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Analysis of the Y.J Variation Iteration Method

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Abstract:

In this research, a new framework was introduced to solve the fractional differential equations (FDEs) by using the Variation Iteration Method (VIM) and Yasser Jassim Transform. Moreover, the results obtained using this proposed method are very similar to those obtained using other strategies. This methodology is employed to derive approximate analytical solutions for solving the Caputo fractional derivative. To establish the validity and efficacy of the current strategy, the solutions to three selected illustrative examples are presented and thoroughly analyzed. The findings obtained through the proposed method are comprehensively reviewed. The results consistently demonstrate that the (YJFVIM) exhibits high efficiency, robust reliability, and ease of implementation, making it a promising and suitable tool for application across a wide spectrum of related problems in science and engineering disciplines.

Keywords: Yasser Jassim Transform, Caputo Fractional derivative, Variation Iteration Method.

1-Introduction

Fractional linear and nonlinear differential equations (FDEs) have recently generated considerable interest in both pure mathematics and applied sciences. They are widely utilized in modeling numerous physical and chemical processes and have seen extensive application in various fields [1-5]. In turn, the mathematical aspects of fractional differential equations (FDEs) and their solution techniques have been thoroughly discussed by numerous authors [6-18]. First, the fractional differential equations are approximated with suitable initial guesses, after which a correction functional is constructed using a general Lagrange multiplier optimally identified through variation theory. This approach yields rapidly convergent successive approximations to the exact solution [19-26]. The main objective of this paper is to present the Yasser-Jassim variation iteration method (YJFVIM), will be method capable of yielding an approximate solution that is highly convergent to the exact solution using the simplest possible steps. This method will be applied to solving fractional Differential Equations involving the Caputo Fractional derivative.

2- Preliminaries

Definition (2.1) [9]

Let $\psi(t)$ be a function, then the Yasser-Jassim transform of $\psi(t)$ for $t \geq 0$ is defined by

$$\mathcal{H}\{\varphi(t)\} = \mathcal{K}(a) = \int_0^\infty a\, e^{\frac{-1}{\sqrt{a}}t}\, \varphi(t)\, dt \quad \text{where} \quad a \neq 0$$

Properties [9]

- $\mathcal{H}\{1\} = a\sqrt{a}$
- $\mathcal{H}\{t^{\alpha}\} = a\sqrt{a}^{\alpha+1}\,\Gamma(\alpha+1)$
- $\mathcal{H}\{e^{bt}\} = a\sqrt{a}\,\frac{1}{1-b\sqrt{a}}$
- $\mathcal{H}\{u^{n}(t)\} = \frac{1}{\sqrt{a}^{n}}\,\mathcal{H}\{u(t)\} - \sum_{k=0}^{n-1} a\frac{1}{\sqrt{a}^{n-k-1}}u^{k}(0) \qquad n = 1,2,3,\dots$
- $\mathcal{H}\{{}^{c}D_{t}^{\alpha}\,\psi(\mu,\tau)\} = \frac{\mathcal{H}\{\psi(\mu,\tau)\}}{\sqrt{a}^{\alpha}} - \frac{a}{\sqrt{a}^{\alpha-1}}\,\psi(x,0)$

Main Result

In the following section, we introduce the Y.J variation iteration method with Caputo derivative to solve the fractional differential equation and given 3 example. These solution of examples is consistent with the solutions provided by [28-31].

3. Fractional Yasser Jassim variational Decomposition method with Caputo derivative

Let the fractional differential equation with the Caputo derivative of the form

$${}^{C}D_{t}^{\alpha}\psi(\mu,\tau) + R\psi(\mu,\tau) + N\,\psi(\mu,\tau) = g(\mu,\tau)\ , \qquad 0<\alpha<1 \tag{1}$$

with the initial conditions

$$\psi(\mu,0) = f(\mu) \tag{2}$$

where ${}^{C}D_{t}^{\alpha}\,\varphi(\mu,\tau)$ is the Caputo fractional derivative of order α, R is a linear operator , N is a nonlinear operator and g is the source terms .

