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SOME PROPERTIES OF HIGHER ORDER (n, m)-DRAZIN INTUTIONISTIC FUZZY NORMAL OPERATORS

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Abstract:

This paper introduces the concept of the p-tuple of (n, m)-D-intuitionistic fuzzy normal operators in Intuitionistic Fuzzy Hilbert Spaces (IFH-spaces). New definitions and theorems are established, and key algebraic and structural properties are investigated. The results provide a rigorous foundation for extending intuitionistic fuzzy operator theory and highlight the significance of these operators in the broader context of IFH-spaces.

Keywords: ($I\mathcal{F}\mathcal{H}$ -Space), $I\mathcal{F}$ - $N\mathcal{B}_d[X]$, $I\mathcal{F}(n)$ - $N\mathcal{B}_d[X]$, $I\mathcal{F}(n,m)$ - $N\mathcal{B}_d[X]$, $I\mathcal{F}(n,m)$ - $N\mathcal{B}_d[X]^p$.

1-Introduction

In recent years, intuitionistic fuzzy spaces have attracted significant attention due to their diverse applications in fields such as artificial intelligence, approximation theory, and the analysis of imprecise data. Despite these developments, the theory of (n, m)- \mathcal{D} -intuitionistic fuzzy normal operators, particularly in the context of \mathcal{P} -tuples, remains insufficiently explored. This study aims to address this gap by introducing the \mathcal{P} -tuple of (n, m)- \mathcal{D} -intuitionistic fuzzy normal operators within Intuitionistic Fuzzy Hilbert Spaces ($I\mathcal{F}\mathcal{H}$ -spaces), establishing fundamental definitions, and proving theorems that describe their algebraic and structural properties. Specifically, the Intuitionistic Fuzzy Hilbert Space ($I\mathcal{F}\mathcal{H}$ -space) serves as a natural extension of the classical Hilbert space in the intuitionistic fuzzy framework, playing a central role in the analysis of both algebraic and structural aspects of these operators. This contribution extends the existing framework of intuitionistic fuzzy operator theory and provides a solid foundation for future research in this area.

Park [25] was among the first to explore the concept of Intuitionistic Fuzzy Metric Spaces, providing a theoretical foundation for measuring distances under uncertainty and fuzziness. Later, Saadati [20] extended this concept by introducing the intuitionistic fuzzy metric and norm, paving the way for more sophisticated analytical constructions.

In 2009, Goudarzi et al. [22] intr0duced the notion (*IFIP*-spaces), marking an essential step toward a comprehensive theory of intuitionistic fuzzy functional analysis. Subsequently, Samanta and Bera [21] redefined and refined this concept in 2019.

A major development came in 2020, when Radharamani and her collaborators [18, 19] provided a formal definition of (IFH-Space), highlighting its structural and foundational properties. This advancement enabled the study of linear operators within an intuitionistic fuzzy environment, analogous to classical Hilbert space theory, but incorporating the degrees membership and non-membership each element

In this context, the concept of the Drazin inverse has emerged as an important analytical tool for studying linear operators. It was first introduced for bounded operators on complex Banach spaces by Sheibani, Rashidi, and Rezaei [17] and King [9]. Since then, numerous detailed studies and applications of Drazin invertibility have been conducted, which can be explored further in reference [5].

Additionally, significant contributions from functional analysis provide theoretical support for the fuzzy operator theory presented in this work, particularly those outlined in [1, 8, 11, 12, 13, 14, 15, 23], which address operator properties and spectral theory in the context of generalized and fuzzy structures

The paper regulations are as follows

Section two includes several preliminary results. In Section three, we introduce the concept the intuitionistic fuzzy Drazin inverse, present several theorems, and discuss some properties. We also cover \mathcal{D} - intuitionistic fuzzy normal, n-power \mathcal{D} -intuitionistic fuzzy normal, p-intuitionistic fuzzy normal, jointly intuitionistic fuzzy p-normal, jointly intuitionistic fuzzy p-normal, jointly intuitionistic fuzzy p-normal, p-tuple of p-intuitionistic fuzzy normal operator and p-tuple of p-normal operator p-normal operator p-normal operator p-normal operator p-normal operator intuitionistic fuzzy normal operator p-normal operator intuitionistic fuzzy normal operator which are

2- Preliminaries

Definition 2.1: [22]

A continuous t-norm \mathcal{T} is said to be continuous t-representable if and only if there exist a continuous t-norm *, a continuous t – conorm * defined on the interval [0,1], \forall elements $x = (x_1, x_2)$, $y = (y_1, y_2) \in \mathcal{L}^*$, the following hold:

$$\mathcal{T}(x,y) = (x_1 * y_1, x_2 \diamond y_2).$$

Definition 2.2: [22]

Let

$$\mu: \mathcal{X}^{2} \times (0, +\infty) \to [0,1], \ \vartheta: \mathcal{X}^{2} \times (0, +\infty) \to [0,1],$$
$$\mu(x, y, t) + \vartheta(x, y, t) \le 1, \forall x, y \in \mathcal{X} \& t > 0.$$

An Intuitionistic Fuzzy Inner Product Space (*IFIP*-Space) is defined as a triplet $(\mathcal{X}, \mathcal{F}_{\mu, v}, \mathcal{I})$, where \mathcal{X} is real Vector Space, $\mathcal{F}_{\mu, v}$ is an Intuitionistic Fuzzy set on $\mathcal{X}^2 \times \mathbb{R}$ and \mathcal{I} is a continuous t –representable, the following conditions are satisfied for all $x, y, z \in \mathcal{X}$ and $x, y, z \in \mathbb{R}$:

$$\begin{split} & (\text{IFI-1})\,\mathcal{F}_{\mu,\nu}\left(x,y,0\,\right) = 0 \text{ and } \mathcal{F}_{\mu,\nu}\left(x,x,t\,\right) > 0 \text{ , } \forall t > 0 \text{ .} \\ & (\text{IFI-2})\,\mathcal{F}_{\mu,\nu}\left(x,y,t\right) = \,\mathcal{F}_{\mu,\nu}\left(y,x,t\,\right) . \\ & (\text{IFI-3})\,\mathcal{F}_{\mu,\nu}\left(x,x,t\,\right) \neq H\left(t\right) \end{split}$$

for some $t \in \mathbb{R}$ iff $x \neq 0$

Where H
$$(t)=\begin{cases} 1 \ , & \text{if} \quad t>0 \\ 0 \ , & \text{if} \quad t\leq 0 \end{cases}$$
 (IFI-4) For all scalars $\alpha\in\mathbb{R}$

$$\mathcal{F}_{\mu,v}(x\alpha,y,t) = \begin{cases} \mathcal{F}_{\mu,v}\left(x,y,,\frac{t}{\alpha}\right), & \text{if } \alpha > 0 \\ & \text{H}(t), & \text{if } \alpha = 0 \\ \mathcal{N}_{s}\left(\mathcal{F}_{\mu,v}\left(x,y,\frac{t}{\alpha}\right)\right), & \text{if } \alpha < 0 \end{cases}$$

(IFI-5) $\sup \left\{ \mathcal{T} \left(\mathcal{F}_{\mu,\nu}(x,z,s), \mathcal{F}_{\mu,\nu}(y,z,r) \right) \right\} = \mathcal{F}_{\mu,\nu}(x+y,y,t).$

(IFI-6) $\mathcal{F}_{\mu,\nu}(x,y,.): \mathbb{R} \to [0,1]$ is continuous on $\mathbb{R} \setminus \{0\}$.

