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Result for the Groups $\mathcal{SL}(2,23)$ and $\mathcal{SL}(2,29)$

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Abstract:

A collection of (n×n) not-sole matrices on F domain this collection combine a group beneath process of matrix multiply invited the general linear group of measure n on domain F, indicate $GL(n,F)$. The kernel of a homomorphism of $GL(n,F)$ to F* is the special linear group and indicate by $\mathcal{SL}(n,F)$is the determinant of these matrices, and this homomorphism. Thus $\mathcal{SL}(n,F)$ contains all matrices of determinant one which is the subgroup of $GL(n,F)$.

The objective of the study is to find the Artin directrix (A.d.) for the groups $\mathcal{SL}(2,23)$ and $\mathcal{SL}(2,29)$ from written the rational valued characters (r.v.ch.) of the rational representations (r.r.) as a linear combination of the induced characters (in.ch.) for these groups.

Keywords: Artin directrix, induced characters table, special linear group.

1-Introduction

Seeker in [1] realizes the representation of the group. The (c.h.t.) character table of (r.r.) for the group $\mathcal{SL}(2,p)$ studied in [2,3]. Authors in [4] survey the (c.h.t.) of (r.r.) of the groups $\mathcal{SL}(2,23)$ and $\mathcal{SL}(2,29)$, in this work we compute the (A.d.) for these groups by apply the same idea in [5]. While the authors in [6-9] compute the (A.d.) for different groups.

2- Primary Notions

Theorem 2.1: [1]

$| \mathcal{SL}(2, q^n) | = q^n \ (q^{2n} - 1)$

Definition 2.2: [2]

Let H be a cyclic subgroup of a group G, and ϕ be a class function of H. Then

$$\phi \uparrow^G = \frac{|C_G(g)|}{|C_H(g)|} \sum_{I=1}^{m} \emptyset(x_i)$$

Definition 2.3: [3]

The Artin character is the (in.ch.) from the p.ch. of cyclic subgroups of the group.

Definition 2.4: [3]

For a finite group G and χ any (r.v.ch.). (A.d.) is the little positive number n,

$$n\chi = \sum_c a_c \phi_c, \in \mathbb{Z}, \phi_c$$

Artin character and indicate A(G).

3- The Results with their Discussion

3.1 The Result for the group $\mathcal{SL}$(2,23)

From [4] the (c.h.t.) of (r.r.) of the groups $\mathcal{SL}$(2,23) is

Table 1. The (c.h.t.) of (r.r.) of the groups $\mathcal{SL}$(2,23)

C_g	1	z	c	zc	a	a^2	b	b^2	b^3	b^4	b^6	b^7
$\lvert C_g \rvert$	1	1	264	264	552	552	552	552	552	552	552	552
$\lvert C_G(g) \rvert$	12144	12144	46	46	22	22	22	22	22	22	22	22
1_G	1	1	1	1	1	1	1	1	1	1	1	1
Ψ	23	23	0	1	1	1	-1	-1	-1	-1	-1	0
$\chi_1+\chi_3+\chi_5+\chi_7+\chi_9$	120	-120	5	-5	1	-1	0	0	0	0	0	0
$\chi_2+\chi_4+\chi_6+\chi_8+\chi_{10}$	120	120	5	5	-1	-1	0	0	0	0	0	0
$\theta_1+\theta_5+\theta_7+\theta_{11}$	88	-88	-4	4	0	0	0	0	0	-4	0	4
$\theta_2+\theta_{10}$	44	44	-2	-2	0	0	0	-2	0	2	4	2
$\theta_3+\theta_9$	44	- 44	-2	2	0	0	0	0	0	4	0	-4
θ_4	22	22	-1	-1	0	0	-1	1	2	1	-2	1
θ_6	22	22	-1	-1	0	0	0	2	0	-2	2	-2
θ_8	22	22	-1	-1	0	0	1	1	-2	1	-2	1
$\xi_1+\xi_2$	24	-24	1	-1	-2	2	0	0	0	0	0	0
$\eta_1+\eta_2$	22	22	-1	-1	0	0	2	-2	2	-2	-2	-2

