

A New Analytical Technique for Solving Ordinary Differential Equations with Improved Efficiency and Accuracy

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Abstract:

This paper develops a new integral transformation, NT, and proposes combining it with the homotopic perturbation method (HPM) to obtain efficient approximate solutions to various classes of partial differential equations, including nonlinear ones. A background on the importance of partial differential equations and the difficulty of finding exact solutions for them is first presented. This was followed by a review of the benefits and multiple uses of the Homotopy method. The generalized form of the combined transformation (HPM-NT) was then derived and applied to four diverse examples: two linear and two nonlinear functions, and two linear and two nonlinear systems. The results demonstrated high accuracy and ease of application, reflecting the effectiveness of the new transformation when combined with the Homotopy method.

Keywords: Integral transform; Differential equations; Laplace transform. NT transform.

1-Introduction

Partial differential equations are an essential tool in the mathematical modeling of many physical and engineering phenomena, such as heat propagation, wave propagation, fluid flow, and quantum systems. Due to the nonlinear and complex nature of some of these equations, obtaining accurate solutions is a significant challenge, necessitating the development of efficient and flexible analytical techniques [15,19]. Among the semi-analytical methods that have proven their effectiveness in this context, the homotopic perturbation method (HPM), introduced by Ji-Huan He in 1999, stands out [1,2]. This method has gained widespread attention, as it has been used to solve a wide range of functional equations, including ordinary and partial differential equations, integral equations, and complex nonlinear equations. This method is based on constructing a convergent infinite series using an embedding operator [20,21]

$p \in [0,1]$ is treated as a small parameter, allowing solutions to be approximated with high accuracy [3,4]. This method has been successfully applied to the Schrödinger equations, nonlinear heat transfer equations, Riccati equations, and many other types, making it a powerful tool for solving nonlinear equations in a variety of fields. In this paper, we introduce a new integral transformation called "NT" and propose combining it with the Homotopy method to form a new joint transformation that combines the advantages of both methods. We first derive the general formula for this joint transformation, and then apply it to a set of illustrative examples, including: Two examples of partial differential equations: one linear and one nonlinear. Two examples of differential systems: one linear and one nonlinear [5,6]. These examples aim to highlight the effectiveness of the new NT transformation when combined with HPM, and its ability to simplify solutions and improve their accuracy [9,10].

Partibilities

Definition .1. [7,8] Suppose $f(t)$ is an integrable function defined for values $t > 0$. We define a new transformation for the function $f(t)$ denoted by the symbol $NT\{f(t)\}$ or $RJ(v)$ in the following form:

$$NT\{f(t)\} = RJ(v) = \int_0^{\infty} e^{-\sqrt{a}t} f(vt) dt. \quad (1.1)$$

Some useful features

$$1. NT(k) = \frac{k}{\sqrt{a}}$$

$$2. NT\{t\} = \frac{v}{(\sqrt{a})^2}$$

$$3. NT\{e^{bt}\} = \frac{1}{\sqrt{a}-bv}$$

$$4. NT(t^n) = \frac{n! v^n}{(\sqrt{a})^{n+1}}$$

$$5. NT\{\sin(bt)\} = \frac{bv}{a+b^2 v^2}$$

$$6. NT\{\sinh(bt)\} = \frac{bv}{a-b^2 v^2}$$

$$7. NT\{\cos(bt)\} = \frac{\sqrt{a}}{a+b^2 v^2}$$

$$8. NT\{\cosh(bt)\} = \frac{\sqrt{a}}{a-b^2 v^2}$$

Some derivations of the New Transformation (NT)

$$(1) NT\{u'(t)\} = \frac{\sqrt{a} NT\{u(t)\}}{v} - \frac{u(0)}{v} \quad (1.2)$$

$$(2) NT\{u''(t)\} = \frac{a NT\{u(t)\}}{v^2} - \frac{\sqrt{a} u(0)}{v^2} - \frac{u'(0)}{v} \quad (1.3)$$

$$(3) NT\{u^n(t)\} = \frac{(\sqrt{a})^n NT\{u(x,t)\}}{v^n} - \frac{(\sqrt{a})^{n-1} u(x,0)}{v^n} - \frac{(\sqrt{a})^{n-2} u_t(x,0)}{v^{n-1}} - \dots - \frac{u_n^{(n-1)}(x,0)}{v} \quad (1.4)$$

In this section we will use a new integral transformation with the homotope method to solve n-order partial differential equations.

