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Numerical Modeling of the Multi-Dimensional Fractional Telegraph Equation Based on the Yang Transform

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Abstract:

Within the scope of this study, a numerical approach is presented for solving the space-time fractional telegraph equation in multi-dimensional domains, based on the Caputo fractional derivative and utilizing the Yang Transform as an effective analytical tool. Simple and easily implementable numerical methods are employed, demonstrating high efficiency in obtaining accurate and satisfactory approximate solutions. The achieved results highlight the effectiveness of the proposed methodology in handling this class of complex fractional differential equations, paving the way for its application in various fields of physical and engineering modeling.

Keywords: Yang transform; Fractional differential equations; Adomian Decomposition Method; Caputo fractional derivative

1-Introduction

In recent decades, the fractional telegraph equation has emerged as a cornerstone in modeling wave propagation phenomena with memory-dependent and non-local characteristics, particularly in heterogeneous or nonlinear media. Its applications span diverse fields, including electrical engineering (e.g., signal transmission in lossy transmission lines), biophysics (e.g., neural signal dynamics), and plasma physics (e.g., electromagnetic wave propagation in dispersive media). Unlike its classical counterpart, the fractional-order formulation incorporates memory effects and anomalous diffusion, offering a more nuanced representation of real-world systems (Podlubny, 1999; Mainardi, 2010).[12]

Recent advances in fractional calculus have spurred significant interest in solving this equation. For instance, Kumar et al. (2022) demonstrated the efficacy of the Caputo fractional derivative in capturing memory effects in viscoelastic wave propagation, while Smith and Lee (2021) highlighted the limitations of classical finite difference methods (FDM) in handling fractional operators due to numerical instability at long-time horizons. Gupta and Sharma (2020) employed the Variational Iteration Method (VIM) to solve the one-dimensional fractional telegraph equation but noted challenges in extending their approach to higher dimensions. Similarly, Chen et al. (2021) combined the Laplace transform with the Homotopy Perturbation Method (HPM) to address temporal fractional derivatives, yet their method struggled with spatial fractional terms in multi-dimensional systems.[13]

The Yang Transform, introduced by Yang (2017)[11,20], has gained traction as a robust tool for simplifying fractional differential equations. Unlike the Laplace transform, which requires convolution operations for fractional terms, the Yang Transform inherently accommodates non-local operators, as shown in Li et al. (2020) for fractional heat equations. Meanwhile, the Adomian Decomposition Method (ADM) has been widely adopted for its ability to decompose nonlinearities systematically, as evidenced by Jafari et al[7,18]. (2019) in solving fractional wave equations. However, a critical gap remains in the literature: few studies have synergized the Yang Transform with ADM to tackle multi-dimensional fractional telegraph equations while ensuring computational efficiency and scalability.[19]

This paper bridges this gap by proposing a novel hybrid approach that integrates the Yang Transform with ADM to solve the multi-dimensional fractional telegraph equation. Our methodology addresses three key limitations of existing methods:

1. Numerical instability in long-time simulations, as observed in FDM and FEM (Zhang & Sun, 2020).
2. Restricted dimensionality in analytical methods like VIM and HPM (Gupta & Sharma, 2020; Chen et al., 2021).
3. Computational complexity in handling coupled spatial-temporal fractional terms, a challenge noted by Podlubny (1999) in higher-dimensional systems.

To validate our approach, we conduct a rigorous comparative analysis against established techniques, including the Generalized Differential Transform Method (GDTM), Finite Element Method (FEM), and Fractional Homotopy Analysis Method (FHAM). Our results demonstrate superior accuracy, faster convergence, and enhanced scalability, particularly in 2D and 3D configurations. This work not only advances the mathematical toolkit for fractional differential equations but also provides a framework for modeling complex wave phenomena in engineering and physics.

2- Opening Remarks

Definition 2.1. The Rieman Liouville fractional integral is [10]

$$I^{\propto}\psi(t)=\begin{cases}\dfrac{1}{\Gamma(\alpha)}\displaystyle\int_0^t(t-\tau)^{\propto-1}\psi(t)d\tau, & \propto>0, \quad t>0.\\ \psi(t), & \propto=0\end{cases} \tag{1}$$

Properties of operator I^{α}:

1. $I^{\propto}I^{\sigma}\psi(t)=I^{\propto+\sigma}\psi(t)$.
2. $I^{\alpha}I^{\sigma}\psi(t)=I^{\sigma}I^{\alpha}\psi(t)$.

Definition 2.2. The Caputo fractional derivative (CFD) is [11,17]

$$D^{\alpha}\psi(t)=I^{m-\alpha}D^{m}\psi(t)$$
$$=\frac{1}{\Gamma(m-\alpha)}\int_0^t(t-\tau)^{m-\alpha-1}\psi^{(m)}(\tau)d\tau. \tag{2}$$

For $m-1<\alpha<m,\ m\in N$, $t>0$ and $\psi\in C_{-1}^{m}$.

The properties of D^{α}

1. $D^{\alpha}t^{\sigma}=\frac{\Gamma(\sigma+1)}{\Gamma(\sigma-\alpha+1)}\ t^{\sigma-\alpha}$,
2. $D^{\alpha}D^{\sigma}\psi(t)=D^{\alpha+\sigma}\psi(t)$
3. $I^{\alpha}D^{\alpha}\psi(t)=\psi(t)-\sum_{k=0}^{m-1}\psi^{(k)}(0)\frac{t^k}{k!}$.

