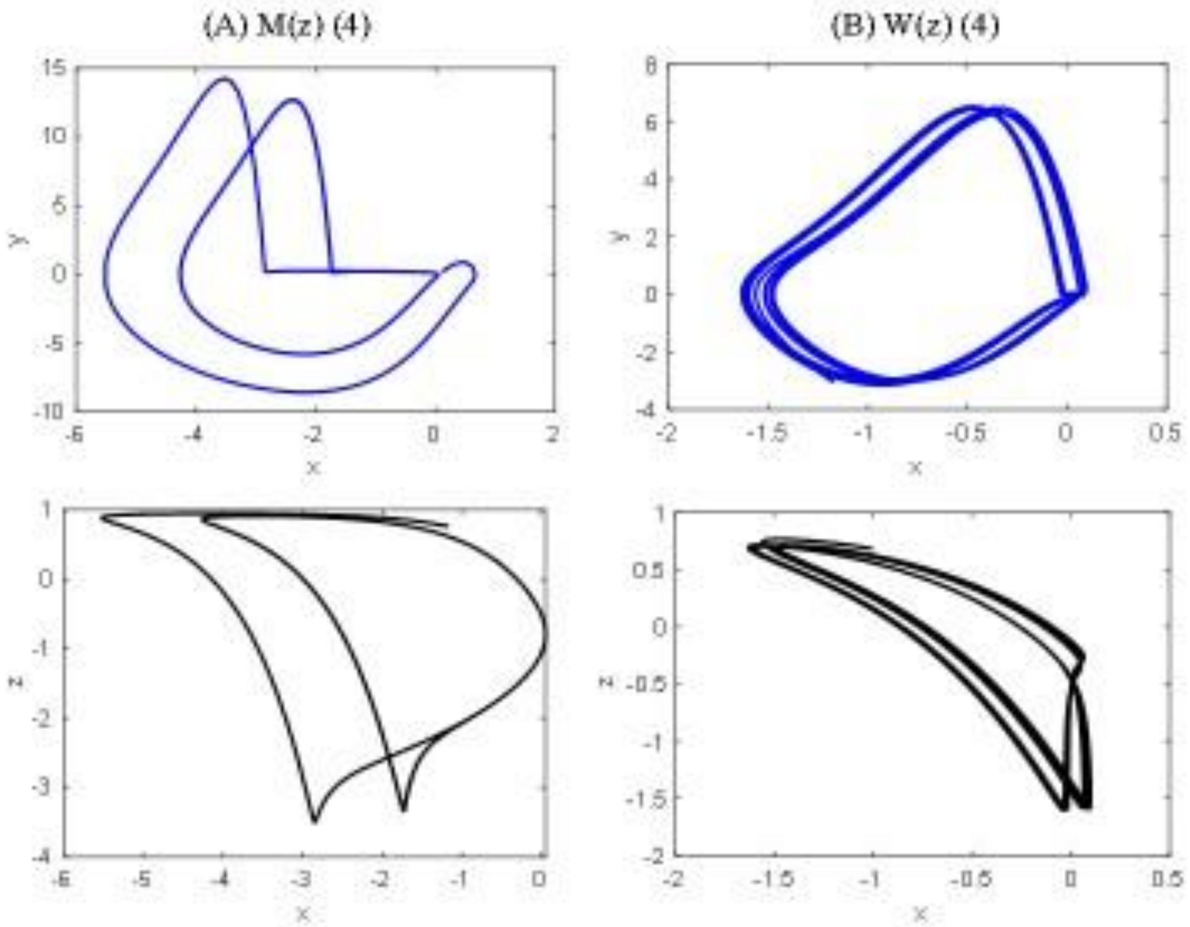

Fig. (11) Phase diagrams of charge-controlled memory resistors based on the simplest chaotic circuit.

