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Some Dominating Applications on Discrete Topological Graphs

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Abstract:

In this paper several types of dominating parameters are applied on a discrete topological graph $G_\tau = (V, E)$ created from the topological space (X, τ), where $V(G_\tau)$ is the set of vertices in G_τ that includes all subset of τ unless X , $\emptyset$, and $E(G_\tau)$ is a set of edges between any two vertices in which any two vertices in G_τ such that $|A| + |B| \geq |X|$; $A \neq B$. Some dominating parameters on G_τ are studied such as: general domination, connected domination, doubly connected domination, and polynomial domination. Besides the minimum dominating set is proved for any topological graph formed by corona operation or join operation between two topological graphs. Also, the inverse domination is discussed.

Keywords: Discrete Topological Graph, Domination number, Inverse domination, Polynomial domination.

1-Introduction

Let $G = (V, E)$ be a simple graph has no isolated vertices, connected, undirected graph. The order of a graph G is the number of the vertices in a given graph and is denoted by $|V(G)|$ The degree of any vertex in a graph is the number of the edges that incident on it [24]. $\Delta(G_\tau)$ is maximum degree and $\delta(G)$ is minimum degree. For any two vertices $w, v \in V(G)$ if there is an edge between them, then the are adjacent. The isolated vertex v of degree zero. If one or more of the vertices of D are adjacent to each vertex of V, then the subset D is referred to as the dominant set [23]. The domination number is the order of a minimum dominating set $\gamma(G)$. Let G be a topoplogical graph with a minimum dominating set D. If $V - D$ is dominating set w.r.t D, then $V - D$ is called inverse dominating, denotrd by γ^{-1} [4,7,18]. Join of G_1 and G_2 is graph having vertex set $V(G_1) \cup V(G_2)$ and edge $E(G_1) \cup E(G_2) \cup \{uv: u \in V(G_1) \text{and } v \in V(G_2)\}$. Corona is taking one copy of G_1 and p_1 copies of G_2 and joining the j^{th} vertex of to all vertices j^{th} copy of G_2[12]. Also, if G[D] is connected, then it is called a connected dominating set. And if both G[D] and $G[V - D]$ are connected D is a doubly connected subgraphs . In 2023 studied of transforming the discrete topology into a graph by Mohammad Abdali and Zainab Naeem, when placed a condition on edges such that $E(G_\tau) = \{A\ B; A \not\subseteq B \text{ and } B \not\subseteq A\}$ see [15]. In this present a new study of transforming the discrete topology into a graph with new condition on edges such that $E(G_\tau) = \{A\,B;\ |A| + |B| \geq |X|;\ A \neq B\}$. The terms "domination number" and "dominating set" were first used by Ore [21] in 1962. Also, many researches have applied different types of domination graph. M. Al-Harere and M. Abdlhusein presented the pitchfork domination, a new model of domination in graphs [9] in 2020. Stability of inverse pitchfork domination was introduced by M. A. Abdhusein [3] in 2021. The definition of arrow domination was put up by Suha Jaber Radhi et al. [22], and its characteristics were determined in 2021. It has been studied many result domination on discrete topological graph et al [13,16,19].

2- Domination in Topological graphs

- **Definition 2.1**: Let X be a nonempty set of order n and let τ be a discrete topology graph on X. The discrete topological graph DTG denoted by $G_\tau = (V, E)$ is a graph has a vertices set $V = \{A; A \in \tau,$ A is nonempty proper subset of X$\}$ and edges set $E = \{AB;\ |A| + |B| \geq |X|\ ; A, B \in V(G) \wedge A \neq B\}$.
- **Theorem 2.2**: Let G_τ be a DTG on a non-empty set X, then $\gamma(G_\tau) = \gamma^{-1}(G_\tau) = 1$.