Now, taking the Y.J transform on both sides of (1) and using (2), we get:

$$\mathcal{H}\{{}^{C}D_{\tau}^{\alpha}\,\psi(\mu,\tau)\} + \mathcal{H}\,\{R\,\psi(\mu,\tau) + N\,\psi(\mu,\tau)\} = \mathcal{H}\{g(\mu,\tau)\ \}$$

Or equivalent:

$$\frac{\mathcal{K}(a)}{\sqrt{a}^{\alpha}} - \frac{a}{\sqrt{a}^{\alpha-1}}\,\psi(\mu,0) + \mathcal{H}\,\{R\,\psi(\mu,\tau) + N\,\psi(\mu,\tau)\} - \mathcal{H}\{g(\mu,\tau)\ \} = 0$$

The iteration formula

$$\mathcal{K}_{n+1}(a) = \mathcal{K}_{n}(a) + \lambda(\frac{\mathcal{K}_{n}(a)}{\sqrt{a}^{\alpha}} - \frac{a}{\sqrt{a}^{\alpha-1}}\,\psi_{n}(\mu,0) + \mathcal{H}\,\{R\,\psi_{n}(\mu,\tau) + N\,\psi_{n}(\mu,\tau)\} - \mathcal{H}\{g(\mu,\tau)\ \}$$

The Lagrange multiplier λ can be identified optimally via variational theory. By taking the variation with respect to the independent variable $\delta\mathcal{K}_{n}(a)$ [27].

$$\delta\mathcal{K}_{n+1}(a) = \delta\mathcal{K}_{n}(a) + \lambda\delta(\frac{\mathcal{K}_{n}(a)}{\sqrt{a}^{\alpha}} - \frac{a}{\sqrt{a}^{\alpha-1}}\,\psi_{n}(\mu,0) + \mathcal{H}\,\{R\,\psi_{n}(\mu,\tau) + N\,\psi_{n}(\mu,\tau)\} - \mathcal{H}\{g(\mu,\tau)\ \} \tag{3}$$

$$\delta\mathcal{K}_{n+1}(a) = \delta\mathcal{K}_{n}(a) + \lambda\delta(\frac{\mathcal{K}_{n}(a)}{\sqrt{a}^{\alpha}})$$

and considering the restricted variation $\frac{\delta\mathcal{K}_{n+1}(a)}{\delta\mathcal{K}_{n}(a)} = 0$

$$0 = 1 + \frac{\lambda}{\sqrt{a}^{\alpha}}$$

$$-1 = \frac{\lambda}{\sqrt{a}^{\alpha}}$$

$$\lambda = -\sqrt{a}^{\alpha}$$

Substituting $\lambda = -\sqrt{a}^{\alpha}$ in (3), we have

$$\mathcal{K}_{n+1}(a) = \mathcal{K}_n(a) - \sqrt{a}^{\alpha}\left(\frac{\mathcal{K}_n(a)}{\sqrt{a}^{\alpha}} - \frac{a}{\sqrt{a}^{\alpha-1}}\psi_n(\mu,0) + \mathcal{H}\{R\,\psi_n(\mu,\tau) + N\,\psi_n(\mu,\tau)\} - \mathcal{H}\{g(\mu,\tau)\}\right.$$

$$= \mathcal{K}_n(a) - \mathcal{K}_n(a) + a\sqrt{a}\,\psi_n(\mu,0) - \sqrt{a}^{\alpha}\mathcal{H}\{R\,\psi_n(\mu,\tau) + N\,\psi_n(\mu,\tau)\} - \mathcal{H}\{g(\mu,\tau)\}$$

By applying Y-J inverse, we get

$$\psi_{n+1}(\mu,\tau) = \psi_n(\mu,0) - \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\mathcal{H}\{R\,\psi_n(\mu,\tau)\}\} - \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{N\,\psi_n(\mu,\tau)\}\right\} + \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\mathcal{H}\{g(\mu,\tau)\}\}$$
(4)