The (in.ch.) table is:

Table 2. The (in.ch.) table for $\mathcal{SL}(2,23)$

C_g	**1**	z	c	zc	a	a^2	b	b^2	b^3	b^4	b^6	b^7
$\lvert C_g \rvert$	**1**	**1**	**264**	**264**	**552**	**552**	**552**	**552**	**552**	**552**	**552**	**552**
$\lvert C_G(g) \rvert$	**12144**	**12144**	**46**	**46**	**22**	**22**	**22**	**22**	**22**	**22**	**22**	**22**
Φ_1	12144	0	0	0	0	0	0	0	0	0	0	0
Φ_2	6072	6072	0	0	0	0	0	0	0	0	0	0
Φ_3	528	0	2	0	0	0	0	0	0	0	0	0
Φ_4	264	792	3	3	0	0	0	0	0	0	0	0
Φ_5	552	1104	0	0	2	0	0	0	0	0	0	0
Φ_6	1104	0	0	0	0	2	0	0	0	0	0	0
Φ_7	506	1012	0	0	0	0	2	0	0	0	0	0
Φ_8	1012	2024	0	0	0	0	0	4	0	0	0	0
Φ_9	1518	0	0	0	0	0	0	0	6	0	0	0
Φ_{10}	2024	4048	0	0	0	0	0	0	0	2	0	0
Φ_{11}	3036	6072	0	0	0	0	0	0	0	0	2	0
Φ_{12}	506	1012	0	0	0	0	0	0	0	0	0	2

Hence,

$$1= 0.5\Phi_{12} + 0.5\Phi_{11} + 0.5\,\Phi_{10} + 0.1666666667\,\Phi_9 + 0.25\,\Phi_8 + 0.5\,\Phi_7 + 0.5\,\Phi_6 + 0.5\,\Phi_5 + 0.3333333333\,\Phi_4 - 1.217555995\Phi_2 + 0.2416831358\,\Phi_1,$$

$$\Psi = -0.5\,\Phi_{11} - 0.5\,\Phi_{10} - 0.1666666667\,\Phi_9 + 0.25\,\Phi_8 + 0.5\,\Phi_7 + 0.5\,\Phi_6 + 0.5\,\Phi_5 + 0.3333333333\,\Phi_4 - 0.5\,\Phi_3 + 0.869400527\,\Phi_2 - 0.2156620553\,\Phi_1,$$

$$\chi_1 + \chi_3 + \chi_5 + \chi_7 + \chi_9 = -0.5\,\Phi_6 + 0.5\,\Phi_5 - 1.666666667\,\Phi_4 + 5\,\Phi_3 + 0.1067193676\,\Phi_2 - 0.2019104084\,\Phi_1,$$

$$\chi_2 + \chi_4 + \chi_6 + \chi_8 + \chi_{10} = -0.5\,\Phi_6 - 0.5\,\Phi_5 + 1.666666667\,\Phi_4 + 0.2885375494\,\Phi_2 - 0.1024374177\,\Phi_1,$$

$$\theta_1 + \theta_5 + \theta_7 + \theta_{11} = 2\,\Phi_{12} - 2\,\Phi_{10} + 1.333333333\,\Phi_4 - 4\,\Phi_3 + 1.108036891\,\Phi_2 - 0.1518445325\,\Phi_1,$$

$$\theta_2 + \theta_{10} = \Phi_{12} + 2\,\Phi_{11} + \Phi_{10} - 0.6666666667\,\Phi_4 - 2.739130435\,\Phi_2 + 0.6793478262\,\Phi_1,$$

$$\theta_3 + \theta_9 = -2\,\Phi_{12} - 2\,\Phi_{10} + 0.6666666667\,\Phi_4 - 2\,\Phi_3 + 1.572463768\,\Phi_2 - 0.2934782608\,\Phi_1,$$

$$\theta_4 = 0.5\,\Phi_{12} - \Phi_{11} + 0.5\,\Phi_{10} + 0.3333333333\,\Phi_9 + 0.25\,\Phi_8 - 0.5\,\Phi_7 - 0.3333333333\,\Phi_4 + 0.6304347826\,\Phi_2 - 0.2019927536\,\Phi_1,$$