Consider the following general nonlinear differential equation:

$$L_t^{(m)}u(x, t) + R(u(x, t)) + N(u(x, t)) = g(x, t) \quad (2.1)$$

Where $L_t^{(m)}(.) = \frac{\partial^m}{\partial t^m}$, R denotes to linear operator, N denotes to nonlinear operator

and $g(x, t)$ is the source term. subject to the initial conditions

$$u^{(k)}(x, 0) = C_k, \quad k = 0, 1, \dots, m-1$$

Taking NT transform on equation (5.1), we obtain

$$NT\{L_t^{(m)}u(x, t)\} + NT\{R(u(x, t))\} + NT\{N(u(x, t))\} = NT\{g(x, t)\} \quad (2.2)$$

By applying the NT transform differentiation property we have

$$\frac{(\sqrt{a})^m NT\{u(x, t)\}}{W^m} - \frac{(\sqrt{a})^{m-1} c_0}{W^m} - \frac{(\sqrt{a})^{m-2} c_1}{W^{m-1}} - \dots - \frac{(\sqrt{a})^{m-n} c_{m-1}}{W} + NT\{R(u(x, t))\} + NT\{N(u(x, t))\} = NT\{g(x, t)\}$$

Or equivalent

$$(\sqrt{a})^m NT\{u(x, t)\} = (\sqrt{a})^{m-1} c_0 + (\sqrt{a})^{m-2} W c_1 + \dots + W^{m-1} c_{m-1} + W^m NT\{g(x, t)\} - W^m NT\{R(u(x, t))\} - W^m NT\{N(u(x, t))\}$$

the source term Operating with NT inverse on both sides of we obtain

$$u(x, t) = g(x, t) - \frac{1}{(\sqrt{a})^m} NT^{-1} [W^m NT\{R(u(x, t))\} + W^m NT\{N(u(x, t))\}] \quad (2.3)$$

Where $G(x, t)$ represents the term arising from and the prescribed.

Now we apply the Homotopy perturbation method:

$$u(x, t) = \sum_{n=0}^{\infty} p^n u_n(x, t)$$

and the nonlinear is term can be decomposed as:

$$N(u(x, t)) = \sum_{n=0}^{\infty} p^n H_n(u)$$

Where $H_n(u)$ are He's polynomials of u_0, u_1, \dots, u_n and it can be calculated by formula

Given below:

$$H_n(u_0, u_1, u_2, \dots) = \frac{1}{n!} \frac{\partial^n}{\partial p^n} \{N(\sum_{n=0}^{\infty} p^n u_n(x, t))\}, \quad n = 0, 1, 2, \dots$$

NOW

$$\sum_{n=0}^{\infty} p^n u_n(x, t) = G(x, t) - p NT^{-1} \frac{1}{(\sqrt{a})^m} [W^m NT\{R(\sum_{n=0}^{\infty} p^n u_n(x, t)) + \sum_{n=0}^{\infty} p^n H_n\}]$$

Comparing the coefficient of like powers p , we obtain

$$p^0: u_0(x, t) = g(x, t)$$

$$p^1: u_1(x, t) = -NT^{-1} \frac{1}{(\sqrt{a})^m} [W^m \{NT(u_0(x, t)) + H_0\}]$$

$$p^2: u_2(x, t) = -NT^{-1} \frac{1}{(\sqrt{a})^m} [W^m NT \{(u_1(x, t)) + H_1\}]$$

⋮

$$p^n: u_n(x, t) = -NT^{-1} \frac{1}{(\sqrt{a})^m} [W^m NT \{NT(u_{n-1}(x, t)) + H_{n-1}\}]$$

Applications :

Example .1: Consider Two-dimensional Wave Equation

$$u_{tt} + u_{xx} = 0 \tag{3.1}$$

with

$$u(x, 0) = -e^x, \quad u_t(x, 0) = 0 \tag{3.2}$$

Taking NT on both sides of (3.1), we have

$$\frac{aNT\{u(x,t)\}}{V^2} - \frac{\sqrt{a}NTu(x,0)}{V^2} - \frac{u_t(x,0)}{V} + NT\{u_{xx}\} = 0 \tag{3.3}$$

or $aNT\{u(x, t)\} = -\sqrt{a}e^x - V^2NT\{u_{xx}\}$

The inverse of NT transform implies that

$$u(x, t) = -e^x - NT^{-1} \left\{ \frac{1}{a} V^2 NT \{u_{xx}\} \right\}$$

Let $u(x, t) = \sum_{n=0}^{\infty} p^n u_n(x, t)$

Now, applying the Homotopy perturbation method we get

$$\sum_{n=0}^{\infty} p^n u_n(x, t) = -e^x - pNT^{-1} \left\{ \frac{1}{a} V^2 NT \left(\sum_{n=0}^{\infty} u_{nxx} \right) \right\}$$

