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INTUITIONISTIC FUZZY RECTANGULAR *n*-NORMED SPACES

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Abstract:

The primary the intention of this article is to present the concept of intuitionistic fuzzy rectangular *n*-normed space as a generalization of fuzzy rectangular *n*-normed space and to prove some theorems on this subject. Afterward, we study the concept of ascending family of α-*n*-norms corresponding to intuitionistic fuzzy rectangular *n*-norm. Also, we present and discuss some basic properties of convergence and completeness for intuitionistic fuzzy rectangular *n*-normed space.

Keywords: Fuzzy rectangular *n*-normed space, Rectangular *n*-normed space, Intuitionistic fuzzy *n*-normed space, Intuitionistic fuzzy rectangular *n*-normed space.

1- Introduction

Atanassove [1] introduced the notion of intuitionistic fuzzy sets, building upon Zadeh [23] concept and many authors have studied and developed this concept [2, 3, 14]. In 2006, Saadati and Park [18] presented the concept of intuitionistic fuzzy normed space. Later, many researchers developed and extended this concept, and studies on it can be found in [13, 19, 21]. Gähler [12] presented the theory of *n*-normed space. Balkunder and Gunawan [5] developed the theory of *n*-normed space. Narayan and Vijayabalaji [16] introduced the definition of fuzzy *n*-normed space. They also studied an as ascending family of *α*-*n*-norms corresponding to fuzzy *n*-normed space and provided some results about it. Vijayabalaji and Thillaigovindan et al. [20] studied and further developed the idea of intuitionistic fuzzy *n*-normed space. Branciari [4] studied the concept of rectangular metric space. Later, Muteer and Mohammed [15] have given a definition of intuitionistic fuzzy rectangular b-normed space and studied some results about it. Bader and Mohammed [6] recently defined the concepts of rectangular *n*-normed space and a fuzzy rectangular *n*-normed space. Some works and results about the convergence of sequences in multiple normed spaces in a fuzzy environment can be found in [7-11, 22].

In this paper, we present the concept of intuitionistic fuzzy rectangular *n*-norm on a linear space as a generalization of fuzzy rectangular *n*-normed space due by Bader and Mohammed [6]. After that we study the concept of ascending family of *α*-*n*-norms corresponding to intuitionistic fuzzy rectangular *n*-norm, and study the completeness for intuitionistic fuzzy rectangular n-normed space.

In this paper, we denote by $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ the sets of real, natural and complex numbers, respectively. Additionally, $\mathbb{H}$ denoted the field of real or complex numbers.

2- Preliminaries

In this section, we reproduce the following definitions due to Bader and Mohammed [6], Schweizer and Sklar [17].

- **Definition 2.1**: [6] Let $\dot{\mathcal{X}}$ be a vector space of dimension d $\geq n$, $n \in \mathbb{N}$. A rectangular n-norm in $\dot{\mathcal{X}}$ is a function $\|.,\ldots,.\|$ in $\dot{\mathcal{X}} \times \dot{\mathcal{X}} \times \ldots \dot{\mathcal{X}} \times \dot{\mathcal{X}} = \dot{\mathcal{X}}^n$ satisfying the following condition for every $w_1, w_2, \ldots, w_n, \mathbb{z}, \mathfrak{h} \in \dot{\mathcal{X}}$
  1. $\| w_1, w_2, \ldots, w_n\| = 0 \Leftrightarrow w_1, w_2, \ldots, w_n$ are linearly dependent,
  2. $\| w_1, w_2, \ldots, w_n\|$ is invariant under any permutation,
  3. $\|\rho w_1, \rho w_2,\ldots, \rho\, w_n\| = |\rho|\, \| w_1, w_2, \ldots, w_n\|$ for any $\rho \in \mathbb{R}$,
  4. $\| w_1, w_2, \ldots, w_n + \mathbb{z} + \mathfrak{h}\| \leq \| w_1, w_2, \ldots, w_n \| + \| w_1, w_2, \ldots, \mathbb{z}\| + \| w_1, w_2, \ldots, \mathfrak{h}\|$.

  $\|.,\ldots,.\|$ is called rectangular n-norm in $\dot{\mathcal{X}}$ and the pair $(\dot{\mathcal{X}}, \|.,\ldots,.\|)$ is called rectangular n-normed space (for short *r-n-NS*).

- **Definition 2.2**: [17] A continuous t-norm $*$ is a binary operation on the interval [0,1], which satisfies the following axioms:
  1. For each e $\in$ [0,1] implies that $e * 1 = e$;
  2. $*$ is associative and commutative;
  3. $*$ is continuous;
  4. For each $e, s, z, d \in [0,1]$ and $e \leq z$ and $s \leq d$ implies that $e * s \leq z * d$.

- **Definition 2.3**: [17] A continuous t-conorm $\lozenge$ is a binary operation on the interval [0,1] which satisfies the following axioms:
  1. For each e $\in$ [0,1] implies that $e \lozenge 0 = e$;
  2. $\lozenge$ is associative and commutative;
  3. $\lozenge$ is continuous;
  4. For each $e, s, z, d \in [0,1]$ and $e \leq z$ and $s \leq d$ implies that $e \lozenge s \leq z \lozenge d$.

