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Stability Analysis of the Numerical Solution of the Generalised Burgers Equation Using Finite Difference Methods

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Abstract:

Stability analysis of the numerical solution of the generalised Burgers equation is considered. Three finite difference methods are examined: the forward explicit Euler, the backward implicit Euler and semi-implicit Crank-Nicholson methods. The Fourier mode (von-Neumann) method is used for stability analysis. The results demonstrated that the forward Euler scheme is conditionally stable, while Crank-Nicholson and implicit schemes are unconditionally stable.

Keywords: Generalised Burgers equation, numerical solution, explicit method, Crank-Nicholson method, implicit method.

1-Introduction

Partial differential equations are very important in applied mathematics [6, 13, 20, 21, 22]. The generalised Burgers equation and its special cases and its related and like equations have studied extensively in the literature [12]. In [3] the authors studied the generalised analytical solutions of the Burgers equation. In 2004 [9], the Adomian decomposition method is used for obtaining approximate solutions of the Burgers-Huxley and Burgers–Fisher equations. In 2005, the researchers in [5] applied a mixed spectral/spline approach based on Chebyshev spectral approximation and collocation methods. Some analytical solutions of the Burgers-Fisher equation and the generalised Burgers-Fisher equation are obtained in [14]. The author in [34] investigated the stationary solutions of the viscous Burgers equation. The variational iteration method (VIM) is used for solving the generalised Burgers-Fisher and Burgers equations.

The researchers in [31] solved the generalised Burgers-Fisher and Burgers equation by the use of homotopy perturbation method. In [28] the Burgers equation with uncertainty in the initial and boundary conditions is examined using polynomial chaos technique. Rahman and coworkers in [30] presented a semi-implicit difference method for solving the one-dimensional Burgers equation. The authors in [16] proposed and used non-standard finite difference scheme for solving the generalised Burgers-Huxley equation. In 2013, Srivastava et al. presented an implicit difference method for solving the one-dimensional coupled Burgers equation [36]. In [10] the authors proposed a numerical method based on weighted average differential quadrature approach for solving Burgers equation. Huxley equation is closely related to the Burgers equation and studied by [18, 19, 24, 25, 33]. The stability of the stationary solutions of the viscous Burgers equation is examined in [34].

In 2014 [27], Olayiwola presented a modified variational iteration method for solving the generalised Burgers-Fisher equation. In [8] the generalised Burgers-Huxley equation is studied and solved numerically using the implicit exponential finite difference scheme. In [17], the researchers presented a heuristic method for solving the generalised Burgers-Fishers equation based on hybrid method combining the Exp-function method and heuristic approach. The homotopy perturbation method with the Sumudu transform Method are used for Solving Burgers equation [7]. Singh et al. [35] in 2016, presented the modified cubic B-spline differential quadrature method for solving the generalised Burgers-Huxley equation. In [2], Burgers equation and the advection-diffusion equations are solved using semi-analytical iteration method. The authors in [4] presented a systematic literature review of the recent advances about solving methods of Burgers equation.

The higher-order solution of the generalised Burgers-Fisher equation is obtained using compact difference and DIRK methods [29]. In 2021 [1], the improved differential transform method is used for obtaining a solitary wave solution of the generalised Burgers-Fisher equation. In [11], the generalised Burgers-Fisher and the generalised Burgers-Huxley Equations are solved using the compact difference method. In 2024 [15], the semi-analytic iterative method and the modified simple equation method are used for solving the Burgers-Fisher equation.

In this paper, we examined and investigated the stability of the numerical solution of three finite difference methods applied to (1) using Fourier mode (von Neumann) method. The considered method are: the forward explicit Euler scheme, the backward implicit Euler scheme and the semi-implicit Crank-Nicholson scheme. The paper is organised as follows: The literature review is given in the introduction. The mathematical model is given in section 2. The numerical stability of the numerical schemes is investigated in section 3. The conclusions are give in section 4.

2-The Problem Setting and The Model

The nonlinear generalised Burgers equation is a famous nonlinear convection-diffusion equation [12]:

$$\frac{\partial u}{\partial t} + \alpha u^{\delta} \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = 0, \tag{1}$$

$\alpha > 0, \quad \delta > 0, \quad -L \leq x \leq L, \quad L > 0, \; t \geq 0,$

with initial and boundary conditions

$$u(x,0) = u_0(x),$$
$$u(-L,t) = f(t), \quad u(L,t) = g(t),$$

which appears in the study of the wave propagation in dissipative systems, it plays an important role in nonlinear physics.