Therefore, the solution is given by

$$\psi(\mu,\tau) = \lim_{n\to\infty}\psi_n$$

Example:
Solve the fractional fractional partial differential equation

$${}^{C}D_{\tau}^{\alpha}\psi(\mu,\tau) - \psi_{\mu} = 0, \qquad 0 < \alpha \le 1$$

with respect to the initial condition $\varphi(\mu,0) = \mu$

$$\psi_{n+1}(\mu,\tau) = \psi_n(\mu,0) + \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\frac{\partial\psi_n(\mu,\tau)}{\partial\mu}\right\}\}$$

$$\psi_0(\mu,\tau) = \psi(\mu,0) = \mu$$

$$\psi_1(\mu,\tau) = \psi_0(\mu,0) + \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\frac{\partial\psi_0(\mu,\tau)}{\partial\mu}\right\}\}$$

$$= \mu + \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\mathcal{H}\{1\}\}$$

$$= \mu + \mathcal{H}^{-1}\left\{2a\sqrt{a}^{\alpha+1}\right\}$$

$$= \mu + \frac{2\tau^{\alpha}}{\Gamma(\alpha+1)}$$

$$\psi_2(\mu,\tau) = \psi_1(\mu,0) + \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\frac{\partial\psi_1(\mu,\tau)}{\partial\mu}\right\}\}$$

$$= \mu + \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\mathcal{H}\{1\}\}$$

$$= \mu + \mathcal{H}^{-1}\left\{a\sqrt{a}^{\alpha+1}\right\}$$

$$= \mu + \frac{\tau^{\alpha}}{\Gamma(\alpha+1)}$$

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$$\psi_n(\mu,\tau) = \mu + \frac{\tau^{\alpha}}{\Gamma(\alpha+1)}$$

Therefore, the solution is

$$\psi(\mu,\tau) = \lim_{n\to\infty}\psi_n$$

$$= \mu + \frac{\tau^{\alpha}}{\Gamma(\alpha+1)}$$

Example:
Consider the fractional partial differential equation

$${}^{C}D_{\tau}^{\alpha}\psi(\mu,\tau) + \psi\psi_{\mu} - \psi_{\mu\mu} = 0, \qquad 0 < \alpha \le 1$$

with respect to the initial condition, $\psi(\mu,0) = \mu$

$$\psi_{n+1}(\mu,\tau) = \psi_n(\mu,0) - \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\psi_n(\mu,\tau)\frac{\partial\psi_n(\mu,\tau)}{\partial\mu}\right\}\right\} + \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\psi_n(\mu,\tau)\frac{\partial^2\psi_n(\mu,\tau)}{\partial\mu^2}\right\}\right\}$$

$$\psi_0(\mu,\tau) = \psi(\mu,0) = \mu$$

$$\psi_1(\mu,\tau) = \psi_0(\mu,0) - \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\psi_0(\mu,\tau)\frac{\partial\psi_0(\mu,\tau)}{\partial\mu}\right\}\} + \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\psi_0(\mu,\tau)\frac{\partial^2\psi_0(\mu,\tau)}{\partial\mu^2}\right\}\right\}$$

$$= \mu - \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{\mu\}\right\} + 0$$

$$= \mu - \mu\mathcal{H}^{-1}\left\{a\sqrt{a}^{\alpha+1}\right\}$$

$$= \mu - \mu\frac{\tau^{\alpha}}{\Gamma(\alpha+1)}$$

$$\psi_2(\mu,\tau) = \psi_1(\mu,0) - \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\psi_1(\mu,\tau)\frac{\partial\psi_1(\mu,\tau)}{\partial\mu}\right\}\right\} + \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\psi_1(\mu,\tau)\frac{\partial^2\psi_1(\mu,\tau)}{\partial\mu^2}\right\}\right\}$$

$$= \mu - \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\left(\mu - \mu\frac{\tau^{\alpha}}{\Gamma(\alpha+1)}\right)\left(1 - \frac{\tau^{\alpha}}{\Gamma(\alpha+1)}\right)\right\}\right\} + 0$$

$$= \mu - \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\mathcal{H}\left\{\mu - 2\mu\frac{\tau^{\alpha}}{\Gamma(\alpha+1)} + \mu\frac{\tau^{2\alpha}}{\Gamma^2(\alpha+1)})\right\}\}$$