- **Definition 2.4**: [6] Let $\dot{\mathcal{X}}$ be avector space over $\mathbb{H}$, $*$ be a continuous t-norm. Then the 3-tuple $(\dot{\mathcal{X}}, \Upsilon, *)$ is called a fuzzy rectangular n-normed space (for short *f-r-n-NS*) in $\dot{\mathcal{X}}$, where Υ is a fuzzy set in $\dot{\mathcal{X}}^n\times(0, \infty)$ satisfying the following condition for every $w_1, w_2, \ldots, w_n, \mathbb{z}, \mathfrak{h} \in \dot{\mathcal{X}}$ and $\ell, \mathfrak{r}, \mathfrak{e} > 0$:
  1. $\Upsilon(w_1, w_2, \ldots, w_n, \ell) = 0$, for all $\ell \in \mathbb{R}$ with $\ell \leq 0$,
  2. $\Upsilon(w_1, w_2, \ldots, w_n, \ell) = 1 \Leftrightarrow w_1, w_2, \ldots, w_n$ are linearly dependent,
  3. $\Upsilon(w_1, w_2, \ldots, w_n, \ell)$ is invariant under any permutation of $w_1, w_2, \ldots, w_n$,
  4. $\Upsilon(\rho w_1, \rho w_2,\ldots, \rho\, w_n, \ell) = \Upsilon(w_1, w_2, \ldots, w_n, \frac{\ell}{|\rho|})$, if $\rho \in \mathbb{H}\backslash\{0\}$,
  5. $\Upsilon(w_1, w_2, \ldots, w_n + \mathbb{z} + \mathfrak{h}, \ell + \mathfrak{r} + \mathfrak{e}) \geq \Upsilon(w_1, w_2, \ldots, w_n, \ell) * \Upsilon(w_1, w_2, \ldots, \mathbb{z}, \mathfrak{r}) * \Upsilon(w_1, w_2, \ldots, \mathfrak{h}, \mathfrak{e})$,
  6. $\Upsilon(w_1, w_2, \ldots, w_n, \ell)$ is anon-decreasing function of $\ell \in \mathbb{R}$, $\lim_{\ell\to\infty}\Upsilon(w_1, w_2, \ldots, w_n, \ell) = 1$.

  Hence, (Υ) is called afuzzy rectangular n-norm in $\dot{\mathcal{X}}$.

- **Definition 2.5**: [6] Let $(\dot{\mathcal{X}}, \Upsilon, *)$ be a *f-r-n-NS*. A sequence $\{ w_n\}$ in $\dot{\mathcal{X}}$ is called convergent to w, if given $\varphi > 0$, $\ell > 0$, $0 < \varphi < 1$, $\exists\, n_0 \in \mathbb{N}$ in which,

$$\Upsilon(w_1, w_2, \ldots, w_{n-1}, w_n - w, \ell) > 1 - \varphi\ , \forall\, n \geq n_0.$$

- **Definition 2.6**: [6] Let ($\dot{\mathcal{X}}$, Υ, $*$) be a *f-r-n-NS*. A sequence $\{w_n\}$ in $\dot{\mathcal{X}}$ is called Cauchy sequence if, a given $\varphi > 0$ with $0 < \varphi < 1$ and $\ell > 0$ $\exists$ $n_0 \in \mathbb{N}$ in which,

$$\Upsilon(w_1, w_2, \ldots, w_{n-1}, w_n - w_s, \ell) > 1 - \varphi, \forall\, n, s \geq n_0.$$

- **Definition 2.7**: [6] A *f-r-n-NS* ($\dot{\mathcal{X}}$, Υ, $*$) is called complete if, every Cauchy sequence converges.

3- Intuitionistic Fuzzy Rectangular *n*-Normed Spaces

In this section we discuss the concept of an intuitionistic fuzzy rectangular *n*-normed space and prove some results related about it.

- **Definition 3.1**: Let $\dot{\mathcal{X}}$ be a vector space over $\mathbb{H}$, $*$ be a continuous t-norm and $\Diamond$ be a continuous t-conorm. Then the *5*-tuple ($\dot{\mathcal{X}}$, Υ, $\dot{\mathcal{F}}$, $*$, $\Diamond$) is called an intuitionistic fuzzy rectangular *n*-normed space (for short, *i-f-r-n-NS*) in $\dot{\mathcal{X}}$, where Υ, $\dot{\mathcal{F}}$ are fuzzy sets in $\dot{\mathcal{X}}^n \times (0, \infty)$ satisfying the following condition for every $w_1, w_2, \ldots, w_n$, $\mathbb{z}$, $\mathfrak{h} \in \dot{\mathcal{X}}$ and $\ell, \mathfrak{r}, \mathfrak{s} > 0$:
  1. $\Upsilon(w_1, w_2, \ldots, w_n, \ell) + И(w_1, w_2, \ldots, w_n, \ell) \leq 1$,
  2. $\Upsilon(w_1, w_2, \ldots, w_n, \ell) = 0$, for all $\ell \in \mathbb{R}$ with $\ell \leq 0$,
  3. $\Upsilon(w_1, w_2, \ldots, w_n, \ell) = 1 \Leftrightarrow$ $w_1, w_2, \ldots, w_n$ are linearly dependent,
  4. $\Upsilon(w_1, w_2, \ldots, w_n, \ell)$ is invariant under any permutation of $w_1, w_2, \ldots, w_n$,
  5. $\Upsilon(\rho w_1, \rho w_2, \ldots, \rho\, w_n, \ell) = \Upsilon(w_1, w_2, \ldots, w_n, \frac{\ell}{|\rho|})$, if $\rho \in \mathbb{H}\backslash\{0\}$,
  6. $\Upsilon(w_1, w_2, \ldots, w_n + \mathbb{z} + \mathfrak{h}, \ell + \mathfrak{r} + \mathfrak{s}) \geq \Upsilon(w_1, w_2, \ldots, w_n, \ell) * \Upsilon(w_1, w_2, \ldots, \mathbb{z}, \mathfrak{r}) * \Upsilon(w_1, w_2, \ldots, \mathfrak{h}, \mathfrak{s})$,
  7. $\Upsilon(w_1, w_2, \ldots, w_n, \ell)$ is anon-decreasing function of $\ell \in \mathbb{R}$ and $\lim_{\ell\to\infty} \Upsilon(w_1, w_2, \ldots, w_n, \ell) = 1$,
  8. $\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) = 1$,
  9. $\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) = 0 \Leftrightarrow w_1, w_2, \ldots, w_n$ are linearly dependent,
  10. $\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell)$ is invariant under any permutation of $w_1, w_2, \ldots, w_n$,
  11. $\dot{\mathcal{F}}(\rho w_1, \rho w_2, \ldots, \rho\, w_n, \ell) = \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \frac{\ell}{|\rho|})$, if $\rho \in \mathbb{H}\backslash\{0\}$,
  12. $\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n + \mathbb{z} + \mathfrak{h}, \ell + \mathfrak{r} + \mathfrak{s}) \leq \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) \Diamond \dot{\mathcal{F}}(w_1, w_2, \ldots, \mathbb{z}, \mathfrak{r}) \Diamond \dot{\mathcal{F}}(w_1, w_2, \ldots, \mathfrak{h}, \mathfrak{s})$,
  13. $\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell)$ is anon-increasing function of $\ell \in \mathbb{R}$ and $\lim_{\ell\to\infty} \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) = 0$.