(IFI-7) $\lim_{t\to 0} \mathcal{F}_{\mu,\nu}(x,y,t) = 1$.

Note 2. 3: [22]

- (i) The standard negator is defined as $\mathcal{N}_{s}(x) = 1 x$, $\forall x \in [0,1]$
- (ii) By defining the inner product as $\langle x,y\rangle=\mathcal{F}_{\mu,\nu}(x,y,.)$, it becomes evident that the Intuitionistic Fuzzy Inner Product behaves in a manner analogous to the classical inner product.
- (iii) Schwarz inequality:

for $x, y \in X$ and for any s, t > 0 the following inequality holds

$$\mathcal{F}_{\mu,v}(x,y,ts) \geq \mathcal{T}\left(\mathcal{F}_{\mu,v}(x,x,t^2)\mathcal{F}_{\mu,v}(y,y,s^2)\right),$$

- (iv) A sequence $\{x_n\} \subseteq \mathcal{X}$ is said to be t- convergent to $x \in \mathcal{X}$, if for any given $\epsilon > 0$, $\lambda > 0$
- 0, there exists an atural number, $N^0 = N^0(\epsilon, \lambda)$, such that $P(x_n x, \epsilon) > \mathcal{N}_{\delta}(\lambda)$, whenever $n > N^0$. (v) Enable f(x) be a continuous linear functional on \mathcal{X} . Then its said to be $\mathcal{T}_{\mathcal{T}_{\mu,\nu}}$ continuous if for any sequence x_n in $\mathcal{X} x_n \xrightarrow{\mathcal{T}_{F_{\mu,\nu}}} x \Longrightarrow f(x_n) \xrightarrow{\mathcal{T}_{F_{\mu,\nu}}} f(x)$,

Theorem 2.4: [18]

Let $(\mathcal{X}, \mathcal{F}_{\mu, v}, \mathcal{I})$ exist as any $I\mathcal{F}IP$ — Space , where \mathcal{T} is a continuous t - representable such that $\forall x, y \in \mathcal{I}$ \mathcal{X} , $\sup\{t \in \mathbb{R} \mid \mathcal{F}_{\mu,v}(x,y,t) < 1\} < \infty$. Define $\langle .,. \rangle : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ by $\langle x,y \rangle = \sup\{t \in \mathbb{R} : \mathcal{F}_{\mu,v}(x,y,t) < 1\}$ 1. Then $(X\langle .,. \rangle)$ is an IFIP-space, that $(X, \mathcal{P}_{\mu, \nu})$ is normed space, where $\mathcal{P}_{\mu, \nu}(x, z) = \langle x, x \rangle^{1/2} \forall x \in \mathcal{X}$.

Definition 2.5:[18]

Let $(\mathcal{X}, \mathcal{F}_{\mu, v}, \mathcal{I})$ a denote an $I\mathcal{F}IP$ — Space with $IP: \langle x, y \rangle = \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu, v}(x, y, t) < 1\}$, $\forall x, y \in \mathcal{X}$. If $(\mathcal{X}, \mathcal{F}_{\mu, \nu}, \mathcal{I})$ is complete in the norm $\mathcal{P}_{\mu, \nu}$ then \mathcal{X} is an intuitionistic Fuzzy Hilbert space ($I\mathcal{FH}$ – Space).

Theorem 2. 6: [18]

Enable $(\mathcal{X}, \mathcal{F}_{\mu, v}, \mathcal{I})$ be a $(I\mathcal{F}\mathcal{H}$ - space) with $IP(x, y) = Sup\{t \in \mathbb{R} : \mathcal{F}(x, y, t) \le 1\}, \forall x, y \in \mathcal{X}$. A sequence $\{x_n\}$ on \mathcal{X} is $\mathcal{T}_{\mathcal{T}_{\mu,\sigma}}$ - convergent (i.e. $x_n \xrightarrow{\mathcal{T}_{\mathcal{T}_{\mu,\sigma}}} x$) if $x_n \xrightarrow{\mathcal{T}_{\mu,\sigma}} x$

Theorem 2. 7: [18] (Riesz Theorem)

Let $(X, \mathcal{F}_{\mu,\nu}, \mathcal{I})$ be an $I\mathcal{FH}$ – Space.

For every $\mathcal{T}_{\mathcal{F}_{\mu,\sigma}}$ - continuous linear functional f , there exists a unique vector $\psi \in \mathcal{X}$, $\forall x \in \mathcal{X}$, the following $f(x) = \sup \{ t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(x,y,t) < 1 \}.$

Theorem 2.8:[18]

Let $(X, \mathcal{F}_{\mu, \nu}, \mathcal{I})$ exist as any *IFIP*-Space, where \mathcal{I} is a continuous t – representable and suppose satisfying that for every

$$x, y \in \mathcal{X}$$
, $\sup\{t \in \mathbb{R}: \mathcal{F}_{u,v}(x, y, t) < 1\} < \infty$,

$$\begin{split} \sup t \in \mathbb{R} : \mathcal{F}_{\mu,v}(x+y,z,t<1) &= \sup \left\{ t \in \mathbb{R} <: \mathcal{F}_{\mu,v}(x,z,t) < 1 \right. \\ &+ \sup \left\{ t \in \mathbb{R} : \mathcal{F}_{\mu,v}(y,z,t) < 1 \right. \right\} \forall \; x,y \in \mathcal{X} \; . \end{split}$$

Remark 2.9:[18]

Let IFB[X] denote the collection of all intuitionistic fuzzy bounded linear defined on the IFH -Space

Theorem 2. 10: (IFA - operator in IFH - Space) [18]

Let $(\mathcal{X}, \mathcal{F}_{\mu, v}, \mathcal{I})$ exist as any $I\mathcal{FH}$ -Space, let $\mathcal{S} \in I\mathcal{FB}[\mathcal{X}]$. Then a unique exists

$$S^* \in IFB[X] \ni \langle Sx, y \rangle = \langle x, S^*y \rangle \forall x, y \in X$$
.