Proof : If $n = 2$, then $G_\tau \cong K_2$ by definition 2.1. Let v be any vertex in G_τ. Since v is adjacent with only one vertex. Thus, the minimum dominating set of a graph G_τ has only vertex say v such that $D = \{v\}$. Hence, $\gamma(G_\tau) = 1$. Now, by same technique of proof above $D^{-1} = \{v'\}$. Hence, $\gamma^{-1}(G_\tau) = 1$. See Figure 2.1. If $n \geq 3$, let v be any vertex has $n - 1$ elements in G_τ [S] and u be any another vertex in G_τ. By condition $|v| + |u| \geq |X|$, then v is adjacent with all vertices in G_τ. So, the vertex v dominates all vertices in a graph G_τ. Thus, the graph has $D = \{v\}$ is a minimum dominating set in G_τ. Therefore, $\gamma(G_\tau) = 1$. As an example see Figure 2.1. Now, by same technique of proof above, we take v' be any another vertex has $n - 1$ elements. Also, the vertex v' dominates all vertices in G_τ. Then, the minimum inverse dominating set is $D^{-1} = \{v'\}$. Therefore, $\gamma^{-1}(G_\tau) = 1$. For an example, see Figure 1.

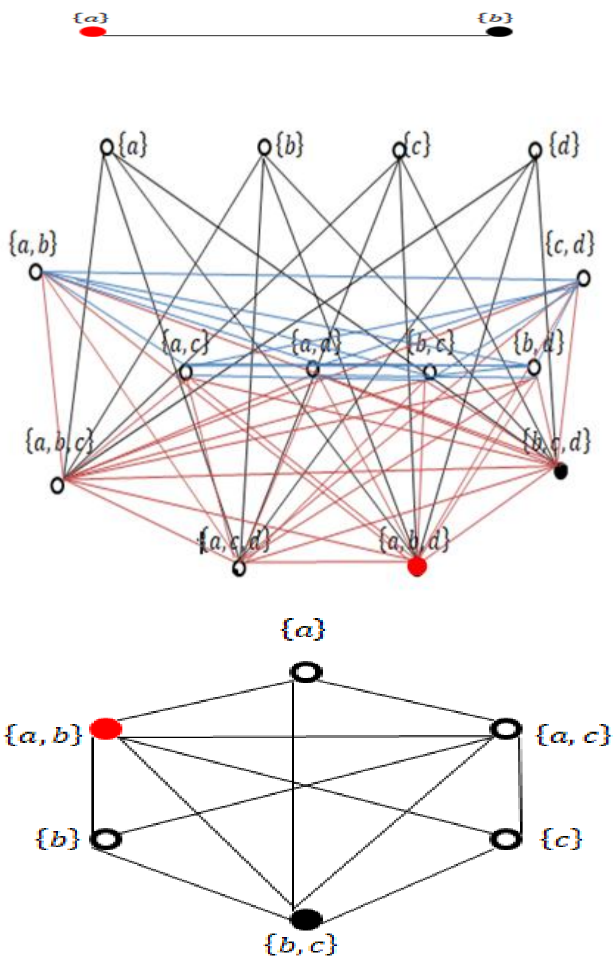


Figure 1. Invers minimum dominating set of G_τ for $|X| = 2, 3, 4, 5$.

- **Theorem 2.3**: Let G_τ be a DTG on a non-empty set X and H_τ be a DTG on a non-empty set Y such that $|X| = n$ and $|Y| = m$, then $\gamma(G_\tau \odot H_\tau) = \gamma^{-1}(G_\tau \odot H_\tau) = 2^n - 2$

Proof By definition of corona operation $G_\tau \odot H_\tau$, every vertex in G_τ is adjacent to all vertices of the corresponding one copy of graph H_τ . Since for every vertex $v_i \in V(G_\tau)$, then v_i is adjacent with all vertices of the i^{th} copy of H_τ. Thus, v_i dominates all vertices of i^{th} copy of H_τ. So, $v_i \in D$. Then, $D = V(G_\tau)$ is the minimum dominating set of a graph $G_\tau \odot H_\tau$. Since the order of a graph G_τ is $2^n - 2$ according to definition 2.1. Thus, the order of minimum dominating set D is $2^n - 2$. If D is not minimum dominating set let D' be a dominating set of $G_\tau \odot H_\tau$ such that $|D'| \leq |D|$. Hence, there exist at least one vertex in $V - D$ which is not dominated by any vertex of D'. Then, D is the dominating set of a graph $G_\tau \odot H_\tau$. Therefore, $\gamma(G_\tau \odot H_\tau) = 2^n - 2$. As an example see Figure 2.2 and Figure 2.3. To proof the inverse dominating set. We take $v_i \in V(H_\tau)$ such that v_i that have $n - 1$ elements. Since v_i dominates all vertices of H_τ according to Theorem 2.2 and dominates one from G_τ . Then, D^{-1} consist of one vertex from every copy of H_τ. Also, the number of all these vertices that belong to D^{-1} formed the minimum invers dominating set. Therefore, $\gamma^{-1}(G_\tau \odot H_\tau) = 2^n - 2$. As an example see Figure 2 and Figure 3.