$$\theta_6 = -\Phi_{12} + \Phi_{11} - \Phi_{10} + 0.5\,\Phi_8 - 0.3333333333\,\Phi_4 - 0.2862318841\,\Phi_2 + 0.06884057973\,\Phi_1,$$

$$\theta_8 = 0.5\,\Phi_{12} - \Phi_{11} + 0.5\,\Phi_{10} - 0.3333333333\,\Phi_9 + 0.25\,\Phi_8 + 0.5\,\Phi_7 - 0.3333333333\,\Phi_4 + 0.4637681159\,\Phi_2 - 0.0769927536\,\Phi_1,$$

$\xi_1+\xi_2 = \Phi_6 - \Phi_5 - 0.3333333333\ \Phi_4 + \Phi_3 + 0.2213438735\ \Phi_2 - 0.1903820817\ \Phi_1,$

$\eta_1+\eta_2 = -\Phi_{12} - \Phi_{11} - \Phi_{10} + 0.3333333333\ \Phi_9 - 0.5\Phi_8 + \Phi_7 - 0.3333333333\ \Phi_4 + 1.547101449\ \Phi_2 - 0.3478260868\ \Phi_1.$

Therefore $\mathcal{A}(\mathcal{SL}(2,23)) = 12144\chi_1$.

3.2 The Result for the group $\mathcal{SL}(2,29)$

From [4] the (c.h.t.) of (r.r.) of the groups $\mathcal{SL}(2,29)$ is

Table 3. The (c.h.t.) of (r.r.) of the groups $\mathcal{SL}(2,29)$

C_g	1	z	c	zc	a	a^2	a^4	b	b^2	b^3	b^6
$\lvert C_g \rvert$	1	1	420	420	870	870	870	812	812	812	812
$\lvert C_G(g) \rvert$	24360	24360	58	58	28	28	28	30	30	30	30
1_G	1	1	1	1	1	1	1	1	1	1	1
Ψ	29	29	0	0	1	1	1	-1	-1	-1	-1
$\chi_1+\chi_3+\chi_5+\chi_9+\chi_{11}+\chi_{13}$	180	-180	6	-6	0	2	-2	0	0	0	0
$\chi_2+\chi_4+\chi_6+\chi_8+\chi_{10}+\chi_{12}$	180	180	6	6	0	-2	-2	0	0	0	0
χ_7	30	-30	1	-1	0	-2	2	0	0	0	0
$\theta_1+\theta_3+\theta_7+\theta_9+\theta_{11}+\theta_{13}$	168	-168	-6	6	0	0	0	0	0	-3	3
$\theta_2+\theta_4+\theta_6+\theta_8+\theta_{12}+\theta_{14}$	168	168	-6	-6	0	0	0	0	0	3	3
θ_5	28	-28	-1	1	0	0	0	-1	1	2	-2
θ_{10}	28	28	-1	-1	0	0	0	1	1	-2	-2
$\xi_1+\xi_2$	30	30	1	1	-2	2	2	0	0	0	0
$\eta_1+\eta_2$	28	-28	-1	1	0	0	0	2	-2	2	-2

The (in.ch.) table is:

Table 4. The (in.ch.) table for $\mathcal{SL}(2,29)$

C_g	1	z	c	zc	a	a^2	a^4	b	b^2	b^3	b^6
$\lvert C_g \rvert$	1	1	420	420	870	870	870	812	812	812	812
$\lvert C_G(g) \rvert$	24360	24360	58	58	28	28	28	30	30	30	30
Φ_1	24360	0	0	0	0	0	0	0	0	0	0
Φ_2	12180	12180	0	0	0	0	0	0	0	0	0
Φ_3	840	0	2	0	0	0	0	0	0	0	0
Φ_4	420	1260	3	3	0	0	0	0	0	0	0
Φ_5	870	1740	0	0	2	0	0	0	0	0	0
Φ_6	1740	3480	0	0	0	4	0	0	0	0	0
Φ_7	870	1740	0	0	0	0	2	0	0	0	0
Φ_8	812	1624	0	0	0	0	0	2	0	0	0
Φ_9	1624	0	0	0	0	0	0	0	2	0	0
Φ_{10}	2436	4872	0	0	0	0	0	0	0	6	0
Φ_{11}	1624	0	0	0	0	0	0	0	0	0	2