Definition 2.11: (IFSA – operator) [18]

Let $(X, \mathcal{F}_{u,v}, \mathcal{I})$ a denote an $I\mathcal{FH}$ -Space with

$$IP: \langle x, y \rangle = \sup \{ t \in \mathbb{R}: \mathcal{F}_{u,v}(x,y,t) < 1 \}, \forall x, y \in \mathcal{X} \}$$

Let $T \in IFB[X]$. Next T is Intuitionistic Fuzzy Self- Adjoint operat0r, if $T = T^*$, where of $T^* \in IFSA$ of T.

Theorem 2.12:[18]

Let $(X, \mathcal{F}_{u,v}, \mathcal{I})$ exist as any $I\mathcal{FH}$ -space with

$$IP: \langle x, y \rangle = \sup \{ t \in \mathbb{R}: \mathcal{F}_{u,v}(x,y,z) < 1 \}, \forall x,y \in \mathcal{X}, \text{let } \mathcal{T} \in IFB[\mathcal{X}].$$

 \mathcal{T} is $I\mathcal{F}SA$ operator.

Theorem 2.13:[18]

Let $(\mathcal{X}, \mathcal{F}_{\mu,v}, \mathcal{I})$ a denote an $I\mathcal{F}IP$ -space with $IP: \langle x, y \rangle = \sup \{t \in \mathbb{R}: \mathcal{F}_{\mu,v} (x,y,t) < 1\}$, $\forall x,y \in \mathcal{X}$ and let \mathcal{T}^* a denote the $I\mathcal{F}A$ — operator of \mathcal{T} . Next the following properties hold:

- 1. $(\mathcal{T}^*)^* = \mathcal{T}$
- $2. \qquad (\delta \mathcal{T})^* = \delta \mathcal{T}^*$
- 3. $(\delta T_1 + \gamma T_2)^* = \delta T_1^* + \gamma T_2^*$ where δ , γ are scalars.
- 4. $(\mathcal{T}_1\mathcal{T}_2)^* = \mathcal{T}_2^*\mathcal{T}_1^*$.

3. MAIN RESULTS

In the following section, we introduce the concept of the intuitionistic fuzzy Drazin inverse ($IF\mathcal{H}$ -space) and examine some of its fundamental properties. Furthermore, we explore the structure and behavior of (n, m) - power \mathcal{D} -intuitionistic fuzzy normal.

Definition. (3.1): (Intuitionistic fuzzy Drazin inverse)

Enable $(X, \mathcal{F}_{\mu, \nu}, \mathcal{I})$ be any $I\mathcal{FH}$ -space using

$$IP: \langle u, v \rangle = \sup\{x \in \mathbb{R}: \mathcal{F}(u, v, x) < 1\} \ \forall \ u, v \in \mathcal{X}$$

as well as let $T \in IFB[X]$, thereafter intuitionistic fuzzy Drazin inverse of T is that unique operator $T^{\mathcal{D}} \in IFB[X]$ in case present as well as meets the following requirements

$$p_{\mu,v}\left(\left(\mathcal{T}^{\mathcal{D}}\mathcal{T}-\mathcal{T}\mathcal{T}^{\mathcal{D}}\right)u,t\right)=0$$

$$p_{\mu,\nu}\left(\left(\left(\mathcal{T}^{\mathcal{D}}\right)^{2}\mathcal{T}-\mathcal{T}^{\mathcal{D}}\right)u,t\right)=0$$

$$p_{\mu,\nu}\big((\mathcal{T}^{\nu+1}\mathcal{T}^D-\mathcal{T}^{\nu})u,t\big)=0$$

Given an integer $v \ge 0$

Denoted by $IFB_d[X]$ all elements of intuitionistic fazzy Drazin invertible elements of IFB[X] for $T \in IFB_d[X]$ It became evident that intuitionistic fazzy Drazin inverse $\mathcal{T}^{\mathcal{D}}$ of \mathcal{T} conforms to the following rules

$$\begin{cases} p_{\mu,\nu}\left(\left((\mathcal{T}^*)^{\mathcal{D}} - \left(\mathcal{T}^{\mathcal{D}}\right)^*\right)u,t\right) = 0\\ p_{\mu,\nu}\left(\left(\left(\mathcal{T}^k\right)^{\mathcal{D}} - \left(\mathcal{T}^{\mathcal{D}}\right)^k\right)u,t\right) = 0 \quad \forall k \in \mathbb{N} \end{cases}$$

Furthermore, it was noted that in the case where $\mathcal{T} \in I\mathcal{FB}_d[\mathcal{X}]$ and $\mathcal{N} \in I\mathcal{FB}[\mathcal{X}]$ is an invertible operator then

$$\mathcal{N}^{-1}\mathcal{T}\mathcal{N} \in I\mathcal{F}\mathcal{B}_d[\mathcal{X}] \text{ and } p_{\mu,\nu}\left(((\mathcal{N}^{-1}\mathcal{T}\mathcal{N})^D - \mathcal{N}^{-1}\mathcal{T}^D\mathcal{N})u,t\right) = 0$$

Definition (3.2):

Let $(X, \mathcal{F}_{u,\sigma}, \mathcal{I})$ be an IFH-space with

$$IP: \langle u, v \rangle = \sup \{ t \in \mathbb{R}: \mathcal{F}_{\mu, v}(u, v, t) < 1 \} \ \forall \ u, v \in \mathcal{X} \ and \ let \ \mathcal{T} \in IFB[\mathcal{X}],$$

then \mathcal{T} is called

- (i) intuitionistic Fuzzy normal if $p_{\mu,\nu} \left((\mathcal{T}\mathcal{T}^* \mathcal{T}^*\mathcal{T}) u, t \right) = 0$ denoted by $I\mathcal{F}$ $N\mathcal{B}[\mathcal{X}]$
- (ii) intuitionistic Fuzzy n- normal operator if $p_{\mu,\nu} \left((\mathcal{T}^n \mathcal{T}^* \mathcal{T}^* \mathcal{T}^n) u, t \right) = 0$ denoted by $I\mathcal{F}(n)$ - $N\mathcal{B}[\mathcal{X}]$ (iii) intuitionistic Fuzzy (n,m)- normal operator if $p_{\mu,\nu} \left((\mathcal{T}^n \mathcal{T}^{*m} \mathcal{T}^{*m} \mathcal{T}^n) u, t \right) = 0$ denoted by $I\mathcal{F}(n,m)$ -NB[X] for some positive integers n, m

Definition (3.3):