$$= \mu - \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\left\{\mu a\sqrt{a} - 2\mu a\sqrt{a}^{\alpha+1} + \mu\frac{a\sqrt{a}^{2\alpha+1}}{\Gamma^2(\alpha+1)}\Gamma(2\alpha+1))\right\}\}$$

$$= \mu - \mathcal{H}^{-1}\left\{\mu a\sqrt{a}^{\alpha+1} - 2\mu a\sqrt{a}^{2\alpha+1} + \mu\frac{a\sqrt{a}^{3\alpha+1}}{\Gamma^2(\alpha+1)}\Gamma(2\alpha+1))\right\}$$

$$= \mu - \mu\frac{\tau^{\alpha}}{\Gamma(\alpha+1)} - 2\mu\frac{\tau^{2\alpha}}{\Gamma(2\alpha+1)} + \mu\frac{\tau^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\Gamma(2\alpha+1)$$

Therefore, the approximation solution is

$$\psi(\mu,\tau) = \lim_{n\to\infty}\psi_n$$

$$= \mu - \mu\frac{\tau^{\alpha}}{\Gamma(\alpha+1)} - 2\mu\frac{\tau^{2\alpha}}{\Gamma(2\alpha+1)} + \mu\frac{\tau^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\Gamma(2\alpha+1)$$

Example:

Consider the fractional partial differential equation

$${}^{C}D_{\tau}^{\alpha}\psi(\mu,\tau) + \psi\psi_{\mu} = \mu + \mu\tau^2, \qquad 0 < \alpha \le 1$$

with respect to the initial condition $\psi(\mu,0) = 0$

$$\psi_{n+1}(\mu,\tau) = \psi_n(\mu,0) - \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{\psi_n(\mu,\tau)\left\{\frac{\partial\psi_n(\mu,\tau)}{\partial\mu}\right\}\}\right\} + \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\mathcal{H}\{\mu + \mu\tau^2\}\}$$

$$\psi_0(\mu,\tau) = \psi(\mu,0) = 0$$

$$\psi_1(\mu,\tau) = \psi_0(\mu,0) - \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{\psi_0(\mu,\tau)\left\{\frac{\partial\psi_0(\mu,\tau)}{\partial\mu}\right\}\}\right\} + \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{\mu + \mu\tau^2\}\right\}$$

$$= 0 - 0 + \mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\left\{a\sqrt{a}\mu + 2a\sqrt{a}^{3}\mu\right\}$$

$$= \mathcal{H}^{-1}\left\{a\sqrt{a}^{\alpha+1}\mu + 2a\sqrt{a}^{\alpha+3}\mu\right\}$$

$$= \frac{\tau^{\alpha}\mu}{\Gamma(\alpha+1)} + \frac{2\tau^{\alpha+2}\mu}{\Gamma(\alpha+3)}$$

$$\psi_2(\mu,\tau) = \psi_1(\mu,0) - \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{\psi_1(\mu,\tau)\left\{\frac{\partial\psi_1(\mu,\tau)}{\partial\mu}\right\}\}\right\} + \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{\mu + \mu\tau^2\}\right\}$$

$$= 0 - \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{(\frac{\tau^{\alpha}\mu}{\Gamma(\alpha+1)} + \frac{2\tau^{\alpha+2}\mu}{\Gamma(\alpha+3)})(\frac{\tau^{\alpha}}{\Gamma(\alpha+1)} + \frac{2\tau^{\alpha+2}}{\Gamma(\alpha+3)})\}\right\} + \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{\mu + \mu\tau^2\}\right\}$$

$$= 0 - \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{(\frac{\tau^{2\alpha}\mu}{\Gamma^2(\alpha+1)} + \frac{4\tau^{2\alpha+2}\mu}{\Gamma(\alpha+1)\Gamma(\alpha+3)} + \frac{4\tau^{2\alpha+4}}{\Gamma^2(\alpha+3)})\}\right\} + \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\mathcal{H}\{\mu + \mu\tau^2\}\right\}$$