Comparing the coefficient of like powers of p we obtain:

$$p^0: u_0(x, t) = -e^x$$

$$\begin{aligned} p^1: u_1(x, t) &= -NT^{-1} \left\{ \frac{1}{a} V^2 NT(u_{0xx}) \right\} = -NT^{-1} \left\{ \frac{1}{a} V^2 NT(-e^x) \right\} \\ &= NT^{-1} \left\{ \frac{1}{(\sqrt{a})^3} V^2 e^x \right\} = \frac{t^2}{2!} e^x \end{aligned}$$

$$p^2: u_2(x, t) = -NT^{-1} \left\{ \frac{1}{a} V^2 NT(u_{1xx}) \right\} = -NT^{-1} \left\{ \frac{1}{a} V^2 NT \left(\frac{t^2}{2!} e^x \right) \right\}$$

$$= -NT^{-1} \left\{ \frac{1}{(\sqrt{a})^5} V^4 e^x \right\} = -\frac{t^4}{4!} e^x$$

$$p^3: u_3(x, t) = -NT^{-1} \left\{ \frac{1}{a} V^2 NT (u_{2xx}) \right\} = -NT^{-1} \left\{ \frac{1}{a} V^2 NT \left(-\frac{t^4}{4!} e^x \right) \right\}$$

$$= NT^{-1} \left\{ \frac{1}{(\sqrt{a})^6} V^6 e^x \right\} = \frac{t^6}{6!} e^x$$

$$\vdots$$

Therefore, the solution is given

$$u(x, t) = -e^x + \frac{t^2}{2!} e^x - \frac{t^4}{4!} e^x + \frac{t^6}{6!} e^x + \dots = -e^x \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \dots \right)$$

$$= -e^x \cdot \text{cost}$$

Example .2: Consider Nonlinear equation

$$u_t + uu_x = 0 \tag{3.4}$$

with

$$u(x, 0) = -x \tag{3.5}$$

Taking the NT transform on both sides of equation (6.4) we have

$$\frac{\sqrt{a} NT\{u(x,t)\}}{V} - \frac{u(x,0)}{V} + NT\{uu_x\} = 0$$

or
$$\sqrt{a} NT\{u(x,t)\} = -x - VNT\{uu_x\}$$

The inverse of NT transform implies that

$$u(x, t) = -X - NT^{-1} \frac{1}{\sqrt{a}} \{VNT \{uu_x\}\}$$

Let

$$u(x, t) = \sum_{n=0}^{\infty} p^n u_n(x, t)$$

$$uu_x = \sum_{n=0}^{\infty} p^n H_n$$

Comparing the coefficient of like powers of p we obtain:

$$p^0: u_0(x, t) = -x$$

$$p^1: u_1(x, t) = -\frac{1}{\sqrt{a}} NT^{-1} \{VNT (H_0)\} = -NT^{-1} \frac{1}{\sqrt{a}} \{VNT (u_0 u_{0x})\}$$

$$\begin{aligned}
 &= -NT^{-1} \frac{1}{\sqrt{a}} \{VNT ((-x)(-1))\} \\
 &= -xNT^{-1} \left\{ \frac{v}{((\sqrt{a})^2)} \right\} = -xt \\
 p^2: u_2(x, t) &= -NT \frac{1}{\sqrt{a}} \{VNT (H_1)\} \\
 &= -NT^{-1} \frac{1}{\sqrt{a}} \{VNT (u_0 u_{1x} + u_1 u_{0x})\} = \\
 &= -NT^{-1} \frac{1}{\sqrt{a}} \{VNT [(xt) + (xt)]\} \\
 &= -NT^{-1} \frac{1}{\sqrt{a}} \{VNT (2xt)\} \\
 \\
 &= -NT^{-1} \frac{1}{\sqrt{a}} \left\{ V \left(2x \frac{v}{((\sqrt{a})^2)} \right) \right\} = -2xNT^{-1} \left\{ \frac{v^2}{((\sqrt{a})^3)} \right\} \\
 &= -xt^2 \\
 \\
 &\vdots
 \end{aligned}$$