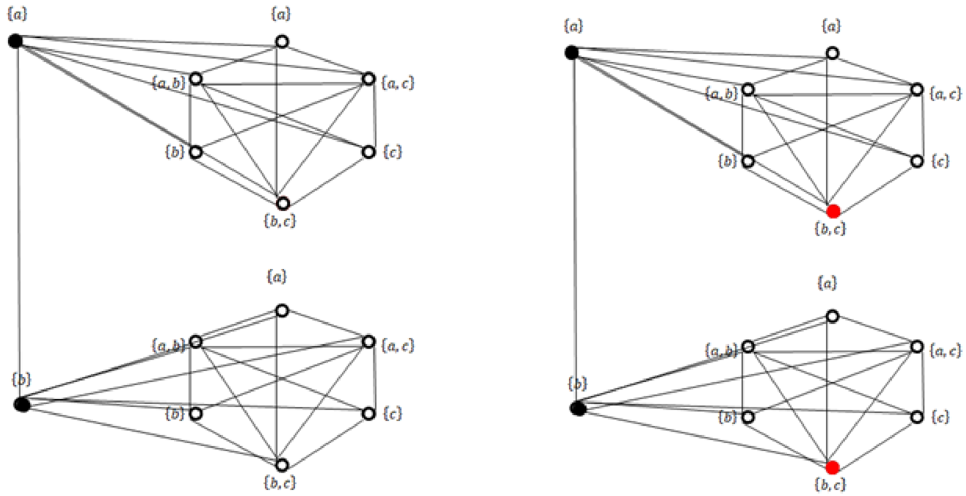


Figure 2. Minimum dominating and inverse dominating sets for $\mathbf{G_\tau \odot H_\tau}$.

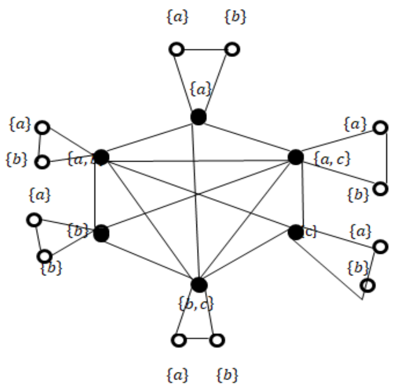


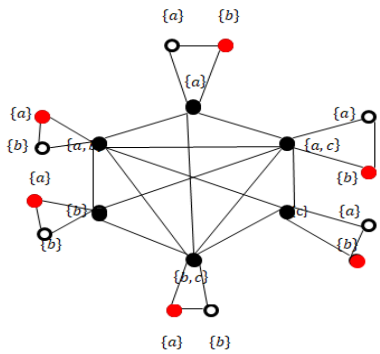
Figure 3. Minimum dominating set and invers dominating set for $\mathbf{G_\tau \odot H_\tau}$.

Theorem 2.4: Let G_τ be a DTG on set X and H_τ be a DTG on set Y such that $|X| = n$ and $|Y| = m$, then $\gamma(G_\tau + H_\tau) = \gamma^{-1}(G_\tau + H_\tau) = 1$.