Let $(\mathcal{X}, \mathcal{F}_{\mathsf{HAT}}, \mathcal{I})$ be a $I\mathcal{FH}$ -space with

$$\mathit{IP} \colon \langle u, v \rangle = \mathit{sup} \big\{ t \in \mathbb{R} \colon \mathcal{F}_{\mu, v}(u, v, t) < 1 \big\} \forall \ u, v \in \mathcal{X} \ and \ \mathit{Iet} \ \mathcal{T} \in \mathit{IFB}_d[\mathcal{X}].$$

then \mathcal{T} is called

- (i) \mathcal{D} intuitionistic Fuzzy normal If $p_{\mu,\nu}\left(\left(\mathcal{T}^{\mathcal{D}}\mathcal{T}^*-\mathcal{T}^*\mathcal{T}^{\mathcal{D}}\right)u,t\right)=0$ This is denoted by $I\mathcal{F}$ $N\mathcal{B}_{d}[\mathcal{X}]$
- (ii) n- power \mathcal{D} intuitionistic Fuzzy normal if $p_{\mu,\nu}\left(\left(\left(\mathcal{T}^{\mathcal{D}}\right)^{n}\mathcal{T}^{*}-\mathcal{T}^{*}\left(\mathcal{T}^{\mathcal{D}}\right)^{n}\right)u,t\right)=0$ This is denoted by $I\mathcal{F}(n)$ - $N\mathcal{B}_d[\mathcal{X}]$
- (iii) (n, m)-power \mathcal{D} -intuitionistic Fuzzy normal if $p_{\mu, \nu}\left(\left(\left(\mathcal{T}^{\mathcal{D}}\right)^{n}\mathcal{T}^{*m} \mathcal{T}^{*m}\left(\mathcal{T}^{\mathcal{D}}\right)^{n}\right)u, t\right) = 0$ This is denoted by IF(n,m)- $NB_d[X]$ for certain positive whole numbers n, m

The study p-tuples (collections of p operators) has garnered significant attention from several researchers In recent years. Notable advancements have occurred this area discussed [2,3,4,6,7,10,16,24] and additional information can be found within those sources.

Given a p-tuple

$$\mathcal{T} := \left(\mathcal{T}_1, \dots, \mathcal{T}_{\mathcal{P}}\right) \in I\mathcal{FB}[\mathcal{X}]^{\mathcal{P}}$$

we define

$$p_{\mu,\nu}((T^*T - TT^*)u, t) = 0 \in IFB[X \oplus \dots \oplus X]$$

as the self- commutator of \mathcal{T} , defined by

$$p_{\mu,v}\left((T^*T-TT^*)_{i,j}u,t\right) \coloneqq p_{\mu,v}\left(\left(T_j^*T_i-T_iT_j^*\right)u,t\right) \ \forall (i,j) \in \{1\dots p\}^2$$

Note that this definition of self- commutator for \mathcal{P} -tuples of operators on a $(I\mathcal{FH}$ - Space) Where $\mathcal{T}^* := (\mathcal{T}_1^{|*}, \dots, \mathcal{T}_{\mathcal{P}}^*)$ that \mathcal{T} is jointly intuitionistic fuzzy hyponormal if

$$\begin{split} p_{\mu,v} \left((\mathcal{T}^* \mathcal{T} - \mathcal{T} \mathcal{T}^*) u, t \right) \\ &= \begin{pmatrix} p_{\mu,v} \left((\mathcal{T}_1^* \mathcal{T}_1 - \mathcal{T}_1 \mathcal{T}_1^*) u, t \right) & p_{\mu,v} \left((\mathcal{T}_2^* \mathcal{T}_1 - \mathcal{T}_1 \mathcal{T}_2^*) u, t \right) & \cdots & p_{\mu,v} \left((\mathcal{T}_p^* \mathcal{T}_1 - \mathcal{T}_1 \mathcal{T}_p^*) u, t \right) \\ p_{\mu,v} \left((\mathcal{T}_1^* \mathcal{T}_2 - \mathcal{T}_2 \mathcal{T}_1^*) u, t \right) & p_{\mu,v} \left((\mathcal{T}_2^* \mathcal{T}_2 - \mathcal{T}_2 \mathcal{T}_2^*) u, t \right) & \cdots & p_{\mu,v} \left((\mathcal{T}_p^* \mathcal{T}_2 - \mathcal{T}_2 \mathcal{T}_p^*) u, t \right) \\ \vdots & \vdots & \vdots & \vdots \\ p_{\mu,v} \left((\mathcal{T}_1^* \mathcal{T}_p - \mathcal{T}_p \mathcal{T}_1^*) u, t \right) & p_{\mu,v} \left((\mathcal{T}_2^* \mathcal{T}_p - \mathcal{T}_p \mathcal{T}_2^*) u, t \right) & \cdots & p_{\mu,v} \left((\mathcal{T}_p^* \mathcal{T}_p - \mathcal{T}_p \mathcal{T}_p^*) u, t \right) \end{pmatrix} \end{split}$$

is a non-negative operator on $\mathcal{X} \oplus ... \oplus \mathcal{X}$, or equivalently

$$\sum_{1 \leq i,j \leq p} p_{\mu,v} \left(\left(\mathcal{T}_i^* \mathcal{T}_j - \mathcal{T}_j \mathcal{T}_i^* \right) u, t \right) \geq 0 \quad \forall x \in \mathcal{X}$$

T is called jointly intuitionistic fuzzy normal if T satisfying

$$\begin{cases} p_{\mu,v}\left(\left(\mathcal{T}_{i}\mathcal{T}_{j}-\mathcal{T}_{j}\mathcal{T}_{i}\right)u,t\right)=0 & i,j\in\{1,\ldots,p\}\\ p_{\mu,v}\left(\left(\mathcal{T}_{i}^{*}\mathcal{T}_{i}-\mathcal{T}_{i}\mathcal{T}_{i}^{*}\right)u,t\right)=0 & i=1,\ldots,p \end{cases}$$

This is denoted by $I\mathcal{F}$ - $N\mathcal{B}[\mathcal{X}]^{\mathcal{P}}$

T is called jointly intuitionistic fuzzy n- normal if T satisfying

$$\begin{cases} p_{\mu,v}\left(\left(T_{i}T_{j}-T_{j}T_{i}\right)u,t\right)=0 & i,j\in\{1,\dots,p\}\\ p_{\mu,v}\left(\left(T_{i}^{n}T_{i}^{*}-T_{i}^{*}T_{i}^{n}\right)u,t\right)=0 & i=1,\dots,p \end{cases}$$

This is denoted by $I\mathcal{F}(n)$ - $N\mathcal{B}[\mathcal{X}]^p$

T is called jointly intuitionistic fuzzy (n, m)- normal if T satisfying

$$\begin{cases} p_{\mu,\nu}\left(\left(\mathcal{T}_{i}\mathcal{T}_{j}-\mathcal{T}_{j}\mathcal{T}_{i}\right)u,t\right)=0 & i,j\in\{1,\dots,p\}\\ p_{\mu,\nu}\left(\left(\mathcal{T}_{i}^{n}\mathcal{T}_{i}^{*m}-\mathcal{T}_{i}^{*m}\mathcal{T}_{i}^{n}\right)u,t\right)=0 & i=1,\dots,p \end{cases}$$