Therefore, the solution is given

$$u(x, t) = -x - xt - xt^2 - \dots = -x(1 + t + t^2 + \dots) = \frac{-x}{1-t}$$

Example 3: Consider

$$\begin{aligned}
 u_t + v_x &= 0 \\
 v_t + u_x &= 0
 \end{aligned} \tag{3.6}$$

With $u(x, 0) = e^x$ $v(x, 0) = e^{-x}$ (3.7)

Taking the NT transform on both sides of equation (3.6) and (3.7), we have

$$\begin{aligned}
 \frac{\sqrt{a} NT\{u(x,t)\}}{v} - \frac{u(x,0)}{v} + NT\{v_x\} &= 0 \\
 \frac{\sqrt{a} NT\{v(x,t)\}}{v} - \frac{v(x,0)}{v} - NT\{u_x\} &= 0
 \end{aligned}$$

Or $\sqrt{a} NT\{u(x, t)\} = e^x - VNT\{v_x\}$
 $\sqrt{a} NT\{v(x, t)\} = e^{-x} - VNT\{u_x\}$

The inverse of NT transform implies that

$$u(x, t) = e^x - NT^{-1} \frac{1}{\sqrt{a}} \{VNT \{v_x\}\}$$

$$v(x, t) = e^{-x} - NT^{-1} \frac{1}{\sqrt{a}} \{VNT \{u_x\}\}$$

$$\text{Let } u(x, t) = \sum_{n=0}^{\infty} p^n u_n(x, t)$$

Now, applying the Homotopy perturbation method, we get

$$\sum_{n=0}^{\infty} p^n u_n(x, t) = e^x - pNT^{-1} \frac{1}{\sqrt{a}} \{VNT (\sum_{n=0}^{\infty} v_{nx})\}$$

$$\sum_{n=0}^{\infty} p^n v_n(x, t) = e^{-x} - pNT^{-1} \frac{1}{\sqrt{a}} \{VNT (\sum_{n=0}^{\infty} u_{nx})\}$$

Comparing the coefficient of like powers of p we obtain:

$$p^0: u_0(x, t) = e^x$$

$$: v_0(x, t) = e^{-x}$$

$$p^1: u_1(x, t) = -NT^{-1} \frac{1}{\sqrt{a}} \{VNT \{v_{0x}\}\}$$

$$= -NT^{-1} \frac{1}{\sqrt{a}} \{VNT(-e^{-x})\}$$

$$= NT^{-1} \left\{ \frac{v}{((\sqrt{a})^2)} e^{-x} \right\}$$

$$= te^{-x}$$

$$: v_1(x, t) = -\frac{1}{\sqrt{a}} NT^{-1} \{VNT \{u_{0x}\}\}$$

$$= -NT^{-1} \frac{1}{\sqrt{a}} \{VNT(e^x)\} = -NT^{-1} \left\{ \frac{v}{((\sqrt{a})^2)} e^x \right\} = -te^x$$

$$p^2: u_2(x, t) = -NT^{-1} \frac{1}{\sqrt{a}} \{VNT \{v_{1x}\}\}$$

$$= -NT^{-1} \frac{1}{\sqrt{a}} \{VNT(-te^x)\} = NT^{-1} \left\{ \frac{v^2}{((\sqrt{a})^3)} e^x \right\} = \frac{t^2}{2!} e^x$$

$$: v_2(x, t) = -NT^{-1} \frac{1}{\sqrt{a}} \{VNT \{u_{1x}\}\}$$

$$= -NT^{-1} \frac{1}{\sqrt{a}} \{VNT(-te^{-x})\}$$

$$= NT^{-1} \left\{ \frac{v^2}{((\sqrt{a})^3)} e^{-x} \right\} = \frac{t^2}{2!} e^{-x}$$

$$p^3: u_3(x, t) = \frac{t^3}{3!} e^{-x}$$

$$: v_3(x, t) = -\frac{t^3}{3!} e^x$$

$$p^4: u_4(x, t) = \frac{t^4}{4!} e^x$$

$$: v_4(x, t) = \frac{t^4}{4!} e^{-x}$$

$$p^5: u_5(x, t) = \frac{t^5}{5!} e^{-x}$$

$$: v_5(x, t) = -\frac{t^5}{5!} e^x$$

Therefore, the solution is given

$$u(x, t) = e^x \cdot \text{cosht} + e^{-x} \cdot \text{sinht}$$

$$v(x, t) = e^{-x} \text{csht} - e^x \text{sinht}$$

Example .4: Consider

$$u_t - v_x w_y = 1$$

$$v_t - w_x u_y = 5$$

$$w_t - u_x v_y = 5 \quad (3.8)$$

$$\text{with } u(x, y, 0) = x + 2y, v(x, y, 0) = x - 2y, w(x, y, 0) = -x + 2y \quad (3.9)$$