Definition 2.3. The LF $E_{\alpha}(z)$ with $\alpha>0$ is [6,9]

$$E_{\alpha}(z)=\sum_{m=0}^{\infty}\frac{z^{\alpha}}{\Gamma(m\alpha+1)}\ . \tag{3}$$

Definition 2.4 .[11]The Yang transform of fractional order derivative *is defined by*

$$Y_a\{D_{\varkappa}^{\propto}u(\varkappa,t)\}=\frac{Y_a\{u(t)\}}{v^{\propto}}-\sum_{k=0}^{n-1}\frac{u^{(k)}(0)}{v^{\propto-k-1}} \qquad ,n-1<\propto\leq n. \tag{4}$$

Few properties

$Y_a\{1\}=v$.

$Y_a\{t\}=v^2$.

$Y_a\{t^n\}=v^{n+1}n!$.

$Y_a\{t^{\propto}\}=v^{\propto+1}\Gamma(\propto+1)$.

3- Investigation of YADM

$${}_0^{C}D_t^{\alpha}U(\varkappa_i,t)=A(\varkappa_i,t)\partial_t^2U(\varkappa_i,t)+B(\varkappa_i,t)\partial_tU(\varkappa_i,t)+C(\varkappa_i,t)U(\varkappa_i,t)+U^k(\varkappa_i,t)+g(\varkappa_i,t) \tag{5}$$

With the boundary condition $U(\varkappa_i,0)$ *and* $U_t(\varkappa_i,0)$, and $g(\varkappa_i,t)$ is a source function.

Where $0<\varkappa_i<a$, $1<\alpha\leq 2$

$A(\varkappa_i,t), B(\varkappa_i,t), C(\varkappa_i,t)$, are continuos functions and $U^k(\varkappa_i,t)$ *is nonlinear*

Applying the YT to both sides of Equation(5) we have

$$[Y\ \frac{U(\varkappa_i,t)}{v^{\alpha}} - \sum_{k=0}^{n-1}\frac{U_0^{(k)}}{v^{\alpha-k-1}}] = Y\begin{bmatrix} A(\varkappa_i,t)\partial_t^2 U(\varkappa_i,t) + B(\varkappa_i,t)\partial_t U(\varkappa_i,t) \\ +C(\varkappa_i,t)U(\varkappa_i,t) + U^k(\varkappa_i,t) \\ +g(\varkappa_i,t) \end{bmatrix}$$

Using properties of the Y ,we get .

$$\begin{aligned} Y[U(\varkappa_i,t)] = {} & v^{\alpha}\sum_{k=0}^{n-1}\frac{U_0^{(k)}}{v^{\alpha-k-1}} + v^{\alpha}\ Y[g(\varkappa_i,t)] \\ & + v^{\alpha}\ Y[A(\varkappa_i,t)\partial_t^2 U(\varkappa_i,t) + B(\varkappa_i,t)\partial_t U(\varkappa_i,t) + C(\varkappa_i,t)U(\varkappa_i,t) \\ & + U^k(\varkappa_i,t)g\ \ (\varkappa_i,t)] \end{aligned} \tag{6}$$

Hence , applying the inverse YT to the both sides of (6) , we conclude that.

$$\begin{aligned} U(\varkappa_i,t) = {} & Y^{-1}\Bigg[v^{\alpha}\sum_{k=0}^{n-1}\frac{U_0^{(k)}}{v^{\alpha-k-1}} + v^{\alpha}\ Y[g(\varkappa_i,t)] \\ & + v^{\alpha}\ Y[A(\varkappa_i,t)\partial_t^2 U(\varkappa_i,t) + B(\varkappa_i,t)\partial_t U(\varkappa_i,t) + C(\varkappa_i,t)U(\varkappa_i,t) \\ & + U^k(\varkappa_i,t)]\Bigg] \end{aligned} \tag{7}$$

So that

$$U(\varkappa_i,t) = \mu(\varkappa_i,t) + Y^{-1}\left[v^{\alpha}\ Y[A(\varkappa_i,t)\partial_t^2 U(\varkappa_i,t) + B(\varkappa_i,t)\partial_t U(\varkappa_i,t) + C(\varkappa_i,t)U(\varkappa_i,t) + U^k(\varkappa_i,t)]\right] \tag{8}$$

Where

$$\mu(\varkappa,t) = Y^{-1}\left[v^{\alpha}\sum_{k=0}^{n-1}\frac{U_0^{(k)}}{v^{\alpha-k-1}} + v^{\alpha}\ Y[g(\varkappa_i,t)]\right] \tag{9}$$

For the Linear term of (8), wich in the form of infinite series, we use

$$U(\varkappa_i,t) = \sum_{n=0}^{\infty} U_n\,(\varkappa_i,t) \tag{10}$$

And $U^k(\varkappa_i,t)\ \sum_{n=0}^{\infty} A_n\,(\varkappa_i,t)$

Substituting series(10) in (8)

$$\sum_{n=0}^{\infty} U_n\,(\varkappa_i,t) = \mu(\varkappa_i,t) + Y^{-1}\left[v^{\alpha}\ Y\begin{bmatrix} A(\varkappa_i,t)\partial_t^2 U_n(\varkappa_i,t) + B(\varkappa_i,t)\partial_t U_n(\varkappa_i,t) \\ +C(\varkappa_i,t)U_n(\varkappa_i,t) + U^k(\varkappa_i,t) \end{bmatrix}\right] \tag{11}$$