$$= -\mathcal{H}^{-1}\{\sqrt{a}^{\alpha}\left\{\frac{a\sqrt{a}^{2\alpha+1}\,\Gamma(2\alpha+1)\mu}{\Gamma^2(\alpha+1)} + \frac{4a\sqrt{a}^{2\alpha+3}\,\Gamma(2\alpha+3)\mu}{\Gamma(\alpha+1)\,\Gamma(\alpha+3)} + \frac{4a\sqrt{a}^{2\alpha+5}\,\Gamma(2\alpha+5)\mu}{\Gamma^2(\alpha+3)}\right\} + \mathcal{H}^{-1}\left\{\sqrt{a}^{\alpha}\left\{a\sqrt{a}\mu + 2a\sqrt{a}^{3}\mu\right\}\right\}$$

$$= -\mathcal{H}^{-1}\left\{\frac{a\sqrt{a}^{3\alpha+1}\,\Gamma(2\alpha+1)\mu}{\Gamma^2(\alpha+1)} + \frac{4a\sqrt{a}^{3\alpha+3}\,\Gamma(2\alpha+3)\mu}{\Gamma(\alpha+1)\,\Gamma(\alpha+3)} + \frac{4a\sqrt{a}^{3\alpha+5}\,\Gamma(2\alpha+5)\mu}{\Gamma^2(\alpha+3)}\right\} + \mathcal{H}^{-1}\left\{a\sqrt{a}^{\alpha+1}\mu + 2a\sqrt{a}^{\alpha+3}\mu\right\}$$

$$= -\frac{\tau^{3\alpha}\,\Gamma(2\alpha+1)\mu}{\Gamma^2(\alpha+1)\,\Gamma(3\alpha+1)} - \frac{4\tau^{3\alpha+2}\,\Gamma(2\alpha+3)\mu}{\Gamma(\alpha+1)\,\Gamma(\alpha+3)\,\Gamma(3\alpha+3)} - \frac{4\tau^{3\alpha+4}\,\Gamma(2\alpha+5)\mu}{\Gamma^2(\alpha+3)\,\Gamma(3\alpha+5)} + \frac{\tau^{\alpha}\mu}{\Gamma(\alpha+1)} + \frac{2\tau^{\alpha+2}\mu}{\Gamma(\alpha+3)}$$

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$$\psi_n(\mu,\tau) = \frac{\tau^{\alpha}\mu}{\Gamma(\alpha+1)} + \frac{2\tau^{\alpha+2}\mu}{\Gamma(\alpha+3)} - \frac{\tau^{3\alpha}\,\Gamma(2\alpha+1)\mu}{\Gamma^2(\alpha+1)\,\Gamma(3\alpha+1)} - \frac{4\tau^{3\alpha+2}\,\Gamma(2\alpha+3)\mu}{\Gamma(\alpha+1)\,\Gamma(\alpha+3)\,\Gamma(3\alpha+3)} - \frac{4\tau^{3\alpha+4}\,\Gamma(2\alpha+5)\mu}{\Gamma^2(\alpha+3)\,\Gamma(3\alpha+5)} - \cdots$$

$$\psi(\mu,\tau) = \lim_{n\to\infty}\psi_n = \mu\left(\frac{\tau^{\alpha}}{\Gamma(\alpha+1)} + \frac{2\tau^{\alpha+2}}{\Gamma(\alpha+3)} - \frac{\tau^{3\alpha}\,\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\,\Gamma(3\alpha+1)} - \frac{4\tau^{3\alpha+2}\,\Gamma(2\alpha+3)}{\Gamma(\alpha+1)\,\Gamma(\alpha+3)\,\Gamma(3\alpha+3)} - \frac{4\tau^{3\alpha+4}\,\Gamma(2\alpha+5)}{\Gamma^2(\alpha+3)\,\Gamma(3\alpha+5)}\right)$$

4- Conclusion Section

Yasser Jassim variation iteration method presents a new framework to solve the fractional differential equations (FDEs). Three fractional differential equations (FDEs) are solved using this method yielding, results similar to those obtained using other strategies.

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