Hence,

$$1= 0.5\,\Phi_{11} + 0.1666666667\,\Phi_{10} + 0.5\,\Phi_9 + 0.5\,\Phi_8 + 0.5\,\Phi_7 + 0.25\,\Phi_6 + 0.5\,\Phi_5 + 0.3333333333\,\Phi_4 - 0.3820197044\,\Phi_2 - 0.3502873563\,\Phi_1,$$

$$\Psi = -0.5\,\Phi_{11} - 0.1666666667\,\Phi_{10} - 0.5\,\Phi_9 - 0.5\,\Phi_8 + 0.5\,\Phi_7 + 0.25\,\Phi_6 + 0.5\,\Phi_5 + 0.2071428571\,\Phi_2 - 0.05595238093\,\Phi_1,$$

$$\chi_1 + \chi_3 + \chi_5 + \chi_9 + \chi_{11} + \chi_{13} = -\Phi_7 + 0.5\,\Phi_6 - 2\,\Phi_4 + 6\,\Phi_3 + 0.1921182266\,\Phi_2 - 0.2610837438\,\Phi_1,$$

$$\chi_2 + \chi_4 + \chi_6 + \chi_8 + \chi_{10} + \chi_{12} = -\Phi_7 - 0.5\Phi_6 - 2\Phi_4 + 0.09359605911\,\Phi_2 - 0.002463054186\,\Phi_1,$$

$$\chi_7 = \Phi_7 - 0.5\Phi_6 - 0.3333333333\,\Phi_4 + \Phi_3 + 0.03201970443\,\Phi_2 - 0.009031198685\,\Phi_1,$$

$$\theta_1 + \theta_3 + \theta_7 + \theta_9 + \theta_{11} + \theta_{13} = 1.5\,\Phi_{11} - 0.5\,\Phi_{10} + 2\,\Phi_4 - 6\,\Phi_3 - 0.02068965517\,\Phi_2 + 0.1689655172\,\Phi_1,$$

$$\theta_2 + \theta_4 + \theta_6 + \theta_8 + \theta_{12} + \theta_{14} = 1.5\,\Phi_{11} + 0.5\,\Phi_{10} - 2\,\Phi_4 + 0.02068965517\,\Phi_2 - 0.1196551724\,\Phi_1,$$

$$\theta_5 = -\Phi_{11} + 0.3333333333\,\Phi_{10} + 0.5\,\Phi_9 - 0.5\,\Phi_8 + 0.3333333333\,\Phi_4 - \Phi_3 + 0.1034482759\,\Phi_2 - 0.005172413812\,\Phi_1,$$

$$\theta_{10} = -\Phi_{11} - 0.3333333333\,\Phi_{10} + 0.5\,\Phi_9 + 0.5\,\Phi_8 - 0.3333333333\,\Phi_4 + 0.1034482759\,\Phi_2 + 0.005172413774\,\Phi_1,$$

$$\xi_1 + \xi_2 = \Phi_7 + 0.5\,\Phi_6 - \Phi_5 + 0.3333333333\,\Phi_4 - 0.1748768473\,\Phi_2 + 0.04720853859\,\Phi_1,$$

$$\eta_1 + \eta_2 = -\Phi_{11} + 0.3333333333\,\Phi_{10} - \Phi_9 + \Phi_8 + 0.3333333333\,\Phi_4 - \Phi_3 - 0.3034482759\,\Phi_2 - 0.2482758621\Phi_1.$$

Therefore $\mathcal{A}(\mathcal{SL}(2,29)) = 24360\chi_1$.

4- Conclusion

In this paper, we compute the (A.d.) for the groups $\mathcal{SL}(2,23)$ and $\mathcal{SL}(2,29)$ from (r.v.ch.) of (r.r).

Conflicts Of Interest

No conflicts in this paper of interest.

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