Proof : Let v be any vertex in G_τ has $n - 1$ elements. Since this vertex is adjacent with all vertices in G_τ by proof of Theorem 2.2. Also, this vertex is adjacent with all vertices of a graph H_τ by definition of join operations in graph $G_\tau + H_\tau$. Where, v is dominates all vertices of $G_\tau + H_\tau$. So, $D = \{v\}$ is the dominating set of $G_\tau + H_\tau$. Therefore, $\gamma(G_\tau + H_\tau) = 1$ is the domination number. As an example see Figure 2.4. Now, to find the invers dominating set of a graph $G_\tau + H_\tau$ we take v' another vertex of G_τ has $n - 1$ elements in G_τ . Then, v is adjacent with all vertices of $G_\tau + H_\tau$. Therefore, the minimum inverse dominating set is $D^{-1} = \{v'\}$. Also, the inverse domination number of a graph $G_\tau + H_\tau$ is $\gamma^{-1}(G_\tau + H_\tau) = 1$. As an example see Figure5.

- **Proposition 2.4.** Assume that G be any graph of order m has no $\gamma_{dm} -$ set and is not null graph and has no isolated vertex, then $\gamma_{dm}(K_1 \odot G) = \gamma(G)$.

Proof. Since G has no $\gamma_{dm} -$ set, but G has $\gamma -$ set and since there exists one vertex in K_1 is adjacent with all vertices of G and G is not null graph and has no isolated vertex, then each vertex in D of G dominates at least one vertex of G, let $v \in K_1$ belongs to $V - D$. Then every vertex in D dominates at least one vertex from D of G and the vertex v of K_1. Then, each vertex of D in G dominates two or more vertices of $V - D$. Then, each vertex of D dominates two or more vertices of $V - D$ and G[D] is disconnected graph such that the vertex v of K_1 belongs to $V - D$. Hence, $\gamma_{dm}(K_1 \odot G) = \gamma(G)$ and $\gamma_{dm} -$ set. For example, see Figure 4.



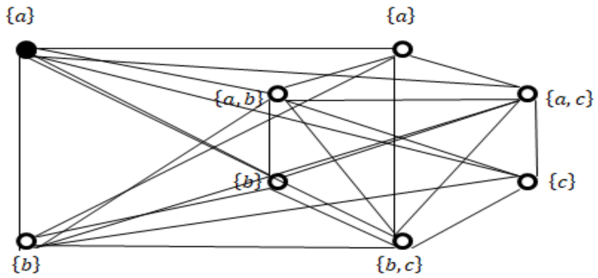


Figure 4. Minimum dominating set in $G_\tau + H_\tau$.

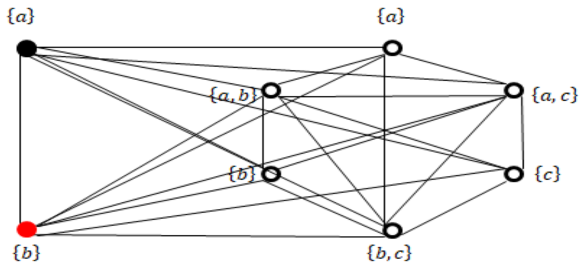


Figure 5. Invers minimum dominating set in $K_2 + G_\tau$ on $|X| = 3$.

- **Observation 2.5:** For any graph G_τ of order $2^n - 2$ has domination number. If $\gamma(G_\tau) \geq \frac{2^n-2}{2}$, then G_τ has no inverse dominating set.
- **Theorem 2.6:** Let $|X| = n$ and G_τ be a DTG on a non-empty set X, then G_τ have connected and doubly connected dominating sets such that:

1) $\gamma_c(G_\tau) = \gamma_c{}^{-1}(G_\tau) = 1$.

2) $\gamma_{cc}(G_\tau) = \gamma_{cc}{}^{-1}(G_\tau) = 1$.

Proof:1. Depending on Theorem 2.2, since $\gamma(G_\tau) = 1$, then G[D] is connected subgraph. Hence, G_τ has connected dominating set and $\gamma_c(G_\tau) = \gamma(G_\tau) = 1$. So that, since $\gamma^{-1}(G_\tau) = 1$, then $G[D^{-1}]$ is connected subgraph and $\gamma^{-1}(G_\tau) = \gamma_c{}^{-1}(G_\tau)1$.

2. Since G_τ has no cut vertex by Proposition 2.15 see [17] and G_τ is connected graph by proof of Theorem 2.9 see [17]. Then, G[D] and G[V − D] are connected subgraphs. Also that, $G[D^{-1}]$ and $G[V - D^{-1}]$ Hence, $\gamma_{cc}(G_\tau) = \gamma_{cc}{}^{-1}(G_\tau) = 1$.