  Hence, (Υ, $\dot{\mathcal{F}}$) is called an intuitionistic fuzzy rectangular *n*-norm in $\dot{\mathcal{X}}$.

- **Definition 3.2**: Let ($\dot{\mathcal{X}}$, Υ, $\dot{\mathcal{F}}$, $*$, $\Diamond$) be an *i-f-r-n-NS*. A sequence $\{w_n\}$ in $\dot{\mathcal{X}}$ is called convergent to *w*, if for each $\varphi \in (0,1)$ and $\ell > 0$, $\exists$ $n_0 \in \mathbb{N}$ in which,

$$\Upsilon(w_1, w_2, \ldots, w_{n-1}, w_n - w, \ell) > 1 - \varphi \text{ and } \dot{\mathcal{F}}(w_1, w_2, \ldots, w_{n-1}, w_n - w, \ell) < \varphi, \forall\, n \geq n_0.$$

(Or equivalently,

$$\lim_{n\to\infty} \Upsilon(w_1, w_2, \ldots, w_{n-1}, w_n - w, \ell) = 1 \text{ and } \lim_{n\to\infty} \dot{\mathcal{F}}(w_1, w_2, \ldots, w_{n-1}, w_n - w, \ell) = 0).$$

- **Definition 3.3**: Let ($\dot{\mathcal{X}}$, Υ, $\dot{\mathcal{F}}$, $*$, $\Diamond$) be an *i-f-r-n-NS*. A sequence $\{w_n\}$ in $\dot{\mathcal{X}}$ is called Cauchy if for each $\varphi \in (0,1)$ and $\ell > 0$, there is $n_0 \in \mathbb{N}$ in which,

$$\Upsilon(w_1, w_2, \ldots, w_{n-1}, w_n - w_s, \ell) > 1 - \varphi, \dot{\mathcal{F}}(, w_1, w_2, \ldots, w_{n-1}, w_n - w_s, \ell) < \varphi, \text{ for all } n, s \geq n_0.$$

(Or equivalently,

$$\lim_{n,s\to\infty} \Upsilon(w_1, w_2, \ldots, w_{n-1}, w_n - w_s, \ell) = 1 \text{ and } \lim_{n,s\to\infty} \dot{\mathcal{F}}(w_1, w_2, \ldots, w_{n-1}, w_n - w_s, \ell) = 0).$$

- **Definition 3.4**: An *i-f-r-n-NS* $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}})$ is called complete if every Cauchy sequence converges.

- **Example 3.5**: Let $(\dot{\mathcal{X}}, \|.,\ldots,.\|)$ be a *r-n-NS*. Define $c * \tilde{e} = c\,.\,\tilde{e}$ and $c \lozenge \tilde{e} = \min\{1, c + \tilde{e}\}$ for each $c, \tilde{e} \in [0, 1]$. Defined as follows:

$$\Upsilon(w_1, w_2, \ldots, w_n, \ell) = \frac{\ell}{\ell + ||w_1, w_2, \cdots, w_n||} \text{ and}$$

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) = \frac{||w_1, w_2, \cdots, w_n||}{\ell + ||w_1, w_2, \cdots, w_n||},\ \ell > 0 \text{ and } w_1, w_2, \ldots, w_n \in \dot{\mathcal{X}}.$$

So $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}}, *, \lozenge)$ is an *i-f-r-n-NS*. Hence $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}}, *, \lozenge)$ is called a standard intuitionistic fuzzy rectangular n-normed space (for short *St-i-f-r-n-NS*) induced by a *r-n-NS* $(\dot{\mathcal{X}}, \|.,\ldots,.\|)$.

Proof.