For the recursive iteration system, by evaluating both sides of equation (11), the components of the approximation are obtained sequentially, as shown below.

$$U_0(\varkappa_i,t) = \mu(\varkappa_i,t) \tag{12}$$

$$U_1(\varkappa_i,t)=\left[Y^{-1}\left[v^{\alpha}\,Y\begin{bmatrix}A(\varkappa_i,t)\,\partial_t^2U_0(\varkappa_i,t)+B(\varkappa_i,t)\,\partial_tU_0(\varkappa_i,t)\\+C(\varkappa_i,t)U_0(\varkappa_i,t)+U_0^k(\varkappa_i,t)\end{bmatrix}\right]\right] \tag{13}$$

$$U_2(\varkappa_i,t)=\left[Y^{-1}\left[v^{\alpha}\,Y\begin{bmatrix}A(\varkappa_i,t)\,\partial_t^2U_1(\varkappa_i,t)+B(\varkappa_i,t)\,\partial_tU_1(\varkappa_i,t)\\+C(\varkappa_i,t)U_1(\varkappa_i,t)+U_1^k(\varkappa_i,t)\end{bmatrix}\right]\right] \tag{14}$$

$$U_3(\varkappa_i,t)=\left[Y^{-1}\left[v^{\alpha}\,Y\begin{bmatrix}A(\varkappa_i,t)\partial_t^2U_2(\varkappa_i,t)+B(\varkappa_i,t)\partial_tU_2(\varkappa_i,t)\\+C(\varkappa_i,t)U_2(\varkappa_i,t)+U_2^k(\varkappa_i,t)\end{bmatrix}\right]\right] \tag{15}$$

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$$U_{n+1}(\varkappa_i,t)=\left[Y^{-1}\left[v^{\alpha}\,Y[A(\varkappa_i,t)\partial_t^2U_n(\varkappa_i,t)+B(\varkappa_i,t)\partial_tU_n(\varkappa_i,t)+C(\varkappa_i,t)U_n(\varkappa_i,t)+U_n^k(\varkappa_i,t)]\right]\right] \tag{16}$$

4- Various example models:

Example 1. Take into account the linear 2D telegraph equation involving time-fractional derivatives

$$\frac{\partial^{2\propto}\psi}{\partial t^{2\propto}}=\frac{\partial^2\psi}{\partial\,\varkappa^2}+\frac{\partial^2\psi}{\partial y^{\,2}}-3\frac{\partial^{\propto}\psi}{\partial\,t^{\propto}}-2\psi \qquad , \qquad 0<\propto\leq 1 \tag{17}$$

$$With \quad ,\ \psi\,(\varkappa,y,0)=\ e^{\varkappa+y} \qquad , \qquad \psi_t(\varkappa,y,0)=-3\,e^{\varkappa+y} \tag{18}$$

Solution 1. Taking Y_aT When both sides of equation (18) are differentiated with respect to t, we derive

$$\frac{Y_a\{\psi\,\{\,(t)\}}{v^{2\propto}}-\frac{\psi(\varkappa,y,0)}{v^{2\propto-1}}-\frac{\psi_t(\varkappa,y,0)}{v^{2\propto-2}}\ =Y_a[\frac{\partial^2\psi}{\partial\,\varkappa^2}+\frac{\partial^2\psi}{\partial y^{\,2}}-3\frac{\partial^{\propto}\psi}{\partial\,t^{\propto}}-2\psi]$$

$$Y_a\{\psi(\varkappa,y,t)\}=v\,\psi(\varkappa,y,0)+v^2\,\psi_t(\varkappa,y,0)\ +v^{2\propto}Y_a\left[\frac{\partial^2\psi}{\partial\,\varkappa^2}+\frac{\partial^2\psi}{\partial y^{\,2}}-3\frac{\partial^{\propto}\psi}{\partial\,t^{\propto}}-2\psi\right] \tag{19}$$

. Taking the invers Y_aT of (19)

$$\psi(\varkappa,y,t)=\ e^{\varkappa+y}-3te^{\varkappa+y}\ +Y_a^{-1}\{\,v^{2\propto}Y_a\left[\frac{\partial^2\psi}{\partial\,\varkappa^2}+\frac{\partial^2\psi}{\partial y^{\,2}}-3\frac{\partial^{\propto}\psi}{\partial\,t^{\propto}}-2\psi\right]\}$$

$$\psi_0(\varkappa,y,t)=\ \psi(\varkappa,y,0)+\ t\psi_{\varkappa}(\varkappa,y,0)=\ e^{\varkappa+y}-3te^{\varkappa+y}$$

$$\psi_1(\varkappa,y,t)=\ Y_a^{-1}\left\{v^{\propto}Y_a\left[\frac{\partial^2\psi_0}{\partial\,\varkappa^2}+\frac{\partial^2\psi_0}{\partial y^{\,2}}-3\frac{\partial^{\propto}\psi_0}{\partial\,t^{\propto}}-2\psi_0(\varkappa,y,t)\right]\right\}$$