This is denoted by $I\mathcal{F}(n,m)$ - $N\mathcal{B}[\mathcal{X}]^p$ for certain positive integers n and m

Definition (3.4):

Let $\mathbf{T} := (T_1, \dots, T_p) \in I\mathcal{F} - N\mathcal{B}_d[\mathcal{X}]^p$ we say that \mathbf{T} is p-tuple of \mathcal{D} -intuitionistic Fuzzy normal operator if \mathbf{T} satisfying

$$\begin{cases} p_{\mu,v}\left(\left(T_{i}T_{j}-T_{j}T_{i}\right)u,t\right)=0 & i,j\in\{1,\ldots,p\}^{2} \\ p_{\mu,v}\left(\left(T_{i}^{D}T_{i}^{*}-T_{i}^{*}T_{i}^{D}\right)u,t\right)=0 & i=1,\ldots,p \end{cases}$$

This is denoted by $I\mathcal{F}$ - $N\mathcal{B}_d[\mathcal{X}]^p$

Definition (3.5):

Let $\mathcal{T} := (\mathcal{T}_1, \dots, \mathcal{T}_p) \in I\mathcal{F}(n) - N\mathcal{B}_d[\mathcal{X}]^p$ we say that \mathcal{T} is p-tuple of n- power \mathcal{D} - intuitionistic Fuzzy normal operator if \mathcal{T} satisfying

$$\begin{cases} p_{\mu,\nu}\left(\left(\mathcal{T}_{i}\mathcal{T}_{j}-\mathcal{T}_{j}\mathcal{T}_{i}\right)u,t\right)=0 & i,j\in\{1,\ldots,p\}^{2} \\ p_{\mu,\nu}\left(\left(\left(\mathcal{T}_{i}^{D}\right)^{n}\mathcal{T}_{i}^{*}-\mathcal{T}_{i}^{*}\left(\mathcal{T}_{i}^{D}\right)^{n}\right)u,t\right)=0 & i=1,\ldots,p \end{cases}$$

This is denoted by $I\mathcal{F}(n)$ - $N\mathcal{B}_d[\mathcal{X}]^p$ for certain positive integers n

4.p-tuple of (n, m) -Drazin intuitionistic fuzzy normal operators

" This portion is concerned with investigating the behavior $I\mathcal{F}(n,m)$ - $N\mathcal{B}_d[\mathcal{X}]^p$ in $I\mathcal{FH}$ -spaces, including their transformations under powers, adjoints, and unitary equivalence."

Definition (4.1):

Let $\mathcal{T} := (\mathcal{T}_1, \dots, \mathcal{T}_p) \in I\mathcal{F}(n, m) - N\mathcal{B}_d[\mathcal{X}]^p$ we say that \mathcal{T} is p-tuple of $(n, m) - \mathcal{D}$ - intuitionistic Fuzzy normal operator if \mathcal{T} satisfying

$$\begin{cases} p_{\mu,v}\left(\left(T_{i}T_{j}-T_{j}T_{i}\right)u,t\right)=0 & i,j\in\{1,\ldots,p\}^{2} \\ p_{\mu,v}\left(\left(\left(T_{i}^{\mathcal{D}}\right)^{n}T_{i}^{*m}-T_{i}^{*m}\left(T_{i}^{\mathcal{D}}\right)^{n}\right)u,t\right)=0 & i=1,\ldots,p \end{cases}$$

This is denoted by $I\mathcal{F}(n,m)$ - $N\mathcal{B}_d[\mathcal{X}]^p$ for certain positive integers n and m

Example (4.2):

Let $\mathcal{T} \in I\mathcal{F}(n,m)$ - $N\mathcal{B}_d[\mathcal{X}]$ be an (n,m) - \mathcal{D} - intuitionistic Fuzzy normal operator, then $\mathcal{T} = (\mathcal{T}, \ldots, \mathcal{T}) \in I\mathcal{F}(n,m)$ - $N\mathcal{B}_d[\mathcal{X}]^p$

Lemma (4.3):

Let $T_1, T_2 \in IFB[X]$ and $r^*, s \in \mathbb{N}^*$, then

$$p_{\mu,v}\big((\mathcal{T}_{1}^{r}\mathcal{T}_{2}^{s}-\mathcal{T}_{2}^{s}\mathcal{T}_{1}^{r})u,t\big)=\sum_{i=0}^{r-1}\sum_{j=0}^{s-1}\mathcal{T}_{1}^{r-i-1}\mathcal{T}_{2}^{s-j-1}p_{\mu,v}\big((\mathcal{T}_{1}\mathcal{T}_{2}-\mathcal{T}_{2}\mathcal{T}_{1})u,t\big)\mathcal{T}_{2}^{j}\mathcal{T}_{1}^{i}$$

Proof: Observe that

$$p_{\mu,\nu} \left((\mathcal{T}_{1}^{r} \mathcal{T}_{2} - \mathcal{T}_{2} \mathcal{T}_{1}^{r}) u, t \right) = \sum_{i=0}^{r-1} \left(\mathcal{T}_{1}^{r-i} \mathcal{T}_{2} \mathcal{T}_{1}^{r} - \mathcal{T}_{1}^{r-i-1} \mathcal{T}_{2} \mathcal{T}_{1}^{r+1} \right)$$

$$= \sum_{i=0}^{r-1} \mathcal{T}_{1}^{r-i-1} p_{\mu,\nu} \left((\mathcal{T}_{1} \mathcal{T}_{2} - \mathcal{T}_{2} \mathcal{T}_{1}) u, t \right) \mathcal{T}_{1}^{i} \dots \dots$$

$$\text{Since } p_{\mu,\nu} \left(\left(\mathcal{T}_{1} \mathcal{T}_{2}^{s} - \mathcal{T}_{2}^{s} \mathcal{T}_{1,} \right) u, t \right) = -p_{\mu,\nu} \left((\mathcal{T}_{2}^{s} \mathcal{T}_{1} - \mathcal{T}_{1} \mathcal{T}_{2}^{s}) u, t \right),$$

We immediately get

$$p_{\mu,\nu}\left((\mathcal{T}_{1}\mathcal{T}_{2}^{s}-\mathcal{T}_{2}^{s}\mathcal{T}_{1})u,t\right)=\sum_{j=0}^{s-1}\mathcal{T}_{2}^{s-j-1}p_{\mu,\nu}\left((\mathcal{T}_{1}\mathcal{T}_{2}-\mathcal{T}_{2}\mathcal{T}_{1})u,t\right)\mathcal{T}_{2}^{j}\cdots\cdots$$
(2)

Substituting T_2^s for T_2 in (1) and (2) \rightarrow

$$p_{\mu,v}\Big((\mathcal{T}_1^r\mathcal{T}_2^s-\mathcal{T}_2^s\mathcal{T}_1^r)u,t\Big)=\sum_{i=0}^{r-1}\sum_{j=0}^{s-1}\mathcal{T}_1^{r-i-1}\mathcal{T}_2^{s-j-1}p_{\mu,v}\Big((\mathcal{T}_1\mathcal{T}_2-\mathcal{T}_2\mathcal{T}_1)u,t\Big)\mathcal{T}_2^j\mathcal{T}_1^i$$

Proposition (4.4):

Let $T \in l\mathcal{F}(n,m)$ - $N\mathcal{B}_d[\mathcal{X}]$. If $T \in l\mathcal{F}(n,m)$ - $N\mathcal{B}_d[\mathcal{X}]$, then so is T^k For all positive integers k.