1. Clearly,

$$\Upsilon(,w_1, w_2, \ldots, w_n\ \ell) + \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) \leq 1.$$

First, in Proposition (3.6) [4], it is shown that Υ satisfies conditions from (2) to (7).
Now, we prove that $\dot{\mathcal{F}}$ satisfies conditions from (8) to (13).

8. Clearly, $\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) = 1.$

9.

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) = 0 \Leftrightarrow \frac{||w_1, w_2, \cdots, w_n||}{\ell + ||w_1, w_2, \cdots, w_n||}$$

$$= 0 \Leftrightarrow ||w_1, w_2, \ldots, w_n\,|| = 0 \Leftrightarrow w_1, w_2, \ldots, w_n,$$

are linearly dependent.

10.

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) = \frac{||w_1, w_2, \cdots, w_n||}{\ell + ||w_1, w_2, \cdots, w_n||}$$

$$= \frac{||w_1, w_2, \cdots, w_n, w_{n-1}||}{\ell + ||w_1, w_2, \cdots, w_n, w_{n-1}||} = \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, w_{n-1}, \ell) = \ldots .$$

11.

$$\dot{\mathcal{F}}(\rho w_1, \rho w_2, \ldots, \rho w_n, \ell) = \frac{||\rho w_1, \rho w_2, \cdots, \rho w_n||}{\ell + ||\rho w_1, \rho w_2, \cdots, \rho w_n||}$$

$$= \frac{||w_1, w_2, \ldots, w_n||}{\frac{\ell}{|\rho|} + ||w_1, w_2, \ldots, w_n||} = \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \frac{\ell}{|\rho|}),\ \forall \rho \in \mathbb{H}\backslash\{0\}.$$

12. Let $w_1, w_2, \ldots, w_n, \mathbb{z}, \mathfrak{h} \in \dot{\mathcal{X}}$, and $\ell, \mathfrak{r}, \mathfrak{e} > 0$

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n + \mathbb{z} + \mathfrak{h}, \ell + \mathfrak{r} + \mathfrak{e}) = \frac{||w_1, w_2, \cdots, w_n + \mathbb{z} + \mathfrak{h}||}{\ell + \mathfrak{r} + \mathfrak{e} + ||w_1, w_2, \cdots, w_n + \mathbb{z} + \mathfrak{h}||}$$

$$\leq \frac{||w_1, w_2, \cdots, w_n|| + ||w_1, w_2, \cdots, \mathbb{z}|| + ||w_1, w_2, \cdots, \mathfrak{h}||}{\ell + \mathfrak{r} + \mathfrak{e} + ||w_1, w_2, \cdots, w_n|| + ||w_1, w_2, \cdots, \mathbb{z}\,|| + ||w_1, w_2, \cdots, \mathfrak{h}||}$$

$$= \frac{||w_1, w_2, \cdots, w_n||}{\ell + \mathfrak{r} + \mathfrak{e} + ||w_1, w_2, \cdots, w_n|| + ||w_1, w_2, \cdots, \mathbb{z}|| + ||w_1, w_2, \cdots, \mathfrak{h}||}$$

$$+\frac{||w_1,w_2,\cdots,\mathbb{z}||}{\ell+\mathfrak{r}+\mathfrak{c}+||w_1,w_2,\cdots,w_n||+||w_1,w_2,\cdots,\mathbb{z}||+||w_1,w_2,\cdots,\mathfrak{h}||}$$

$$+\frac{||w_1,w_2,\cdots,\mathfrak{h}||}{\ell+\mathfrak{r}+\mathfrak{c}+||w_1,w_2,\cdots,w_n||+||w_1,w_2,\cdots,\mathbb{z}||+||w_1,w_2,\cdots,\mathfrak{h}||}$$

$$\leq \frac{||w_1,w_2,\cdots,w_n||}{\ell+||w_1,w_2,\cdots,w_n||}+\frac{||w_1,w_2,\cdots,\mathbb{z}||}{\mathfrak{r}+||w_1,w_2,\cdots,\mathbb{z}||}+\frac{||w_1,w_2,\cdots,\mathfrak{h}||}{\mathfrak{c}+||w_1,w_2,\cdots,\mathfrak{h}||}$$

$$= \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) + \dot{\mathcal{F}}(w_1, w_2, \ldots, \mathbb{z}, \mathfrak{r}) + \dot{\mathcal{F}}(w_1, w_2, \ldots, \mathfrak{h}, \mathfrak{c})$$

Since

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n + \mathbb{z} + \mathfrak{h}, \ell + \mathfrak{r} + \mathfrak{c}) \leq 1$$

then

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n + \mathbb{z} + \mathfrak{h}, \ell + \mathfrak{r} + \mathfrak{c})$$

$$\leq \min\{1, \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) + \dot{\mathcal{F}}(w_1, w_2, \ldots, \mathbb{z}, \mathfrak{r}) + \dot{\mathcal{F}}(w_1, w_2, \ldots, \mathfrak{h}, \mathfrak{c})\}$$

then

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n + \mathbb{z} + \mathfrak{h}, \ell + \mathfrak{r} + \mathfrak{c}) \leq$$

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) \lozenge \dot{\mathcal{F}}(w_1, w_2, \ldots, \mathbb{z}, \mathfrak{r}) \lozenge \dot{\mathcal{F}}(w_1, w_2, \ldots, \mathfrak{h}, \mathfrak{c}).$$