Taking the NT transform on both sides of equation (3.8), we have

$$\frac{\sqrt{a} \text{NT}\{u(x,y,t)\}}{z} - \frac{u(x,y,0)}{z} - \text{NT}\{(v_x w_y)\} = \text{NT}(1)$$

$$\frac{\sqrt{a} \text{NT}\{v(x,y,t)\}}{z} - \frac{v(x,y,0)}{z} - \text{NT}\{(w_x u_y)\} = \text{NT}(5)$$

$$\frac{\sqrt{a} \text{NT}\{u(x,y,t)\}}{z} - \frac{u(x,y,0)}{z} - \text{NT}\{(u_x v_y)\} = \text{NT}(5)$$

Solve in the same way as before from equation (2.3). We get:

$$u(x, y, t) = x + 2y + 3t$$

$$v(x, y, t) = x - 2y + 3t$$

$$w(x, y, t) = -x + 2y + 3t$$

Conclusion

In this paper, a new integral transformation (NT) is proposed and combined with the homotopic perturbation method (HPM) to form a joint transformation that enhances the ability to find accurate approximate solutions to complex partial differential equations. The examples studied demonstrate the effectiveness of the joint transformation in handling both linear and nonlinear models. This opens new horizons for its application in broader fields. In the future, this approach could be expanded to include

higher-dimensional systems or systems with more complex initial and boundary conditions, and it could be compared with other analytical and semi-analytical methods in terms of efficiency and accuracy.

Conflicts Of Interest

The authors declare no conflicts of interest.

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References

1. **Liao, S.** (2003). *Beyond perturbation: Introduction to the homotopy analysis method*. Chapman and Hall/CRC. <https://doi.org/10.1201/9780203491164>
2. **He, J. H.** (2006). Homotopy perturbation method for solving boundary value problems. *Physics Letters A*, 350(1-2), 87–88. <https://doi.org/10.1016/j.physleta.2005.10.005>
3. **Marinca, V., & Herişanu, N.** (2008). Application of optimal homotopy asymptotic method for solving nonlinear equations arising in heat transfer. *International Communications in Heat and Mass Transfer*, 35(6), 710–715. <https://doi.org/10.1016/j.icheatmasstransfer.2008.01.010>
4. **Jassim, H. K., & Talab, R.** (2025). A novel integral transform (NT) and its applications to fractional differential equations. *International Journal of Science, Mathematics and Technology Learning*, 33(2), 378–389. doi.org/10.32792/jeps.v15i2.673.
5. **Jassim, H. K., & Nasser, R. T.** (2025). Rida-Jassim integral transform: A tool for solving linear and non-linear differential equations. *Boletim da Sociedade Paranaense de Matemática*, 43(1), 1–10. doi.org/10.5269/bspm.v72i01.7201
6. **Odibat, Z. T., & Momani, S.** (2008). A reliable treatment of the differential transformation method for solving differential equations. *Computers & Mathematics with Applications*, 55(4), 761–772. <https://doi.org/10.1016/j.camwa.2007.05.007>
7. **Nayfeh, A. H.** (2000). *Perturbation methods*. Wiley Classics Library. <https://doi.org/10.1002/9783527617609>
8. **Swain NR, Jassim HK.** Solving Multidimensional Fractional Telegraph Equation Using Yang Hussein Jassim Method. *Iraqi J Comput Sci Math*. 2025;6(2):2. doi.org/10.52866/2788-7421.1238.
9. **Swain NR, Jassim HK.** Innovation of Yang Hussein Jassim's Method for Nonlinear Telegraph Equations. *Partial Differ Equ Appl Math*. 2025:101182. doi.org/10.1016/j.padiff.2025.101182
10. **Alomari, A. K., Noorani, M. S. M., & Nazar, R.** (2019). A comparative study of homotopy analysis method and homotopy perturbation method for a nonlinear problem. *Results in Physics*, 15, 102567. <https://doi.org/10.1016/j.rinp.2019.102567>