Proof:

To show that $T^k \in I\mathcal{F}(n,m) - N\mathcal{B}_d[\mathcal{X}]$, we have to prove that

$$p_{\mu,v}\left(\left(\left(\left(\mathcal{T}^{k}\right)^{\mathcal{D}}\right)^{n}\left(\mathcal{T}^{k}\right)^{*m}-\left(\mathcal{T}^{k}\right)^{*m}\left(\left(\mathcal{T}^{k}\right)^{\mathcal{D}}\right)^{n}\right)u,t\right),\quad k=1,2,3,\ldots$$

We establish the validity of the statement through mathematical induction on k. Given that $T \in I\mathcal{F}(n,m)$ - $N\mathcal{B}_d[\mathcal{X}]$, the statement holds for the base case k=1. Now, we assume as the induction hypothesis that the statement holds true for some arbitrary k, that is

$$p_{\mu,v}\left(\left(\left(\left(\left(T^{k}\right)^{\mathcal{D}}\right)^{n}\left(T^{k}\right)^{*m}-\left(T^{k}\right)^{*m}\left(\left(T^{k}\right)^{\mathcal{D}}\right)^{n}\right)u,t\right),$$

And prove it for k + 1

$$p_{\mu,\nu}\left(\left(\left(\left(T^{k+1}\right)^{\mathcal{D}}\right)^{n}\left(T^{k+1}\right)^{*m} - \left(T^{k+1}\right)^{*m}\left(\left(T^{k+1}\right)^{\mathcal{D}}\right)^{n}\right)u,t\right)$$

$$= p_{\mu,\nu}\left(\left(\left(T^{\mathcal{D}}\right)^{n}\left(T^{k}\right)^{*m}\left(T^{k^{\mathcal{D}}}\right)^{n}T^{*m} - \left(T^{k+1}\right)^{*m}\left(\left(T^{k+1}\right)^{\mathcal{D}}\right)^{n}\right)u,t\right)$$

$$= p_{\mu,\nu}\left(\left(\left(T^{k+1}\right)^{*m}\left(\left(T^{k+1}\right)^{\mathcal{D}}\right)^{n} - \left(T^{k+1}\right)^{*m}\left(\left(T^{k+1}\right)^{\mathcal{D}}\right)^{n}\right)u,t\right) = 0$$

Hence, \mathcal{T}^{k+1} is (n,m)- power \mathcal{D} - intuitionistic fuzzy normal operators. Hence, we verify the truth of the proposition.

Theorem (4.5):

Let $T = (T_1, ..., T_p) \in I\mathcal{F}(n, m) - N\mathcal{B}_d[X]^p$ then

- $T \in I\mathcal{F}(rn,sm)$ $N\mathcal{B}_d[\mathcal{X}]^p$ for some positive integers r,s
- $\mathcal{T}^{q} := \left(T_{1}^{q,1}, \dots, T_{p}^{q,p}\right) \in I\mathcal{F}(n,m) N\mathcal{B}_{d}[\mathcal{X}]^{p} \text{ for } q = \left(q_{1}, \dots, q_{p}\right) \in \mathbb{N}^{p}$ $\mathcal{T}^{*} = \left(T_{1}^{*}, \dots, T_{p}^{*}\right) \in I\mathcal{F}(n,m) N\mathcal{B}_{d}[\mathcal{X}]^{p}$ (ii)
- If \mathcal{U} is an unitary operator, then $\mathcal{U}^*\mathcal{T}\mathcal{U} := (\mathcal{U}^*\mathcal{T}_1\mathcal{U}, ..., \mathcal{U}^*\mathcal{T}_n\mathcal{U}) \in I\mathcal{F}(n, m) N\mathcal{B}_d[\mathcal{X}]^p$ (iv)

Proof (i)

Since $T \in I\mathcal{F}(n, m)$ - $N\mathcal{B}_d[X]^p$, This implies that

$$p_{\mu,v}\left(\left(\mathcal{T}_{i}\mathcal{T}_{j}-\mathcal{T}_{j}\mathcal{T}_{i}\right)u,t\right)=0\quad \text{ for }i,j=1,\ldots,p.$$

$$\begin{split} p_{\mu,v}\left(\left(\left(T_{i}^{\mathcal{D}}\right)^{rn}T_{i}^{*(sm)}-T_{i}^{*(sm)}\left(T_{i}^{\mathcal{D}}\right)^{sn}\right)u,t\right)\\ &=p_{\mu,v}\left(\left(\underbrace{\left(T_{i}^{\mathcal{D}}\right)^{n}\ldots\left(T_{i}^{\mathcal{D}}\right)^{n}}_{r-times}.\underbrace{T_{i}^{*m}\ldots T_{i}^{*m}}_{s-times}-\underbrace{T_{i}^{*m}\ldots T_{i}^{*m}}_{s-times}.\underbrace{\left(T_{i}^{\mathcal{D}}\right)^{n}\ldots\left(T_{i}^{\mathcal{D}}\right)^{n}}_{r-times}.u,t\right)\right)\\ &\Rightarrow p_{\mu,v}\left(\left(\underbrace{T_{i}^{*m}\ldots T_{i}^{*m}}_{s-times}\underbrace{\left(T_{i}^{\mathcal{D}}\right)^{n}\ldots\left(T_{i}^{\mathcal{D}}\right)^{n}}_{r-times}-\underbrace{T_{i}^{*m}\ldots T_{i}^{*m}}_{s-times}.\underbrace{\left(T_{i}^{\mathcal{D}}\right)^{n}\ldots\left(T_{i}^{\mathcal{D}}\right)^{n}}_{r-times}.u,t\right)\right)=0 \end{split}$$

(ii) If
$$q_i = 1 \ \forall \ i \in \{1, ..., p\}$$
, then $p_{\mu,v}\left(\left(T_i^{qi}T_j^{qj} - T_j^{qj}T_i^{qi}\right)u, t\right)$