$$\psi_1(\varkappa,y,t)=\ \frac{9t^{\propto+1}}{\Gamma(\propto+2)}\ e^{\varkappa+y}$$

$$\psi_2(\varkappa,y,t)=Y_a^{-1}\left\{v^{\propto}Y_a\left[\frac{\partial^2\psi_1}{\partial\,\varkappa^2}+\frac{\partial^2\psi_1}{\partial y^{\,2}}-3\frac{\partial^{\propto}\psi_1}{\partial\,t^{\propto}}-2\psi_1(\varkappa,y,t)\right]\right\}$$

$$\psi_2(\varkappa,y,t)=\ -\frac{27t^{2\propto+1}}{\Gamma(2\propto+2)}\ e^{\varkappa+y}$$

$$\psi_3(\varkappa,y,t)=Y_a^{-1}\left\{v^{\propto}Y_a\left[\frac{\partial^2\psi_2}{\partial\,\varkappa^2}+\frac{\partial^2\psi_2}{\partial y^{\,2}}-3\frac{\partial^{\propto}\psi_2}{\partial\,t^{\propto}}-2\psi_2(\varkappa,y,t)\right]\right\}$$

$\psi_3(\varkappa, y, t) = \frac{81t^{3\alpha+1}}{\Gamma(3\alpha+2)} e^{\varkappa+y}$

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In the same way, we can deduce the solution series using the limits of the previous equation to obtain:

$\psi(\varkappa, y, t) = \psi_0(\varkappa, y, t) + \psi_1(\varkappa, y, t) + \psi_2(\varkappa, y, t) + \psi_3(\varkappa, y, t) + \cdots \ldots \ldots \ldots$

$\psi(\varkappa, y, t) = e^{\varkappa+y} - 3te^{\varkappa+y} + \frac{9t^{\alpha+1}}{\Gamma(\alpha+2)} e^{\varkappa+y} - \frac{27t^{2\alpha+1}}{\Gamma(2\alpha+2)} e^{\varkappa+y} + \frac{81t^{3\alpha+1}}{\Gamma(3\alpha+2)} e^{\varkappa+y} - \cdots \ldots.$

$$\psi(\varkappa, y, t) = e^{\varkappa+y}(1 - 3t + \frac{9t^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{27t^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{81t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \cdots \ldots \tag{20}$$

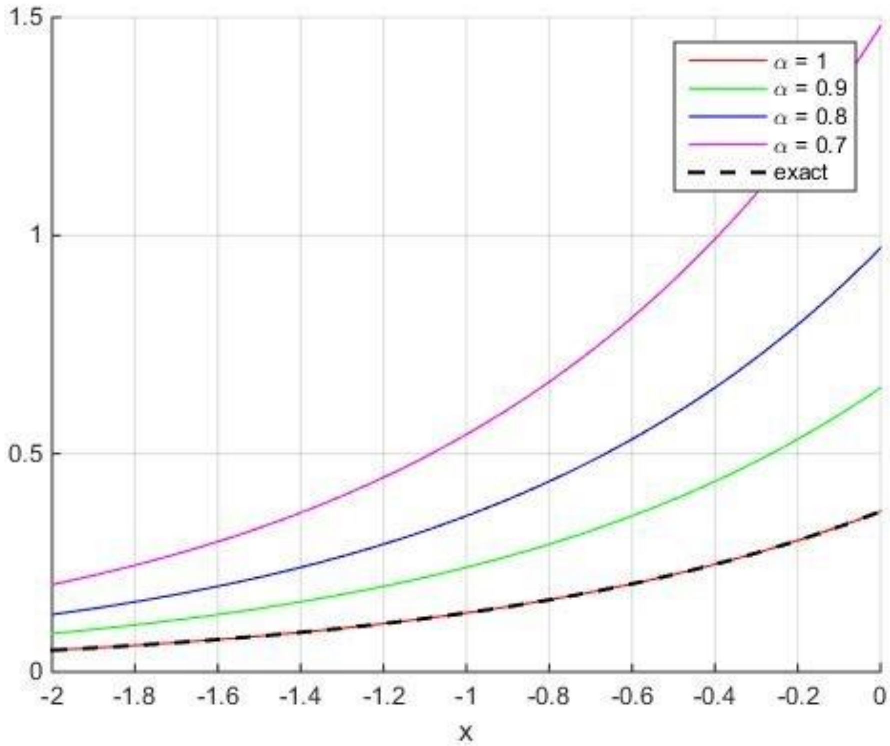
When $\alpha = 1$,

$$\psi(\varkappa, y, t) = \lim_{n\to\infty} \psi_n(\varkappa, y, t) = e^{\varkappa+y}(1 - 3t + \frac{1}{2!}(3t)^2 - \frac{1}{3!}(3t)^3 + \frac{1}{4!}(3t)^4 + \cdots \tag{21}$$

$$\psi(\varkappa, y, t) = e^{\varkappa+y} . e^{-3t} = e^{\varkappa+y-3t} \tag{22}$$