13. For all $\ell_1, \ell_2 \in \mathbb{R}$, if $\ell_1 < \ell_2 \leq 0$, then by our definition,

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell_1) = \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell_2) = 1.$$

Suppose $\ell_2 > \ell_1 > 0$, then

$$\frac{||w_1,w_2,\cdots,w_n||}{\ell_2+||w_1,w_2,\cdots,w_n||}-\frac{||w_1,w_2,\cdots,w_n||}{\ell_1+||w_1,w_2,\cdots,w_n||}$$

$$=\frac{||w_1,w_2,\cdots,w_n||(\ell_2-\ell_1)}{(\ell_2+||w_1,w_2,\cdots,w_n||)(\ell_1+||w_1,w_2,\cdots,w_n||)} \geq 0$$

For all $w_1, w_2, \ldots, w_n \in \dot{\mathcal{X}}$, implies

$$\frac{||w_1,w_2,\cdots,w_n||}{\ell_2+||w_1,w_2,\cdots,w_n||} \geq \frac{||w_1,w_2,\cdots,w_n||}{\ell_1+||w_1,w_2,\cdots,w_n||}$$

which in turn implies,

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell_2) \geq \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell_1).$$

Thus $\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell)$ is anon-increasing function.

As well,

$$\lim_{\ell\to\infty} \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) = \lim_{\ell\to\infty} \frac{||w_1,w_2,\cdots,w_n||}{\ell+||w_1,w_2,\cdots,w_n||} = 0.$$

Thus $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}}, *, \Diamond)$ is an *i-f-r-n-NS*.

- **Theorem 3.6**: Let $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}})$ be an *i-f-r-n-NS* in which,

1. $c * c = c$, $c \Diamond c = c$, for all $c \in [0,1]$.
2. $\Upsilon(w_1, w_2, \dots, w_n, \ell) > 0$, $\dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) < 1$, $\forall \ell > 0$ implies $w_1, w_2, \dots, w_n$ are linearly dependent.

Define

$$||w_1, w_2, \dots, w_n||_\alpha = \inf\{t: \Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1 - \alpha, \alpha \in (0,1)\}.$$

Hence $\{||.,\dots,.||_\alpha: \alpha \in (0,1)\}$ be an ascending family of rectangular n-norms in $\dot{\mathcal{X}}$. These rectangular n-norms are called rectangular α-n-norms in $\dot{\mathcal{X}}$ corresponding to the intuitionistic fuzzy rectangular n-norm in $\dot{\mathcal{X}}$.

Proof.

1. Let

$$||w_1, w_2, \dots, w_n||_\alpha = 0.$$

This implies,

$$\inf\{\ell: \Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1 - \alpha, \alpha \in (0,1)\} = 0.$$

Then $\forall\ \alpha \in (0,1)$,

$$\Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha > 0 \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1 - \alpha < 1.$$

implies that $w_1, w_2, \dots, w_n$ are linearly dependent.

Conversely, assume that $w_1, w_2, \dots, w_n$ are linearly dependent.

This implies, by (3) and (9)

$$\Upsilon(w_1, w_2, \dots, w_n, \ell) = 1 \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) = 0, \forall\ \ell > 0,$$

we get

$$\inf\{\ell: \Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1 - \alpha, \alpha \in (0,1)\} = 0,$$

This implies $||w_1, w_2, \dots, w_n||_\alpha = 0$.

2. As

$$\Upsilon(w_1, w_2, \dots, w_n, \ell) \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell)$$

are invariant under any permutation, it follows that $||w_1, w_2, \dots, w_n||_\alpha$ is invariant under any permutation.

3. if $k \neq 0$, then

$$||kw_1, kw_2, \dots, kw_n||_\alpha$$

$$= \inf\{a: \Upsilon(kw_1, kw_2, \dots, kw_n, a) \geq \alpha \text{ and } \dot{\mathcal{F}}(kw_1, kw_2, \dots, kw_n, a) \leq 1 - \alpha, \alpha \in (0,1)\}$$

$$= \inf\{a: \Upsilon(w_1, w_2, \dots, w_n, \tfrac{a}{|k|}) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \tfrac{a}{|k|}) \leq 1 - \alpha, \alpha \in (0,1)\}.$$

Let $\ell = \frac{a}{|k|}$, then

$$||kw_1, kw_2, \dots, kw_n||_\alpha$$

$$= \inf\{|k|\ell: \Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1 - \alpha, \alpha \in (0,1)\}$$

$$= |k|\ ||w_1, w_2, \dots, w_n||_\alpha$$

If k = 0, then

$$\|kw_1, kw_2,\dots, kw_n\|_\alpha = \|w_1, w_2, \dots, w_n, 0\|_\alpha = 0$$

$$= 0\|w_1, w_2, \dots, w_n\|_\alpha = |k|\|w_1, w_2, \dots, w_n\|_\alpha, \forall k \in \mathbb{R}.$$