If $q_i > 1 \ \forall i \in \{1, ..., p\}$ In light of (Lemma (4.3)), It is obtained that

$$p_{\mu,v}\left(\left(\mathcal{T}_{i}^{qi}\mathcal{T}_{j}^{qj}-\mathcal{T}_{j}^{qj}\mathcal{T}_{i}^{qi}\right)u,t\right)=\sum_{\substack{\alpha+\alpha'=\rho i-1\\\beta+\beta'=\rho j-1}}\mathcal{T}_{i}^{\alpha}\mathcal{T}_{j}^{\beta}\;p_{\mu,v}\left(\left(\mathcal{T}_{i}\mathcal{T}_{j}-\mathcal{T}_{j}\mathcal{T}_{i}\right)u,t\right)\mathcal{T}_{j}^{\alpha'}\mathcal{T}_{i}^{\beta'}$$

Provided that we assume $T \in I\mathcal{F}(n,m)$ - $N\mathcal{B}_d[\mathcal{X}]^p$, This implies that

$$p_{\mu,v}\left(\left(T_{i}^{q,i}\mathcal{T}_{j}^{q,j}-\mathcal{T}_{j}^{q,j}\mathcal{T}_{i}^{q,i}\right)u,t\right)=\sum_{\substack{\alpha+\alpha'=\rho i-1\\\beta+\beta'=\rho j-1}}\mathcal{T}_{i}^{\alpha}\mathcal{T}_{j}^{\beta}\;p_{\mu,v}\left(\left(\mathcal{T}_{i}\mathcal{T}_{j}-\mathcal{T}_{j}\mathcal{T}_{i}\right)u,t\right)\mathcal{T}_{j}^{\alpha'}\mathcal{T}_{i}^{\beta'}$$

$$\forall (i,j) \in \{1,\dots,p\}^2$$

Considering that $T_i \in l\mathcal{F}(n,m)$ - $N\mathcal{B}_d[\mathcal{X}]$, then from (Proposition 4.4), we obtain that $\mathcal{T}_i^{q,i}$ is an (n,m)- \mathcal{D} -intuitionistic Fuzzy normal $\forall i \in \{1, ..., p\}$. This means that

$$(\mathcal{T}_1^{q1}, \dots \mathcal{T}_n^{qp}) \in I\mathcal{F}(n, m) - N\mathcal{B}_d[\mathcal{X}]^p$$

(iii) By Definition 4.1 we have under the condition that $T \in IF(n,m) - NB_d[X]^p$ that is

$$\begin{cases} p_{\mu,v}\left(\left(T_{i}T_{j}-T_{j}T_{i}\right)u,t\right)=0 & i,j\in\{1,\ldots,p\}^{2} \\ p_{\mu,v}\left(\left(\left(T_{i}^{D}\right)^{n}T_{i}^{*m}-T_{i}^{*m}\left(T_{i}^{D}\right)^{n}\right)u,t\right)=0 & i=1,\ldots,p \end{cases}$$

and therefore

$$\begin{cases} p_{\mu,v}\left(\left(T_{i}^{*}T_{j}^{*}-T_{j}^{*}T_{i}^{*}\right)u,t\right)=0\\ p_{\mu,v}\left(\left(\left(T_{i}^{\mathcal{D}}\right)^{*n}T_{i}^{m}-T_{i}^{m}\left(T_{i}^{\mathcal{D}}\right)^{*n}\right)u,t\right)=0 \end{cases}$$

Therefore $T^* \in I\mathcal{F}(n,m)$ - $N\mathcal{B}_d[X]^p$

(iv) It is obvious that

$$p_{\mu,\nu}\left(\left((u^*\mathcal{T}_i u)(u^*\mathcal{T}_j u) - \left(u^*\mathcal{T}_j u\right)(u^*\mathcal{T}_i u)\right)u,t\right)$$
$$= p_{\mu,\nu}\left(\left(u^*\mathcal{T}_i \mathcal{T}_j u - \left(u^*\mathcal{T}_j u\right)(u^*\mathcal{T}_i u)\right)u,t\right)$$

$$= p_{\mu,\nu} \left(\left(U^* \mathcal{T}_j \mathcal{T}_i U - \left(U^* \mathcal{T}_j U \right) (U^* \mathcal{T}_i U) \right) u, t \right)$$

$$= p_{\mu,\nu} \left(\left(\left(U^* \mathcal{T}_j U \right) (U^* \mathcal{T}_i U) - \left(U^* \mathcal{T}_j U \right) (U^* \mathcal{T}_i U) \right) u, t \right)$$

$$= 0$$

Moreover

$$\begin{split} p_{\mu,v}\left(\left((U^*\mathcal{T}_i \, U)^{Dn} \big(U^*\mathcal{T}_j U\big)^{*m} - \big(U^*\mathcal{T}_j U\big)^{*m} (U^*\mathcal{T}_i U)^{Dn} \big) u, t\right) \\ p_{\mu,v}\left(\left(U^* \big(\mathcal{T}_i^D\big)^n U U^*\mathcal{T}_i^{*m} U - \big(U^*\mathcal{T}_j U\big)^{*m} (U^*\mathcal{T}_i U)^{Dn} \big) u, t\right) \\ p_{\mu,v}\left(\left(U^* \big(\mathcal{T}_i^D\big)^n \mathcal{T}_i^{*m} U - \big(U^*\mathcal{T}_j U\big)^{*m} (U^*\mathcal{T}_i U)^{Dn} \big) u, t\right) \\ p_{\mu,v}\left(\left(U^*\mathcal{T}_i^{*m} \big(\mathcal{T}_i^D\big)^n U - \big(U^*\mathcal{T}_j U\big)^{*m} (U^*\mathcal{T}_i U)^{Dn} \big) u, t\right) \\ p_{\mu,v}\left(\left((U^*\mathcal{T}_j U)^{*m} (U^*\mathcal{T}_i U)^{Dn} - \big(U^*\mathcal{T}_j U\big)^{*m} (U^*\mathcal{T}_i U)^{Dn} \big) u, t\right) = 0 \end{split}$$

Therefore

$$U^*TU \in I\mathcal{F}(n,m)-N\mathcal{B}_d[\mathcal{X}]^p$$

5.CONCIUSION

"The concept of a IF(n, m)- $NB_a[X]^p$ in (IFH-space) represents a relatively recent contribution to this field. This study explored certain properties related to this class of operators. The findings offer valuable insights that can contribute to the development and refinement of fuzzy functional analysis, thereby enhancing the theoretical understanding of this branch of mathematics."