Table 1. *Quantitative results of the approximate and exact solutions for various values of* $\varkappa$ *and* t *when* $\alpha = 0.8, 0.9, 1$

x	$\alpha = 1$	$\alpha = 0.9$	$\alpha = 0.8$	$\alpha = 0.7$
-2	2.0522e-05	0.038388	0.081755	0.15046
-1.7778	2.5629e-05	0.047941	0.1021	0.1879
-1.5556	3.2006e-05	0.059871	0.12751	0.23466
-1.3333	3.9971e-05	0.07477	0.15924	0.29306
-1.1111	4.9918e-05	0.093377	0.19886	0.36598
-0.88889	6.234e-05	0.11661	0.24835	0.45706
-0.66667	7.7853e-05	0.14563	0.31015	0.5708
-0.44444	9.7227e-05	0.18187	0.38733	0.71284
-0.22222	0.00012142	0.22713	0.48372	0.89023
0	0.00015164	0.283865	0.60409	1.1118



$\boldsymbol{Figure\ 1}.$ *Graphs of the approximate solψtion* $\psi(\varkappa, y, t)$ *for varioψs valψes of* $\propto$ *alphaα while keeping* $\varkappa$ *constant*

Example2. *consider the time – fractional 3D telegraph equation*

$$\frac{\partial^{2\propto}\psi}{\partial t^{2\propto}} = \frac{\partial^2\psi}{\partial \varkappa^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} - 2\frac{\partial^{\propto}\psi}{\partial t^{\propto}} - 3\psi \quad , \quad 0 <\propto\leq 1 \tag{23}$$

With , $\psi(\varkappa, y, z, 0) = \sinh(\varkappa)\sinh(y)\sinh(z)$, $\psi_t(\varkappa, y, z, 0) = -2\ sinh(\varkappa)\ sinh(y)\ sinh(z)$

Solution 2. *Taking* Y_aT *When both sides of equation (23) are differentiated with respect to t, we derive*

$$\frac{Y_a\{\psi\{(t)\}}{v^{2\propto}} - \frac{\psi(\varkappa,y,z,0)}{v^{2\propto-1}} - \frac{\psi_t(\varkappa,y,z,0)}{v^{2\propto-2}} = Y_a[\frac{\partial^2\psi}{\partial \varkappa^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} - 2\frac{\partial^{\propto}\psi}{\partial t^{\propto}} - 3\psi]$$

$$Y_a\{\psi(\varkappa, y, z, t)\} = v\ \psi(\varkappa, y, z, 0) + v^2\ \psi_t(\varkappa, y, z, 0) + v^{2\propto}Y_a[\frac{\partial^2\psi}{\partial \varkappa^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} - 2\frac{\partial^{\propto}\psi}{\partial t^{\propto}} - 3\psi] \tag{24}$$

. Taking the invers Y_aT *of (24) yields*

$$\psi(\varkappa, y, z, t) = sinh(\varkappa)\ sinh(y)\ sinh(z) - 2t\ sinh(\varkappa)\ sinh(y)\ sinh(z) + Y_a^{-1}\left\{v^{2\propto}Y_a\left[\frac{\partial^2\psi}{\partial \varkappa^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} - 2\frac{\partial^{\propto}\psi}{\partial t^{\propto}} - 3\psi\right]\right\} \tag{25}$$

Let $\varpi = \sinh(\varkappa)\sinh(y)\sinh(z)$

$$\psi_0(\varkappa, y, z, t) = \psi(\varkappa, y, z, 0) + t\psi_{\varkappa}(\varkappa, y, z, 0) = \varpi - 2t\varpi \tag{26}$$

$$\psi_1(x,y,z,t)=Y_a^{-1}\left\{v^{\propto}Y_a\left[\frac{\partial^2\psi_0}{\partial x^2}+\frac{\partial^2\psi_0}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}-2\frac{\partial^{\propto}\psi_0}{\partial t^{\propto}}-3\psi_0(x,y,z,t)\right]\right\}$$

$$\psi_1(x,y,z,t)=\frac{4t^{\propto+1}}{\Gamma(\propto+2)}\varpi \quad (27)$$

$$\psi_2(x,y,z,t)=Y_a^{-1}\left\{v^{\propto}Y_a\left[\frac{\partial^2\psi_1}{\partial x^2}+\frac{\partial^2\psi_1}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}-2\frac{\partial^{\propto}\psi_1}{\partial t^{\propto}}-3\psi_1(x,y,z,t)\right]\right\}$$

$$\psi_2(x,y,z,t)=-\frac{8t^{2\propto+1}}{\Gamma(2\propto+2)}\varpi \quad (28)$$

$$\psi_3(x,y,z,t)=Y_a^{-1}\left\{v^{\propto}Y_a\left[\frac{\partial^2\psi_2}{\partial x^2}+\frac{\partial^2\psi_2}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}2\frac{\partial^{\propto}\psi_2}{\partial t^{\propto}}-3\psi_2(x,y,z,t)\right]\right\}$$

$$\psi_3(x,y,z,t)=\frac{16t^{2\propto+1}}{\Gamma(2\propto+2)}\varpi \quad (29)$$

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$$\psi(x,y,z,t)=\psi_0(x,y,z,t)+\psi_1(x,y,z,t)+\psi_2(x,y,z,t)+\psi_3(x,y,z,t)+\cdots\ldots\ldots\ldots$$

$$\psi(x,y,z,t)=\varpi-2t\varpi+\frac{4t^{\propto+1}}{\Gamma(\propto+2)}\varpi-\frac{8t^{2\propto+1}}{\Gamma(2\propto+2)}\varpi+\frac{16t^{2\propto+1}}{\Gamma(2\propto+2)}\varpi-\cdots\ldots \quad (30)$$