4. We have

$$\|w_1, w_2, \dots, w_n\|_\alpha + \|w_1, w_2, \dots, \mathbb{z}\|_\alpha + \|w_1, w_2, \dots, \mathfrak{h}\|_\alpha$$

$$= \inf\{\ell: \Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1-\alpha\}$$

$$+ \inf\{\mathfrak{r}: \Upsilon(w_1, w_2, \dots, \mathbb{z}, \mathfrak{r}) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, \mathbb{z}, \mathfrak{r}) \leq 1-\alpha\}$$

$$+ \inf\{\mathfrak{e}: \Upsilon(w_1, w_2, \dots, \mathfrak{h}, \mathfrak{e}) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, \mathfrak{h}, \mathfrak{e}) \leq 1-\alpha\}$$

$$\geq \inf\{\ell + \mathfrak{r} + \mathfrak{e}: \Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1-\alpha,$$

$$\Upsilon(w_1, w_2, \dots, \mathbb{z}, \mathfrak{r}) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, \mathbb{z}, \mathfrak{r}) \leq 1-\alpha,$$

$$\Upsilon(w_1, w_2, \dots, \mathfrak{h}, \mathfrak{e}) \geq \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, \mathfrak{h}, \mathfrak{e}) \leq 1-\alpha)\}$$

$$\geq \inf\{n: \Upsilon(w_1, w_2, \dots, w_n, \ell) * \Upsilon(w_1, w_2, \dots, \mathbb{z}, \mathfrak{r}) * \Upsilon(w_1, w_2, \dots, \mathfrak{h}, \mathfrak{e}) \geq \alpha * \alpha * \alpha$$

and

$$\dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \lozenge \dot{\mathcal{F}}(w_1, w_2, \dots, \mathbb{z}, \mathfrak{r}) \lozenge \dot{\mathcal{F}}(w_1, w_2, \dots, \mathfrak{h}, \mathfrak{e})$$

$$\leq (1-\alpha) \lozenge (1-\alpha) \lozenge (1-\alpha) \leq 1-\alpha\}, n = \ell + \mathfrak{r} + \mathfrak{e}$$

$$\geq \inf\{\ell + \mathfrak{r} + \mathfrak{e}: \Upsilon(w_1, w_2, \dots, w_n + \mathbb{z} + \mathfrak{h}, \ell + \mathfrak{r} + \mathfrak{e}) \geq \alpha \text{ and}$$

$$\dot{\mathcal{F}}(w_1, w_2, \dots, w_n + \mathbb{z} + \mathfrak{h}, \ell + \mathfrak{r} + \mathfrak{e}) \leq 1-\alpha\}$$

$$= \|w_1, w_2, \dots, w_n + \mathbb{z} + \mathfrak{h}\|_\alpha$$

Therefore,

$$\|w_1, w_2, \dots, w_n + \mathbb{z} + \mathfrak{h}\|_\alpha$$

$$\leq \|w_1, w_2, \dots, w_n\| + \|w_1, w_2, \dots, \mathbb{z}\|_\alpha + \|w_1, w_2, \dots, \mathfrak{h}\|_\alpha$$

Let $0 < \alpha_1 < \alpha_2 < 1$. Then

$$\|w_1, w_2, \dots, w_n\|_{\alpha_1} = \inf\{\ell: \Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha_1 \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1-\alpha_1\}$$

$$\|w_1, w_2, \dots, w_n\|_{\alpha_2} = \inf\{\ell: \Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha_2 \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1-\alpha_2\}$$

As, $\alpha_1 < \alpha_2$,

$$\{\ell: \Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha_2 \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1-\alpha_2\}$$

$$\subset \{\ell: \Upsilon(w_1, w_2, \dots, w_n, \ell) \geq \alpha_1 \text{ and } \dot{\mathcal{F}}(w_1, w_2, \dots, w_n, \ell) \leq 1-\alpha_1\}$$

implies,

$\inf\{\ell: \Upsilon(w_1, w_2, \ldots, w_n, \ell) \geq \alpha_2$ and $\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) \leq 1 - \alpha_2\}$

$\geq \inf\{\ell: \Upsilon(w_1, w_2, \ldots, w_n, \ell) \geq \alpha_1$ and $\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n, \ell) \leq 1 - \alpha_1\}$

which implies,

$$\|w_1, w_2, \ldots, w_n\|_{\alpha_2} \geq \|w_1, w_2, \ldots, w_n\|_{\alpha_1}.$$

Therefore, it is acomplete proof.

- **Theorem 3.7**: In an *i-f-r-n-NS* $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}})$ asequence converges to w iff

$$\Upsilon(w_1, w_2, \ldots, w_n - w, \ell) \to 1, \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) \to 0 \text{ as } n \to \infty.$$

Proof. Fix $\ell > 0$. Suppose that $\{w_n\}$ converges to w. Then for a given $\ell > 0$, $0 < \varphi < 1$, $\exists$ an integer $n_0 \in \mathbb{N}$ in which,

$$\Upsilon(w_1, w_2, \ldots, w_n - w, \ell) > 1 - \varphi \text{ and } \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) < \varphi, \forall\, n \geq n_0$$

Thus,

$$1 - \Upsilon(w_1, w_2, \ldots, w_n - w, \ell) < \varphi \text{ and } \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) < \varphi \text{ and hence,}$$

$$\Upsilon(w_1, w_2, \ldots, w_n - w, \ell) \to 1, \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) \to 0 \text{ as } n \to \infty.$$

Conversely, if $\forall\, \ell > 0$,

$$\Upsilon(w_1, w_2, \ldots, w_n - w, \ell) \to 1, \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) \to 0 \text{ as } n \to \infty.$$

Then, $\forall\, \varphi$, $0 < \varphi < 1$, $\exists\, n_0 \in \mathbb{N}$ in which,

$$1 - \Upsilon(w_1, w_2, \ldots, w_n - w, \ell) < \varphi, \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) < \varphi, \ \forall\, n \geq n_0.$$

Therefore,

$$\Upsilon(w_1, w_2, \ldots, w_n - w, \ell) > 1 - \varphi, \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) < \varphi, \ \forall\, n \geq n_0.$$

Consequently $\{w_n\}$ converges to w in $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}})$.