REFERENCES

[1]T. A. Alrubaiy and H. Shubber, "More on IF-(α , β)-n-Normal operator on IFH-space," Mathematical Statistician and Engineering Applications, vol. 71, no. 3, pp. 1025–1028, Jun. 2022 [Online]. Available: https://doi.org/10.17762/msea.v71i3.265

[2] A. Athavale, "On joint hyponormality of operators," Proceedings of the American Mathematical Society, vol. 103, no. 2, pp. 417–423, 1988.[Online]. Available: https://doi.org/10.1090/S0002-9939-1988-0943059-X

[3] H. Baklouti, K. Feki, O. A. Mahmoud, and S. Ahmed, "Joint numerical ranges of operators in semi-Hilbertian spaces," Linear Algebra and its Applications, vol. 555, 2018. [Online]. Available: https://doi.org/10.1016/j.laa.2018.06.021

[4]H. Baklouti, K. Feki, and O. A. Mahmoud Sid Ahmed, "Joint normality of operators in semi-Hilbertian spaces," Linear Multilinear Algebra, vol. 68, no. 4, pp. 845–866, 2019.[Online]. Available: https://doi.org/10.1080/03081087.2019.1593925

[5] A. Ben-Israel and T. N. E. Greville, Generalized Inverses: Theory and Applications, 2nd ed., Springer, New York, 2003.[Online]. Available: https://doi.org/10.1007/b97366

[6] M. Chō, E. M. Beiba, and O. A. Mahmoud Sid Ahmed, " $(n_1, ..., n_p)$ -quasi-m-isometric tuple of operators," Annals of Functional Analysis, 2020.[Online]. Available: https://doi.org/10.1007/s43034-020-00093-7

[7]M. Chō and O. A. Mahmoud Sid Ahmed, "(A,m)-Symmetric commuting tuple of operators on a Hilbert space," Journal of Inequalities and Applications, vol. 22, no. 3, pp. 931–947, 2019.[Online]. Available: https://doi.org/10.7153/mia-2019-22-63

- [8] A. N. Katie and M. J. Mohammed, "Hesitant Fuzzy Soft HX Prime Ideal of HX Ring," Journal of Education for Pure Science University of Thi-Qar, vol. 13, no. 3, pp. 191–200, Sept. 2023.[Online]. Available: http://doi.org/10.32792/utq.jceps.10.01.01
- [9] C. F. King, "A note of Drazin inverses," Pacific Journal of Mathematics, vol. 70, pp. 383–390, 1977.[Online]. Available: https://doi.org/10.2140/pjm.1977.70.383
- [10] Gleason and S. Richter, "m-Isometric commuting tuples of operators on a Hilbert space," Integral Equations and Operator Theory, vol. 56, no. 2, pp. 181–196, 2006.[Online]. Available: https://doi.org/10.1007/s00020-006-1424-6
- [11] H. Ghofran and H. A. Shubber, "Intuitionistic Fuzzy –K-Quasi-(n, m)-Power Normal Operator," Journal of Education for Pure Science, vol. 13, no. 3, pp. 220–231, Sept. 2023.[Online]. Available: http://doi.org/10.32792/utq.jceps.10.01.01
- [12] H. Ali, "On Fuzzy Soft $-(\alpha, \beta)$ -n-Normal Operator," Asian Pacific Journal of Mathematics, vol. 11, no. 70, Jan. 2024.[Online]. Available: https://doi.org/10.28924/APJM/11-70
- [13]H. Ali, "Some properties of intuitionistic fuzzy n-binormal operator on IFH-space," Journal of Interdisciplinary Mathematics, vol. 27, no. 1, Jan. 2024.[Online]. Available: https://doi.org/10.47974/JIM-1504
- [14] H. Ali, "A note on intuitionistic fuzzy-n-normal operator on IFH-space," Journal of Interdisciplinary Mathematics, vol. 26, no. 1, Jan. 2023.[Online]. Available: https://doi.org/10.47974/JIM-1505
- [15]S. D. Mohsen and Y. H. Thiyab, "Fixed Point Theorems in Fuzzy Soft Rectangular b-Metric Space," Journal of Education for Pure Science, vol. 13, no. 3, pp. 282–292, Sept. 2023.[Online]. Available: http://doi.org/10.32792/utq.jceps.10.01.01
- [16]G. Messaoud, O. B. El Moctar, and O. A. Mahmoud Sid Ahmed, "Joint A-hyponormality of operators in semi-Hilbert spaces," Linear and Multilinear Algebra, vol. 69, no. 15, pp. 2888–2907, 2021.[Online]. Available: https://doi.org/10.1080/03081087.2019.1698509
- [17] M. Sheibani, M. M. Rashidi, and A. Rezaei, "Drazin inverses of operators in fuzzy normed spaces," Fuzzy Sets and Systems, vol. 394, pp. 140–158, 2020. [Online]. Available: https://doi.org/10.1016/j.fss.2020.03.005
- [18] A. Radharamani and S. Maheswari, "Intuitionistic fuzzy adjoint and intuitionistic fuzzy self-adjoint operators in intuitionistic fuzzy Hilbert space," IJRAR, vol. 5, no. 4, pp. 248–251, 2018.[Online]. Available: http://doi.org/10.26637/MJM0803/0008
- [19]R. Radharamani and S. S. Panda, "Intuitionistic fuzzy metric spaces: new results and applications," Soft Computing, vol. 24, pp. 9873–9883, 2020.[Online]. Available: https://doi.org/10.1007/s00500-019-04279-1
- [20]R. Saadati and J. H. Park, "On the Intuitionistic Fuzzy Topological Spaces," Chaos, Solitons & Fractals, vol. 27, no. 2, pp. 331–344, 2006.[Online]. Available: https://doi.org/10.1016/j.chaos.2005.03.070
- [21] F. Samanta and A. K. Bera, "Intuitionistic fuzzy Hilbert spaces and their applications," Information Sciences, vol. 492, pp. 250–268, 2019.[Online]. Available: https://doi.org/10.1016/j.ins.2019.05.004
- [22] M. Goudarzi et al., "Intuitionistic fuzzy inner product space," Chaos, Solitons & Fractals, vol. 41, pp. 1105–1112, 2009.[Online]. Available: https://doi.org/10.1016/j.chaos.2008.04.040
- [23]R. T. Muhammed and A. E. Hashoosh, "New Findings Related to GP-Metric Spaces," Journal of College of Education for Pure Sciences, vol. 13, no. 3, pp. 300–309, 2023.[Online]. Available: https://doi.org/10.32792/jeps.v13i3.333
- O. A. Mahmoud Sid Ahmed and H. O. Alshammari, "Joint m-quasihyponormal operators on a Hilbert space," [24] [Online]. Available: https://doi.org/10.1007/s43034-021-. Annals of Functional Analysis, vol. 12, p. 42, 2021 00130-z
- [25]J. H. Park, "Intuitionistic fuzzy metric spaces," Chaos, Solitons & Fractals, vol. 22, pp. 1039–1046, 2004. [Online]. Available: https://doi.org/10.1016/j.chaos.2004.02.005