3-The Numerical Stability

The numerical scheme is called stable if the error committed at any stage of the numerical computations damps out gradually without affecting these computations but if the resulting errors propagated in the next stages, then in this case, the numerical scheme is called unstable [23].

3.1. Numerical Stability of The Forward Euler Scheme Using Fourier Method

The general principle for this technique is replacing the exact solution by the FDM solution at time t by the

$$\frac{\Psi(t+\triangle t)e^{i\gamma x}-\Psi(t)e^{i\gamma x}}{\Delta t} = \frac{\Psi(t)e^{i\gamma(x+\Delta x)}-2\Psi(t)e^{i\gamma x}+\Psi(t)e^{i\gamma(x-\Delta x)}}{(\Delta x)^2}-\alpha\left[\frac{\Psi(t)e^{i\gamma x(+\Delta x)}-\Psi(t)e^{i\gamma(x-\Delta x)}}{2\Delta x}\right],$$

quantity $\Psi(t)e^{i\gamma x}$ where $\gamma > 0,\ i=\sqrt{-1}$. To apply von Neumann method on (1) we resort to linearized stability analysis [26] to obtain

$$\frac{\partial u}{\partial t}+\alpha\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=0$$

implies that

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}-\alpha\frac{\partial u}{\partial x}$$

and using the explicit scheme, we have

$$\frac{u_i^{j+1}-u_i^j}{\Delta t}=\frac{u_{i+1}^j-2u_i^j+u_{i-1}^j}{(\Delta x)^2}-\alpha\frac{u_{i+1}^j-u_{i-1}^j}{2\Delta x}, \qquad (2)$$

substituting $u_i^j = \Psi(t)e^{i\gamma x}$ in (2), we get

$$\left[\frac{\Psi(t+\triangle t)-\Psi(t)}{\Delta t}\right]e^{i\gamma x}=\frac{\Psi(t)}{(\Delta x)^2}\left[e^{i\gamma\Delta x}+e^{-i\gamma\Delta x}-2\right]e^{i\gamma x} -\frac{\alpha\Psi(t)}{2\Delta x}\left[e^{i\gamma\Delta x}-e^{\,-i\gamma\Delta x}\right]e^{i\gamma x},$$

this leads to

$$\Psi(t+\triangle t)-\Psi(t)=r\Psi(t)=\left[e^{i\gamma\Delta x}+e^{-i\gamma\Delta x}-2\right]e^{i\gamma x}-\frac{\alpha\Psi(t)}{2\Delta x}\left[e^{i\gamma\Delta x}-e^{\,-i\gamma\Delta x}\right]e^{i\gamma x},$$

where $r=\triangle t/(\triangle x)^2$, and by simplifying the equation in above, we have

$$\Psi(t+\triangle t)-\Psi(t) = r\Psi(t)[2cos\gamma\triangle x+isin\gamma\triangle x+cos\gamma\triangle x-isin\gamma\triangle x-2]]-\frac{\alpha\Psi(t)}{2\Delta x}[cos\gamma\triangle x+isin\gamma\triangle x-cos\gamma\triangle x+isin\gamma\triangle x-2]$$

$$\Psi(t+\triangle t)-\Psi(t)=r\Psi(t)[2cos\gamma\triangle x-2]-\frac{\alpha\Psi(t)}{2\Delta x}[2isin\gamma\triangle x]$$

Since the real part is what leads to stability, we neglect the imaginary part and get

$$\Psi(t+\triangle t)-\Psi(t)=2r\Psi(t)[cos\gamma\triangle x-1](t+\triangle t)-\Psi(t)=-2r\Psi(t)[1-cos\gamma\triangle x]$$
$$\Psi(t+\triangle t)-\Psi(t)=-2r\Psi(t)[1-(1-2\sin^2(\gamma\triangle x/2)]\,\Psi(t+\triangle t)-\Psi(t)$$
$$=-4r\Psi(t)\sin^2(\gamma\triangle x/2)$$

this leads to

$$\Psi(t+\triangle t)=\Psi(t)-4r\Psi(t)\sin^2(\gamma\triangle x/2)\Psi(t+\triangle t)=[1-4rsin^2(\gamma\triangle x/2)]$$