- **Definition 2.7:** [8] Let G be a topological graph on set X and G is simple graph and D(G, i) the family of dominating sets of a graph G with cardinality i, defined domination polynomial as :

$$D(G,X) = \sum_{i=\gamma(G)}^{k} d(G,i)x^i$$

where $\gamma(G)$ is a minimum domination number with k vertices in G and $k \geq 2$.

- **Example 2.18:** Let $|X| = n$ and G_τ be a DTG on set X. Then, the domination polynomial of G_τ given as follows:

1. If $|X| = 2$, then

$D(G_\tau, X) = \sum_{i=\gamma(G_\tau)}^{k} d(G_\tau, i)x^i$ where $\gamma(G_\tau) = 1$.

$D(G_\tau, X) = \sum_{i=1}^{2} d(G_\tau, i)x^i$.

The number of dominating sets of order one equal 2 : {a} and {b}

The number of dominating sets of order two equal 1: $\{\{a\},\{b\}\}$

$D(G_\tau, X) = 2x^1 + x^2$.

2. If $|X| = 3$, then

$D(G_\tau, X) = \sum_{i=\gamma(G_\tau)}^{k} d(G_\tau, i)x^i$ where $\gamma(G_\tau) = 1$.

The number of dominating sets of order one equal 3: $\{a, b\}$ and $\{a, c\}$ and $\{b, c\}$. Therefore, $d(G_\tau, 1) = 3$

The number of dominating sets of order two equal 12 : $\{\{a, b\}, \{a, c\}\}$, $\{\{a, b\}, \{b, c\}\}$, $\{\{a, c\}, \{b, c\}\}, \{\{a, b\}, \{a\}\}, \{\{a, b\}, \{b\}\}, \{\{a, b\}, \{c\}\}, \{\{b, c\}, \{a\}\}, \{\{b, c\}, \{b\}\}, \{\{b, c\}, \{c\}\}$,

$\{\{a, c\}, \{a\}\}, \{\{a, c\}, \{b\}\}, \{\{a, c\}, \{c\}\}$. Therefore, $d(G_\tau, 2) = 12$.

The number of dominating sets of order three equal 20:

$\{\{a, b\}, \{a, c\}, \{b, c\}\}, \{\{a, b\}, \{a, c\}, \{a\}\}, \{\{a, b\}, \{a, c\}, \{b\}\}, \{\{a, b\}, \{a, c\}, \{c\}\}, \{\{a, b\}, \{a\}, \{b, c\}\}$

$, \{\{a, b\}, \{b\}, \{b, c\}\}, \{\{a, b\}, \{c\}, \{b, c\}\}, \{\{b, c\}, \{a, c\}, \{a\}\}, \{\{b, c\}, \{a, c\}, \{b\}\}, \{\{b, c\}, \{a, c\}, \{c\}\}$

$\{\{a\}, \{b\}, \{c\}\}, \{\{a, b\}, \{b\}, \{a\}\}, \{\{b, c\}, \{a\}, \{b\}\}, \{\{a\}, \{a, c\}, \{b\}\}, \{\{a, b\}, \{a\}, \{c\}\}$,

$\{\{b, c\}, \{a\}, \{c\}\}, \{\{a\}, \{a, c\}, \{c\}\}, \{\{a, b\}, \{b\}, \{c\}\}, \{\{b, c\}, \{b\}, \{c\}\}, \{\{c\}, \{a, c\}, \{b\}\}$.

Therefore, $d(G_\tau, 3) = 20$

$$D(G_\tau, X) = \sum_{i=1}^{3} d(G_\tau, i)x^i = d(G_\tau, 1)x^1 + d(G_\tau, 2)x^2 + d(G_\tau, 3)x^3.$$

Then, the domination polynomial is $D(G_\tau, X) = 3x^1 + 12x^2 + 20x^3$.

3- Conclusion

We conclude that some of domination in the DTG. Such as: the general domination, corona domination set, join operations between two graphs, connected domination, doubly connected domination, and polynomial domination. Also, some of parameters were shown to have inverse dominance.

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