4-Conclusion

Comparing the two alternative formulas for M, we find that chaos in M(z) arose from the beginning and gradually disappeared; this supports the time series and chaotic gravity that gradually disappears until it approaches the periodic duality. Regarding the formula W(z), we find that chaos arises when the value of b increases and decreases and eventually disappears. This supports the time series in addition to the chaotic gravity that starts with a periodic duality and then the chaotic gravity appears and returns to the periodic duality, which indicates that the formula M(z) shows stability with the appearance and disappearance of chaos. The opposite happens when W(z) shows instability in showing chaos. These results are added to the results that were studied in the first formula where a single-coil chaotic attractor was found. As for the second formula, it is the first study conducted on the Muthuswamy-Chua circle where it is in a periodic state on the left side, while on the right side we have chaotic attraction, which we find that both formulas give chaos and stability, but the first formula M(z) is much better than the second formula W(z).

References

[1] L. Chua, "Memristor-the missing circuit element," IEEE Trans. circuit theory, vol. 18, no. 5, pp. 507–519, 1971.

[2] X. Xie, L. Zou, S. Wen, Z. Zeng, and T. Huang, "A flux-controlled logarithmic memristor model and emulator," Circuits, Syst. Signal Process., vol. 38, no. 4, pp. 1452–1465, 2019.

[3] H. Chang, Y. Li, G. Chen, and F. Yuan, "Extreme multistability and complex dynamics of a memristor-based chaotic system," Int. J. Bifurc. Chaos, vol. 30, no. 08, p. 2030019, 2020.

[4] M. Shahsavari, "Memristor technology and applications: An overview," 2013.

[5] B. Muthuswamy and L. O. Chua, "Simplest chaotic circuit," Int. J. Bifurc. Chaos, vol. 20, no. 05, pp. 1567–1580, 2010.

[6] R. S. Williams, "How we found the missing memristor," IEEE Spectr., vol. 45, no. 12, pp. 28–35, 2008.

[7] F. Corinto, M. Forti, and L. O. Chua, Nonlinear Circuits and Systems with Memristors: Nonlinear Dynamics and Analogue Computing via the Flux-Charge Analysis Method. Springer Nature, 2020.

[8] F. Corinto, P. P. Civalleri, and L. O. Chua, "A theoretical approach to memristor devices," IEEE J. Emerg. Sel. Top. Circuits Syst., vol. 5, no. 2, pp. 123–132, 2015, doi: 10.1109/JETCAS.2015.2426494.

[9] B. Muthuswamy, "Implementing memristor based chaotic circuits," Int. J. Bifurc. Chaos, vol. 20, no. 05, pp. 1335–1350, 2010.

[10] M. Di Marco, M. Forti, L. Pancioni, and A. Tesi, "Drift of invariant manifolds and transient chaos in memristor Chua's circuit," Electron. Lett., vol. 59, no. 19, 2023, doi: 10.1049/ell2.12948.

[11] M. A. Rahma, E. A. Mohammed, and H. L. Saadon, "Linear and Nonlinear Optical Properties of Natural Dyes Freestanding Films and Application in Optical Limiting," Univ. Thi-Qar J. Sci., vol. 7, no. 2, pp. 139–143, 2020.

[12] M. S. Abdelouahab and R. Lozi, "Bifurcation analysis and chaos in simplest fractional-order electrical circuit," 3rd Int. Conf. Control. Eng. Inf. Technol. CEIT 2015, no. May, pp. 1–5, 2015, doi: 10.1109/CEIT.2015.7233122.

[13] J. M. Ginoux, B. Muthuswamy, R. Meucci, S. Euzzor, A. Di Garbo, and K. Ganesan, "A physical memristor based Muthuswamy–Chua–Ginoux system," Sci. Rep., vol. 10, no. 1, pp. 1–10, 2020, doi: 10.1038/s41598-020-76108-z.

[14] K. C. Iarosz et al., "Chaotic dynamics in memristive circuits," Rev. Bras. Ensino Física, vol. 45, p. e20230116, 2023.