this results in

$$\Psi(t+\triangle t)/\Psi(t)=1-4rsin^2(\gamma\triangle x/2)=\xi,$$

this results in

$$\Psi(t+\triangle t)/\Psi(t)=\xi \qquad (3)$$

where ξ is the factor of error amplification. When the solution moves from level $\Psi(t)$ to the level $\Psi(t+\triangle t)$, then the quantity $|\Psi(t+\triangle t)-\Psi(t)|$ should start in decreasing or the function $\Psi(t)$ is a bounded function. From (3), and for the function $\Psi(t)$ to be bounded, we need to have

$$|\Psi(t+\triangle t)/\Psi(t)|\leq 1\Rightarrow|\xi|\leq 1,$$

implies

$$|1-4rsin^2(\gamma\triangle x/2)|\leq 1, \qquad (4)$$

from inequality (4), we get

$$-1\leq 1-4rsin^2(\gamma\triangle x/2)\leq 1,$$

and by taking the right-hand side of the inequality in above we have the following inequality

$$1-4rsin^2(\gamma\triangle x/2)\leq 1. \qquad (5)$$

From inequality (5), we obtain $-4rsin^2(\gamma\triangle x/2)\leq 0$, this leads to $-4rsin2(\gamma\triangle x/2)$, implies $r>0$, and since $r=\triangle t/(\triangle x)^2$, i.e., r cannot be negative, i.e., the inequality (5) implies that $r>0$, and this always true. In order to satisfy the inequality (3.4), we need to

$$-1\leq 1-4rsin^2(\gamma\triangle x/2),$$

implies that

$$-2\leq -4rsin^2(\gamma\triangle x/2),$$

so,

$$2\geq 4rsin^2(\gamma\triangle x/2),$$

thus,

$$4rsin^2(\gamma \triangle x/2) \leq 2,$$

and since $sin^2(\triangle x/2)$ is one for some value of γ, then the quantity in above can be reduced to

$$r \leq 1/2 \qquad (6)$$

and since $r = \triangle t/(\triangle x)^2$, from inequality (6), we get

$$\Delta t \leq \frac{(\Delta x)^2}{2} \qquad (7)$$

The inequalities (6) and (7) represent the imposed bounds on the forward Euler scheme to be a stable scheme.

3.2. Stability Analysis of Crank-Nicholson Scheme Using Fourier Method

Using Crank-Nicholson scheme for (1), we get

$$\frac{u_i^{j+1} - u_i^j}{\Delta t} = \frac{u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1}}{2(\Delta x)^2} + \frac{u_{i+1}^j - 2u_i^j + u_{i-1}^j}{2(\Delta x)^2} - \alpha \frac{u_{i+1}^j - u_{i-1}^j}{2\Delta x}, \qquad (8)$$

by substituting $u_i^j = \Psi(t)e^{i\gamma x}$ in (8), we have

$$\frac{\Psi(t+\Delta t)e^{i\gamma x} - \Psi(t)e^{i\gamma x}}{\Delta t}$$
$$= \frac{\Psi(t+\Delta t)e^{i\gamma(x+\Delta x)} - 2\Psi(t+\Delta t)e^{i\gamma x} - \Psi(t+\Delta t)e^{i\gamma(x-\Delta x)}}{2(\Delta x)^2}$$
$$+ \frac{\Psi(t)e^{i\gamma(x+\Delta x)} - 2\Psi(t)e^{i\gamma x} - \Psi(t)e^{i\gamma(x-\Delta x)}}{2(\Delta x)^2} - \alpha\left(\frac{\Psi(t)e^{i\gamma(x+\Delta x)} - \Psi(t)e^{i\gamma(x-\Delta x)}}{2\Delta x}\right),$$

implies that

$$\left(\frac{\Psi(t+\Delta t) - \Psi(t)}{\Delta t}\right) e^{i\gamma x} = \frac{\Psi(t+\Delta t)}{2(\Delta x)^2}\left(e^{i\gamma\Delta x} + e^{-i\gamma\Delta x} - 2\right)e^{i\gamma x}$$
$$+ \frac{\Psi(t)}{2(\Delta x)^2}\left(e^{i\gamma\Delta x} + e^{-i\gamma\Delta x} - 2\right)e^{i\gamma x} - \frac{\alpha\Psi(t)}{2\Delta x}\left(e^{i\gamma\Delta x} - e^{-i\gamma\Delta x}\right)e^{i\gamma x},$$