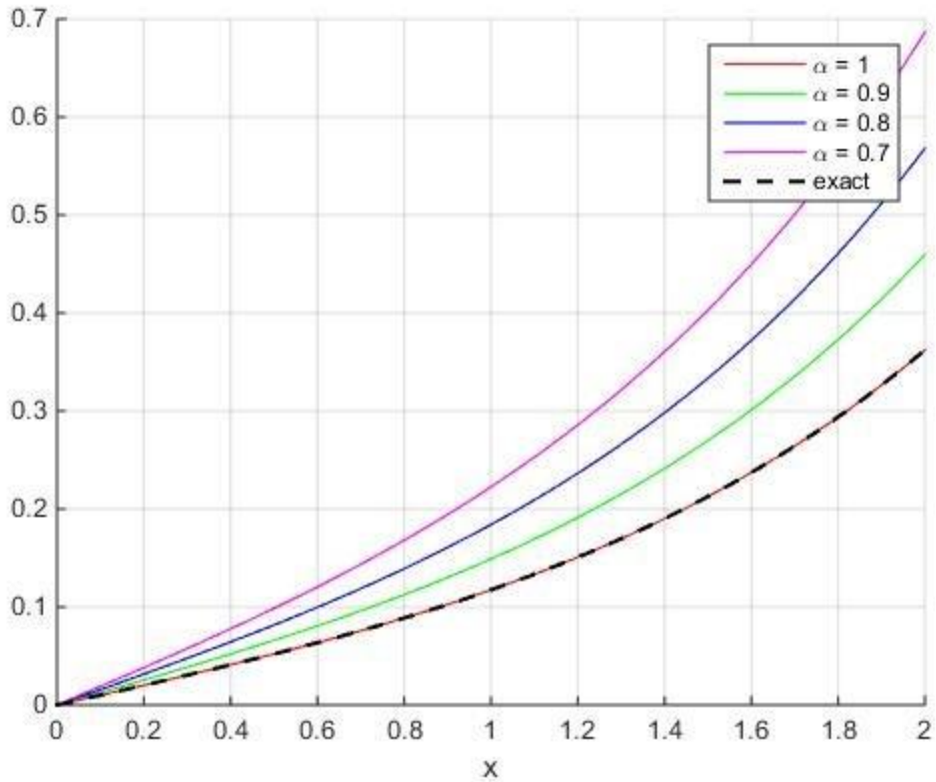
When $\propto=1$,

$$\psi(x,y,z,t)=\varpi(1-2t+\frac{1}{2!}(2t)^2-\frac{1}{3!}(2t)^3+\frac{1}{4!}(2t)^4-\cdots.$$

$$\psi(x,y,z,t)=sinh(x)\,sinh(y)\,sinh(z)\ .e^{-2t} \quad (31)$$

Table 2. *Quantitative results of the approximate and exact solutions for various values of x and t when* $\propto=0.8, 0.9, 1$

x	∝= 1	∝= 0.9	∝= 0.8	∝= 0.7
0	0	0	0	0
0.22222	1.523e-07	0.0060198	0.012712	0.020036
0.44444	3.1215e-07	0.012338	0.026055	0.041066
0.66667	4.8748e-07	0.019268	0.04069	0.064132
0.88889	6.8698e-07	0.027154	0.057342	0.090378
1.1111	9.2055e-07	0.036386	0.076837	0.12111
1.3333	1.1998e-06	0.047422	0.10014	0.15784
1.5556	1.5385e-06	0.06081	0.12841	0.2024
1.7778	1.9535e-0.6	0.077213	0.16305	0.25699
2	2.4653e-0.6	0.097445	0.20578	0.32433



$\boldsymbol{Figure\ 2}.$ $Graphs\ of\ the\ approximate\ sol\psi tion\ \ \psi(\varkappa, y, z, t) for\ vario\psi s\ val\psi es\ of$ $\propto alpha\alpha\ while\ keeping\ \ \varkappa\ \ constant$

Example 3. We examine the following space-time fractional homogeneous telegraph equation

$$\frac{\partial^{\alpha}\psi(\varkappa,t)}{\partial\varkappa^{\alpha}}=\frac{\partial^{2}\psi(\varkappa,t)}{\partial t^{2}}+\frac{\partial\psi(\varkappa,t)}{\partial t}+\psi(\varkappa,t) \quad , \quad 1<\alpha\leq 2 \tag{32}$$