- **Proposition 3.8**: Let $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}})$ be an *i-f-r-n-NS*. If $\{w_n\}$ is aconvergent sequence in $\dot{\mathcal{X}}$, then it is a Cauchy sequence.

Proof. Let $\{w_n\}$ be aCauchy sequence in $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}})$ converging to $w \in \dot{\mathcal{X}}$.
Let $\ell > 0$, $\varphi \in (0,1)$. Choose $\alpha \in (0,1)$ in which,

$$(1 - \alpha) * (1 - \alpha) > 1 - \varphi \text{ and } \alpha \lozenge \alpha < \varphi.$$

Since $\{w_n\}$ converges to w, there is $n_0 \in \mathbb{N}$ in which,

$$\Upsilon(w_1, w_2, \ldots, w_n - w, \tfrac{\ell}{2}) > 1 - \alpha, \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \tfrac{\ell}{2}) < \alpha, \forall\, n \geq n_0.$$

Now,

$$\Upsilon(w_1, w_2, \ldots, w_n - w_s, \ell) = \Upsilon(w_1, w_2, \ldots, w_n - w + w - w_s, \tfrac{\ell}{2} + \tfrac{\ell}{2})$$

$$\geq \Upsilon(w_1, w_2, \ldots, w_n - w, \tfrac{\ell}{2}) * \Upsilon(w_1, w_2, \ldots, w_s - w, \tfrac{\ell}{2})$$

$> (1 - \alpha) * (1 - \alpha) > 1 - \varphi, \forall\, n, s \geq n_0.$

Similarly, it may be demonstrated that,

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w_s, \ell) < \varphi.$$

Consequently, $\{w_n\}$ be a Cauchy sequence in $\dot{\mathcal{X}}$.

- **Remark 3.9**: The converse of above theorem is not true. Below, we provide an example to illustrate this.

- **Example 3.10**: Let ($\dot{\mathcal{X}}$, ||.,…,.||) be a rectangular *n*-normed space and ($\dot{\mathcal{X}}$, Υ, $\dot{\mathcal{F}}$, $*$, $\Diamond$) be a standard intuitionistic fuzzy rectangular *n*-norm in $\dot{\mathcal{X}}$ induced by a rectangular *n*-norm. Let $\{w_n\}$ be a sequence in $\dot{\mathcal{X}}$. Then:

1. $\{w_n\}$ be aCauchy sequence in ($\dot{\mathcal{X}}$, ||.,…,.||) $\Leftrightarrow$ $\{w_n\}$ be a Cauchy sequence in ($\dot{\mathcal{X}}$, Υ, $\dot{\mathcal{F}}$).

2. $\{w_n\}$ be a convergent sequence in ($\dot{\mathcal{X}}$, ||.,…,.||) $\Leftrightarrow$ $\{w_n\}$ be aconvergent sequence in ($\dot{\mathcal{X}}$, Υ, $\dot{\mathcal{F}}$).

Proof.

1. $\{w_n\}$ is a Cauchy sequence in ($\dot{\mathcal{X}}$, ||.,…,.||)

$$\Leftrightarrow \lim_{n,s\to\infty} \| w_1, w_2, \ldots, w_n - w_s\| = 0$$

$$\Leftrightarrow \lim_{n,s\to\infty} \Upsilon(w_1, w_2, \ldots, w_n - w_s, \ell) = \lim_{n,s\to\infty} \frac{\ell}{\ell + ||w_1, w_2, \cdots, w_n - w_s||} = 1 \text{ and}$$

$$\lim_{n,s\to\infty} \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w_s, \ell) = \lim_{n,s\to\infty} \frac{||w_1, w_2, \cdots, w_n - w_s||}{\ell + ||w_1, w_2, \cdots, w_n - w_s||} = 0$$

$$\Leftrightarrow \lim_{n,s\to\infty} \Upsilon(w_1, w_2, \ldots, w_n - w_s, \ell) = 1, \lim_{n,s\to\infty} \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w_s, \ell) = 0$$

$\Leftrightarrow$ $\{w_n\}$ be a Cauchy sequence in ($\dot{\mathcal{X}}$, Υ, $\dot{\mathcal{F}}$).

2. $\{w_n\}$ be a convergent sequence in ($\dot{\mathcal{X}}$, ||.,…,.||)

$$\Leftrightarrow \lim_{n\to\infty} \|w_1, w_2, \ldots, w_n - w\| = 0$$

$$\Leftrightarrow \lim_{n\to\infty} \Upsilon(w_1, w_2, \ldots, w_n - w, \ell) = \lim_{n\to\infty} \frac{\ell}{\ell + ||w_1, w_2, \cdots, w_n - w||} = 1$$

and

$$\lim_{n\to\infty} \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) = \lim_{n\to\infty} \frac{||w_1, w_2, \cdots, w_n - w||}{\ell + ||w_1, w_2, \cdots, w_n - w||} = 0$$

$$\Leftrightarrow \lim_{n\to\infty} \Upsilon(w_1, w_2, \ldots, w_n - w, \ell) = 1, \lim_{n\to\infty} \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) = 0$$

$\Leftrightarrow$ $\{w_n\}$ be aconvergent sequence in ($\dot{\mathcal{X}}$, Υ, $\dot{\mathcal{F}}$).