this leads to

$$\Psi(t+\Delta t) - \Psi(t) = \frac{r\Psi(t+\Delta t)}{2}\left(e^{i\gamma\Delta x} + e^{-i\gamma\Delta x} - 2\right)$$
$$+ \frac{r\Psi(t)}{2}\left(e^{i\gamma\Delta x} + e^{-i\gamma\Delta x} - 2\right) - \frac{\alpha\Psi(t)}{2\Delta x}\left(e^{i\gamma\Delta x} - e^{-i\gamma\Delta x}\right),$$

where $r = \triangle t/(\triangle x)^2$, and by simplifying the equation in above, we have

$$\Psi(t+\triangle t)-\Psi(t)=\frac{\mathrm{r}\Psi(t+\Delta t)}{2}(cos\gamma\triangle x+isin\gamma\triangle x+cos\gamma\triangle x-isin\gamma\triangle x-2)$$
$$+\frac{\mathrm{r}\Psi(t)}{2}(cos\gamma\triangle x+isin\gamma\triangle x+cos\gamma\triangle x-isin\gamma\triangle x-2)$$
$$-\frac{\alpha\triangle t\Psi(t)}{2\Delta \mathrm{x}}(cos\gamma\triangle x+isin\gamma\triangle x+cos\gamma\triangle x-isin\gamma\triangle x).$$

Hence,

$$\Psi(t+\triangle t)-\Psi(t)=-r\Psi(t+\triangle t)[cos\gamma\triangle x-1]$$
$$+r\Psi(t)[cos\gamma\triangle x-1]-\frac{\alpha\Delta t\Psi(t)}{(\Delta x)^2}[isin\gamma\triangle x]$$

Since the real part is what leads to stability, we neglect the imaginary part and we obtain

$$\Psi(t+\triangle t)-\Psi(t) = -r\Psi(t+\triangle t)[1-cos\gamma\triangle x]-r\Psi(t)[1- cos\gamma\triangle x]$$

$$\Psi(t+\triangle t)-\Psi(t)$$
$$= -r\Psi(t+\triangle t)[1-(1-2\sin^2(\gamma\triangle x/2)]- r\Psi(t)[1 - (1 - 2\sin^2(\gamma\triangle x/2)]$$
$$\Psi(t+\triangle t) - \Psi(t) = -2r\Psi(t+\triangle t)\sin^2(\gamma\triangle x/2)-2r\Psi(t)\sin^2(\gamma\triangle x/2)$$

thus,

$$\Psi(t+\triangle t)+2r\Psi(t+\triangle t)\sin^2(\gamma\triangle x/2) = \Psi(t)-2r\Psi(t)\sin^2(\gamma\triangle x/2)$$

hence,

$$[1+2rsin^2(\gamma\triangle x/2)]\Psi(t+\triangle t) = [1-2rsin^2(\gamma\triangle x/2)]\Psi(t)$$

Implies that

$$\Psi(t+\Delta t)/\Psi(t)=\frac{1-2rsin^2(\gamma\triangle x/2)}{1+2rsin^2(\gamma\triangle x/2)},$$

this leads to

$$\Psi(t+\Delta t)/\Psi(t)=\frac{1-2rsin^2(\gamma\triangle x/2)}{1+2rsin^2(\gamma\triangle x/2)}=\xi,$$

For stability, we need

$$|\Psi(t+\triangle t)/\Psi(t)| \leq 1 \Rightarrow |\xi| \leq 1,$$

this results in

$$\left|\frac{1-2rsin^2(\gamma\triangle x/2)}{1+2rsin^2(\gamma\triangle x/2)}\right|\leq 1,\forall\, r,\alpha,\sigma,\Delta t, \tag{9}$$

and since $|\xi| \leq 1$, then the Crank-Nicholson method is unconditionally stable.

3.3. Stability Analysis of Implicit Backward Euler Scheme Using von Neumann Method

Using the implicit scheme for (1), we get

$$\frac{u_i^{j+1}-u_i^j}{\Delta t}=\frac{u_{i+1}^{j+1}-2u_i^{j+1}+u_{i-1}^{j+1}}{(\Delta \mathrm{x})^2}-\alpha\frac{u_{i+1}^{j+1}-u_{i-1}^{j+1}}{2\Delta \mathrm{x}}, \tag{10}$$