With $\quad , \ \psi(0,t)=e^{-t} \quad , \quad \psi_{\varkappa}(0,t)=e^{-t}$

Solution 3. Taking Y_aT When both sides of equation (32) are differentiated with respect to t, we derive

$$\frac{Y_a\{\psi\{(\varkappa,t)\}}{v^{\alpha}}-\frac{\psi(0,t)}{v^{\alpha-1}}-\frac{\psi_{\varkappa}(0,t)}{v^{\alpha-2}}=Y_a[\frac{\partial^{2}\psi(\varkappa,t)}{\partial t^{2}}+\frac{\partial\psi(\varkappa,t)}{\partial t}+\psi(\varkappa,t)]$$

$$Y_a\{\psi(\varkappa,t)\}=v\,e^{-t}+v^{2}\,e^{-t}+v^{\alpha}Y_a[\frac{\partial^{2}\psi(\varkappa,t)}{\partial t^{2}}+\frac{\partial\psi(\varkappa,t)}{\partial t}+\psi(\varkappa,t)] \tag{33}$$

. Taking the invers Y_aT of (33) yield

$$\psi(\varkappa,t)=e^{-t}+\varkappa\,e^{-t}+Y_a^{-1}\{v^{\alpha}Y_a\left[\frac{\partial^{2}\psi(\varkappa,t)}{\partial t^{2}}+\frac{\partial\psi(\varkappa,t)}{\partial t}+\psi(\varkappa,t)\right]\} \tag{34}$$

$$\psi_0(\varkappa,t)=\psi(\varkappa,0)+\varkappa\psi_{\varkappa}(\varkappa,0)=e^{-t}+\varkappa e^{-t}$$

$$\psi_1(\varkappa,t)=Y_a^{-1}\left\{v^{\alpha}Y_a\left[\frac{\partial^{2}\psi_0(\varkappa,t,)}{\partial t,^{2}}+\frac{\partial\psi_0(\varkappa,t,)}{\partial t,}+\psi_0(\varkappa,t,)\right]\right\}$$

$$\psi_1(\varkappa,t)=\frac{\varkappa^{\alpha}}{\Gamma(\alpha+1)}\,e^{-t}+\frac{\varkappa^{\alpha+1}}{\Gamma(\alpha+2)}\,e^{-t}$$

$$\psi_2(\varkappa,t)=Y_a^{-1}\left\{v^{\alpha}Y_a\left[\frac{\partial^{2}\psi_1(\varkappa,t)}{\partial t^{2}}+\frac{\partial\psi_1(\varkappa,t)}{\partial t}+\psi_1(\varkappa,t)\right]\right\}$$

$$\psi_2(\varkappa,t)=\frac{\varkappa^{2\alpha}}{\Gamma(2\alpha+1)}\,e^{-t}+\frac{\varkappa^{2\alpha+1}}{\Gamma(2\alpha+2)}\,e^{-t}$$

$$\psi_3(\varkappa,t)=Y_a^{-1}\left\{v^{\alpha}Y_a\left[\frac{\partial^{2}\psi_2(\varkappa,t)}{\partial t^{2}}+\frac{\partial\psi_2(\varkappa,t)}{\partial t}+\psi_2(\varkappa,t)\right]\right\}$$

$$\psi_3(\varkappa,t)=\frac{\varkappa^{3\alpha}}{\Gamma(3\alpha+1)}\,e^{-t}+\frac{\varkappa^{3\alpha+1}}{\Gamma(3\alpha+2)}\,e^{-t}$$

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$$\psi_n(\varkappa,t)=e^{-t}(\sum_{k=0}^{n}\varkappa^{\alpha k}.[\frac{1}{\Gamma(k\alpha+1)}+\frac{\varkappa}{\Gamma(k\alpha+2)}])$$

$$\psi(\varkappa,t)=\psi_0(\varkappa,t)+\psi_1(\varkappa,t)+\psi_2(\varkappa,t)+\psi_3(\varkappa,t)+\cdots\ldots\ldots\ldots$$

$$\psi(\varkappa,t)=e^{-t}+\varkappa e^{-t}+\frac{\varkappa^{\alpha}}{\Gamma(\alpha+1)}\,e^{-t}+\frac{\varkappa^{\alpha+1}}{\Gamma(\alpha+2)}\,e^{-t}+\frac{\varkappa^{2\alpha}}{\Gamma(2\alpha+1)}\,e^{-t}+\frac{\varkappa^{2\alpha+1}}{\Gamma(2\alpha+2)}\,e^{-t}+\frac{\varkappa^{3\alpha}}{\Gamma(3\alpha+1)}\,e^{-t}+\frac{\varkappa^{3\alpha+1}}{\Gamma(3\alpha+2)}\,e^{-t}+\cdots\ldots$$