- **Theorem 3.11**: If asequence $\{w_n\}$ in an *i-f-r-n-NS* ($\dot{\mathcal{X}}$, Υ, $\dot{\mathcal{F}}$) is aconvergent, it is limit is unique.

Proof.

$\lim_{n\to\infty} w_n = w$ and $\lim_{n\to\infty} w_n = z$. Also, $\ell, r \in \mathbb{R}^+$.

Now

$$\lim_{n\to\infty} w_n = w \Rightarrow \lim_{n\to\infty} \Upsilon(w_1, w_2, \ldots, w_n - w, \ell) = 1,\ \lim_{n\to\infty} \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) = 0$$

$$\lim_{n\to\infty} w_n = \mathbb{Z} \Rightarrow \lim_{n\to\infty} \Upsilon(w_1, w_2, \ldots, w_n - \mathbb{Z}, \mathfrak{r}) = 1,\ \lim_{n\to\infty} \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - \mathbb{Z}, \mathfrak{r}) = 0,$$

$$\Upsilon(w_1, w_2, \ldots, w - \mathbb{Z}, \ell + \mathfrak{r}) = \Upsilon(w_1, w_2, \ldots, w - w_n + w_n - \mathbb{Z}, \ell + \mathfrak{r})$$

$$\geq \Upsilon(w_1, w_2, \ldots, w_n - w, \ell) * \Upsilon(w_1, w_2, \ldots, w_n - \mathbb{Z}, \mathfrak{r})$$

Taking limit, we get

$$\Upsilon(w_1, w_2, \ldots, w - \mathbb{Z}, \ell + \mathfrak{r})$$

$$\geq \lim_{n\to\infty} \Upsilon(w_1, w_2, \ldots, w_n - w, \ell) * \lim_{n\to\infty} \Upsilon(w_1, w_2, \ldots, w_n - \mathbb{Z}, \mathfrak{r}) = 1$$

$$\Rightarrow \Upsilon(w_1, w_2, \ldots, w - \mathbb{Z}, \ell + \mathfrak{r}) = 1 \Rightarrow w - \mathbb{Z} = 0 \Rightarrow w = \mathbb{Z}.$$

Similarly, it may be demonstrated that,

$$\dot{\mathcal{F}}(x_1, w_2, \ldots, w - \mathbb{Z}, \ell + \mathfrak{r}) = 0.$$

Therefore, it is acomplete proof.

- **Theorem 3.12**: An *i-f-r-n-NS* $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}})$ in which every Cauchy sequence has aconvergent subsequence. Then $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}})$ is complete.

Proof. Let $\{w_n\}$be a Cauchy sequence in $(\dot{\mathcal{X}}, \Upsilon, \dot{\mathcal{F}})$ and $\{w_{n_s}\}$ be asubsequence of $\{w_n\}$ that converges to w. Prove that $\{w_n\}$ converges to w. Now let $\ell > 0$ and $\varphi \in (0,1)$, choose $\alpha \in (0,1)$ in which,

$$(1-\alpha) * (1-\alpha) > 1 - \varphi \text{ and } \alpha \lozenge \alpha < \varphi.$$

Since $\{w_n\}$ be a Cauchy sequence, there is $n_0 \in \mathbb{N}$ in which,

$$\Upsilon(w_1, w_2, \ldots, w_n - w_s, \tfrac{\ell}{2}) > 1 - \alpha \text{ and } \dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w_s, \tfrac{\ell}{2}) < \alpha,\ \forall\, n, s \geq n_0.$$

Since $\{w_{n_s}\}$ converges to w, there is $i_s > n_0$ in which,

$$\Upsilon(w_1, w_2, \ldots, w_{i_s} - w, \tfrac{\ell}{2}) > 1 - \alpha,\ \dot{\mathcal{F}}(w_1, w_2, \ldots, w_{i_s} - w, \tfrac{\ell}{2}) < \alpha$$

Now,

$$\Upsilon(w_1, w_2, \ldots, w_n - w, \ell) = \Upsilon(w_1, w_2, \ldots, w_n - w_{i_s} + w_{i_s} - w, \tfrac{\ell}{2} + \tfrac{\ell}{2})$$

$$\geq \Upsilon(w_1, w_2, \ldots, w_n - w_{i_s}, \tfrac{\ell}{2}) * \Upsilon(w_1, w_2, \ldots, w_{i_s} - w, \tfrac{\ell}{2})$$

$$> (1-\alpha) * (1-\alpha) > 1 - \varphi.$$

Similarly, it may be demonstrated that,

$$\dot{\mathcal{F}}(w_1, w_2, \ldots, w_n - w, \ell) < \varphi.$$

Consequently, $\{w_n\}$ converges to w in $\dot{\mathcal{X}}$, so $\dot{\mathcal{X}}$ is complete.

4- Conclusion

Motivated by the concept of fuzzy rectangular *n*-norm, as introduced by Bader and Mohammed [6], in this paper, we define the concept of intuitionistic fuzzy rectangular *n*-normed space. After that, we study and establish the concept of α-*n*-norm corresponding to the intuitionistic fuzzy rectangular *n*-normed space. We also study some basic properties of completeness and convergence for intuitionistic fuzzy rectangular *n*-normed space and conclude that the concept of convergence remains the same, but the norm is different. This means that the distance between intuitionistic fuzzy *n*-normed space [20] and intuitionistic fuzzy rectangular *n*-normed space may be written in the same from, but it is not actually identical.

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