by substituting $u_i^j=\Psi(t)e^{i\gamma x}$ in (10), we have

$$\frac{\Psi(t+\Delta t)e^{i\gamma x}-\Psi(t)e^{i\gamma x}}{\Delta t}=\frac{\Psi(t+\Delta t)e^{i\gamma(x+\Delta \mathrm{x})}-2\Psi(t+\Delta t)e^{i\gamma x}-\Psi(t+\Delta t)e^{i\gamma(x-\Delta \mathrm{x})}}{(\Delta \mathrm{x})^2}-\alpha\left(\frac{\Psi(t)e^{i\gamma(x+\Delta \mathrm{x})}-\Psi(t)e^{i\gamma(x-\Delta \mathrm{x})}}{2\Delta \mathrm{x}}\right),$$

implies that

$$\left(\frac{\Psi(t+\Delta t)-\Psi(t)}{\Delta t}\right)e^{i\gamma x}=\frac{\Psi(t+\Delta t)}{(\Delta \mathrm{x})^2}\left(e^{i\gamma\Delta \mathrm{x}}+e^{-i\gamma\Delta \mathrm{x}}-2\right)e^{i\gamma x}-\frac{\alpha\Psi(t)}{2\Delta \mathrm{x}}\left(e^{i\gamma\Delta \mathrm{x}}-e^{-i\gamma\Delta \mathrm{x}}\right)e^{i\gamma x},$$

this leads to

$$\Psi(t+\triangle t)-\Psi(t)=\mathrm{r}\Psi(t+\triangle t)\left(e^{i\gamma\Delta \mathrm{x}}+e^{-i\gamma\Delta \mathrm{x}}-2\right)-\frac{\alpha\triangle t\Psi(t)}{2\Delta \mathrm{x}}\left(e^{i\gamma\Delta \mathrm{x}}-e^{-i\gamma\Delta \mathrm{x}}\right),$$

where $r=\triangle t/(\triangle x)^2$, and by simplifying the equation in above, we have

$$\Psi(t+\triangle t)-\Psi(t)=\mathrm{r}\Psi(t+\triangle t)(cos\gamma\triangle x+isin\gamma\triangle x+cos\gamma\triangle x-isin\gamma\triangle x-2)-\frac{\alpha\triangle t\Psi(t)}{2\Delta \mathrm{x}}(cos\gamma\triangle x+isin\gamma\triangle x-cos\gamma\triangle x+isin\gamma\triangle x).$$

Since the real part is what leads to stability, we neglect the imaginary part and get

$$\Psi(t+\triangle t)-\Psi(t)=-2r\Psi(t+\triangle t)[1-cos\gamma\triangle x],$$

Implies that

$\Psi(t+\triangle t)-\Psi(t)=-2r\Psi(t+\triangle t)[1-(1-2\sin^2(\gamma\triangle x/2)],$

This results in

$\Psi(t+\triangle t)-\Psi(t)=-4r\Psi(t+\triangle t)\sin^2(\gamma\triangle x/2).$

Thus,

$$\Psi(t + \triangle t) + 4r\Psi(t + \triangle t)\sin^2(\gamma \triangle x/2) = \Psi(t),$$

hence,

$$[1 + 4rsin^2(\gamma \triangle x/2)]\Psi(t + \triangle t) = \Psi(t),$$

implies that

$$\Psi(t + \Delta t)/\Psi(t) = \frac{1}{1 + 4rsin^2(\gamma \triangle x/2)},$$

this leads to

$$\Psi(t + \Delta t)/\Psi(t) = \frac{1}{1 + 4rsin^2(\gamma \triangle x/2} = \xi$$

For stability, we need

$$|\Psi(t + \triangle t)/\Psi(t)| \leq 1 \Rightarrow |\xi| \leq 1,$$

this results in

$$\left|\frac{1}{1 + 4rsin^2(\gamma \triangle x/2}\right| \leq 1, \forall\, r, \alpha, \sigma, \Delta t, \quad (11)$$

and since $|\xi| \leq 1$, then the Implicit scheme is unconditionally stable.

4. Conclusions

The generalised Burgers equation (1) is solved numerically using three finite difference schemes, the forward Euler method, the implicit backward Euler method and the Crank-Nicholson method. The results showed that the Crank-Nicholson and backward methods are more accurate and better than the forward scheme. Additionally, the stability of the numerical solutions of the schemes is investigated using Fourier mode (von Neumann) method. The stability analysis showed that the forward method is conditionally stable if:

$$r \leq \frac{1 - \eta\, sin(\gamma\, h/2)cos(\gamma\, h/2)}{2}.$$

While the backward and Crank-Nicholson methods are unconditionally stable, we solved the problem via an extensive set of numerical examples.

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