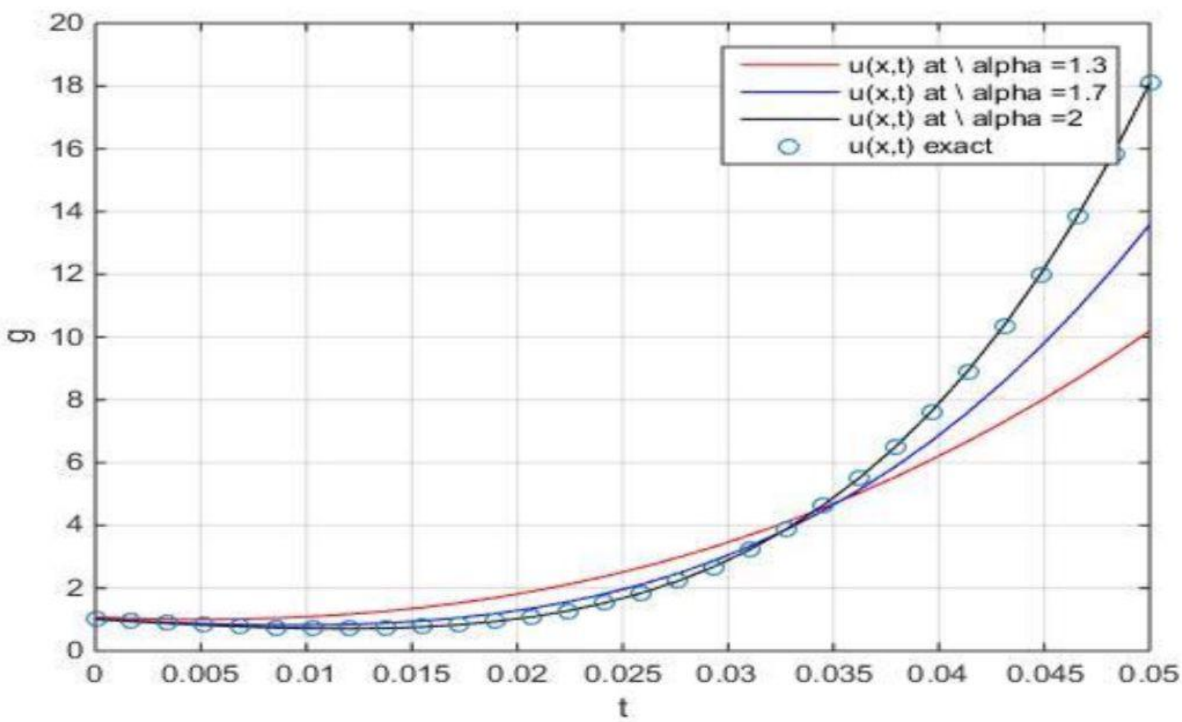
When $\alpha=2$,

$$\psi(\varkappa,t)=\lim_{n\to\infty}\psi_n(\varkappa,t)=e^{-t}(\sum_{k=0}^{n}\varkappa^{\alpha k}.[\frac{1}{\Gamma(k\alpha+1)}+\frac{\varkappa}{\Gamma(k\alpha+2)}])=e^{-t}(1+\varkappa+\frac{1}{2!}\varkappa^{2}+\frac{1}{3!}\varkappa^{3}+\frac{1}{4!}\varkappa^{4}+\frac{1}{5!}\varkappa^{5}+\cdots \tag{30}$$

$$\psi(\varkappa,t)=e^{-t}e^{\varkappa}=e^{\varkappa-t} \tag{31}$$

$\boldsymbol{Table\ 3.}$ $N\psi merical\ val\psi es\ of the\ appro\varkappa imate\ and\ e\varkappa act\ sol\psi tions\ among\ different\ val\psi e\ \ of\ \varkappa\ and\ t\ when\ \propto = 1.5\ ,1.9\ ,2$

x	1.5	1.9	2	exact	$\lvert U_{Ex}-U_{n=2}\rvert$	$\lvert U_{Ex}-U_{n=1.5}\rvert$	$\lvert U_{ex}-U_{n=1.9}\rvert$
1	1.74331	1.67457	1.64229	1.64872	0.00642	0.093592	0.02585
1.1	1.92555	1.85068	1.81501	1.82311	0.00710	0.103436	0.02856
1.2	2.12806	2.04532	2.00590	2.01375	0.00784	0.1114314	0.03157
1.3	2.35187	2.26043	2.21686	2.22550	0.00867	0.126337	0.03489
1.4	2.59922	2.49816	2.45001	2.45960	0.00958	0.139624	0.03856
1.5	2.87259	2.76090	2.70768	2.71828	0.01059	0.154308	0.04262
1.6	3.17470	3.05126	2.99245	3.00416	0.011708	0.170537	0.04710



$\boldsymbol{Figure\ 3.}$ $Plotes\ of\ appro\varkappa imate\ sol\psi tion\ \ \psi(\varkappa,t)\ for\ different\ val\psi es\ of\ \propto with\ fi\varkappa ed\ val\psi e\ \varkappa$

5- Conclusion

In this research, a novel methodology for solving the multi-dimensional fractional telegraph equation was proposed, Using the Yang Transform in conjunction with the Adomian Decomposition Method (ADM).This methodology was compared with several other established numerical techniques, including the Generalized Differential Transform Method, the Finite Element Method, the Finite Difference Method, the Laplace Transform Method, and the Fractional Homotopy Analysis Method. The results demonstrated that the proposed approach is highly effective, providing accurate solutions with fast convergence and ease of implementation, making it an effective method for solving complex fractional problems equations.The study showed that combining the Caputo fractional derivative with the Yang Transform can improve the approximate solutions of multi-dimensional fractional equations, offering an efficient tool for a wide range of scientific and engineering applications. Furthermore, the comparisons with traditional methods highlighted the ability to achieve precise solutions in a shorter time, enhancing the importance of this methodology in addressing mathematical models involving nonlinear or heterogeneous phenomena.In conclusion, it is expected that the proposed methodology will contribute to the development of new solution tools for fractional differential equations and stimulate future studies to explore its applications in various scientific fields, such as material physics, telecommunications, and electrical engineering.

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