11. **Abu Arqub, O., Abo-Hammour, Z., Al-Badarnah, R., & Momani, S.** (2013). A reliable analytical method for solving higher-order initial value problems. *Discrete Dynamics in Nature and Society*, 2013, Article 673829. /doi.org/10.1155/2013/673829
12. **Abu Arqub, O., El-Ajou, A., Bataineh, A. S., & Hashim, I.** (2013). A representation of the exact solution of generalized Lane-Emden equations using a new analytical method. *Abstract and Applied Analysis*, 2013, Article 378593. doi.org/10.1155/2013/378593
13. **Alzaki, L. K.** (2024). Analytical approximations for a system of fractional partial differential equations. *Progress in Fractional Differentiation and Applications*, 10(1), 8189. doi.org/10.18576/pfda/100108
14. **Arqub, O. A.** (2013). Series solution of fuzzy differential equations under strongly generalized differentiability. *Journal of Advanced Research in Applied Mathematics*, 5(1), 31–52. doi.org/10.5373/jaram.1156.1012A
15. **He, J. H.** (1998). Approximate solution of nonlinear differential equations with convolution product nonlinearities. *Computer Methods in Applied Mechanics and Engineering*, 167(1–2), 69–73. doi.org/10.1016/S0045-7825(98)00109-1.
16. **H. Jafari, and H. K. Jassim**, Local Fractional Series Expansion Method for Solving Laplace and Schrodinger Equations on Cantor Sets within Local Fractional Operators, *International Journal of Mathematics and Computer Research*, 2(11)(2014), 736-744.
17. **Hussein, M. A.** (2023). Analysis of fractional differential equations with Atangana-Baleanu fractional operator. *Progress in Fractional Differentiation and Applications*, 9(4), 681–686. doi.org/10.18576/pfda/090408
18. **Hussein, M. A., Jassim, H. K., Salman, A. T., & Zayir, M. Y.** (2023). An efficient homotopy permutation technique for solving fractional differential equations using Atangana-Baleanu-Caputo operator. *AIP Conference Proceedings*, 2845, 060008. doi.org/10.1063/5.0160659
19. **Jassim, H. K., Salman, A. T., Ahmad, H., Zayir, M. Y., & Shuaa, A. H.** (2023). Exact analytical solutions for fractional partial differential equations via an analytical approach. *AIP Conference Proceedings*, 2845, 060007. doi.org/10.1063/5.0160658
20. **Kumar, D., Singh, J., & Qurashi, M. A.** (2023). A computational study of local fractional Helmholtz and coupled Helmholtz equations in fractal media. In *Lecture Notes in Networks and Systems* (Vol. 666, pp. 286–298). Springer. doi.org/10.1007/978-3-031-32166-8_28
21. **Liu, J., Nadeem, M., & Iambor, L. F.** (2023). Application of Yang homotopy perturbation transform approach for solving multi-dimensional diffusion problems with time-fractional derivatives. *Scientific Reports*, 13, 21855. doi.org/10.1038/s41598-023-49036-x
22. **Mohsin, N. H., Hussein, M. A., & Ali, F. H.** (2023). A new analytical method for solving nonlinear Burger's and coupled Burger's equations. *Materials Today: Proceedings*, 80(3), 3193–3195. /doi.org/10.1016/j.matpr.2021.07.205
23. **Salman, A. T., Jassim, H. K., & Zayir, M. Y.** (2023). Solving nonlinear fractional PDEs by Elzaki homotopy perturbation method. *AIP Conference Proceedings*, 2834, 080101. doi.org/10.1063/5.0153667
24. **Singh, J., Kumar, D., & Baleanu, D.** (2023). Fractal dynamics and computational analysis of local fractional Poisson equations arising in electrostatics. *Communications in Theoretical Physics*, 75(12), 125002. //doi.org/10.1088/1572-9494/ad02a8
25. **Singh, J., Kumar, D., & Hammouch, Z.** (2024). New approximate solutions to some of nonlinear PDEs with Atangana-Baleanu-Caputo operator. *Progress in Fractional Differentiation and Applications*, 10(1), 91–98. doi.org/10.18576/pfda/100109
26. **Zayir, M. Y., Jassim, H. K., Salman, A. T., & Hussein, M. A.** (2023). Approximate analytical solutions of fractional Navier-Stokes equation. *AIP Conference Proceedings*, 2834, 080100. /doi.org/10.1063/5.0153666
27. **Zayir, M. Y., Salman, A. T., Jassim, H. K., & Hussein, M. A.** (2023). Solving fractional PDEs by using FADM within Atangana-Baleanu fractional derivative. *AIP Conference Proceedings*, 2845, 060004. doi.org/10.1063/5.0160655