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An Enhanced Analytical Framework for the Solution of Partial Differential Equations

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Abstract:

This paper employed the Yasser Jassim (YJ) Transform with the Dafter Jafari Method (DJM) and the Hussein Jassim Method (HJM) to solve partial differential equations, generating two relationships and subsequently analyzing instances of both linear and nonlinear equations. The findings demonstrated that the integration of the transform and the methodologies considerably streamline the solution of partial differential equations, producing results consistent with those achieved via direct integration.

Keywords: Yasser Jassim Transform, Partial Differential Equations, Daftardar Jafari Method, Hussein Jassim Method, Analytical Scheme.

1-Introduction Section

Partial Differential Equations (PDEs) are crucial for simulating many processes in heat conduction, wave propagation, fluid dynamics, electromagnetism, and multiple other domains in engineering, physics, and natural sciences. The inherent complexity and significant nonlinearity of PDEs sometimes make accurate analytical solutions challenging or impossible, leading researchers to develop efficient analytical and semi-analytical methods for their resolution [1,2].

In recent decades, various analytical techniques have emerged to tackle linear and nonlinear partial differential equations (PDEs), including the Adomian Decomposition Method, the Variational Iteration Method, the Homotopy Perturbation Method, and the Homotopy Analysis Method. Despite the significant success of these solutions, many still involve complex computing processes, repetitive integrations, or rigid assumptions, hence limiting their applicability to a broader array of problems [7–11].

Integral transformations have shown to be effective tools for simplifying differential equations by converting them into algebraic or more tractable forms. Conventional transforms, such as the Laplace and Fourier transforms, together with newly designed integral transforms, have been widely employed to solve various ordinary and partial differential equations. The Yasser Jassim Transform has recently attracted considerable attention for its effectiveness in solving various differential equations while reducing computational complexity [3–6].

This study aims to amalgamate the Yasser Jassim Transform with the Dafter Jafari Method and the Hussein Jassim Method to offer two improved solution strategies for partial differential equations, motivated by the demand for more direct and efficient analytical methods. The proposed methods utilize the advantages of the transform to eliminate direct integrations while maintaining the iterative structure of DJM and HJM, leading to a significant simplification of the solution process.

The effectiveness and reliability of the proposed formulations are demonstrated by numerous illustrative examples involving linear and nonlinear partial differential equations. The results obtained exhibit remarkable agreement with those derived from direct integration and other established analytical approaches, so confirming the accuracy and effectiveness of the suggested systems.

2- Description of the Yasser Jassim Transform (YJT)

Definition 2.1 [3]. Let $u(t)$ be an integral function defined for $t \geqslant 0$, we define a Yasser Jassim transform of $u(t)$ by the formula.

$$H\{u(t)\} = K(a) = a\int_0^\infty e^{-\frac{1}{\sqrt{a}}t}\, u(t)dt \qquad , \ where\ a \neq 0$$

And a is Yasser Jassim transform parameters.

The inverse Yasser Jassim transform is used to obtain the original function $u(t)$.

$$u(t) = H^{-1}\{K(a)\}.$$

The Yasser Jassim transform possesses the following significant characteristics:

(i) For functions $u_1(t)$ and $u_2(t)$ Yasser Jassim transformations and constants that are defined $c_1, c_2 \in R$, then

$$H[c_1u_1(t) + c_2u_2(t)] = c_1H[u_1(t)] + c_2H[u_2(t)].$$

(ii) The Yasser Jassim transform of the 1^{th} derivative of the function $u(t)$ is

$$H[u'(t)] = \frac{1}{\sqrt{a}}H[u(t)] - au(0).$$

(iii) The Yasser Jassim transform of the 2^{th} derivative of the function $u(t)$ is

$$H[u''(t)] = \frac{1}{a}H[u(t)] - \sqrt{a}\,u(0) - au'(0).$$

(iv) The Yasser Jassim transform of the function $u(t)$ derivative, n^{th}, is

$$H\left[u^{(n)}(t)\right] = \frac{1}{(\sqrt{a})^n}\, H[u(t)] - \sum_{k=0}^{n-1} a\frac{1}{(\sqrt{a})^{n-k-1}}\, u^{(k)}(0) \ , \ n = 1,2,3, \dots$$

gives some helpful details about the Yasser Jassim transformations of a few fundamental functions.

$\mathrm{H}[1] = a\sqrt{a}$.

$\mathrm{H}[e^{bt}] = \frac{a\sqrt{a}}{1-b\sqrt{a}}$.

$\mathrm{H}[t^n] = n!\, a(\sqrt{a})^{n+1}$.

$\mathrm{H}[sin(bt)] = \frac{a^2 b}{1+ab^2}$.

$\mathrm{H}[cos(bt)] = \frac{a\sqrt{a}}{1+ab^2}$.

$\mathrm{H}[sinh(bt)] = \frac{a^2 b}{1-ab^2}$.

$\mathrm{H}[cosh(bt)] = \frac{a\sqrt{a}}{1-ab^2}$.

3- Analysis of method

3.1- Yasser Jassim Transform Dafter Jafari Method (YJDJM)

Examine the general nonlinear differential equation that follows:

$$u^n(x,t) + L(u(x,t)) + N(u(x,t)) = g(x,t), \qquad n \in N \tag{3.1.1}$$

using the initial condition $u_t^{(i)}(x,0) = \lambda_i(x),\ i = 0,1,2 \dots, n.$
where $u(x,t)$ denotes the unknown function. $u^{(n)} = \frac{\partial^n u}{\partial t^n}$, L is a linear operator, N represents nonlinear terms and $g(x,t)$ is taken as thesource term.

The Yasser Jassim transform on both sides of (3.1.1) is the first step in the YJDJM, and we obtain

$$\mathrm{H}[u^n(x,t)] = \mathrm{H}[g(x,t) - L(u(x,t)) - N(u(x,t))]. \tag{3.1.2}$$

Utilising the initial conditions and simplifying the aforementioned equation results in

$$\frac{1}{(\sqrt{a})^n}\mathrm{H}[u(x,t)] - \sum_{k=0}^{n-1} \frac{a}{(\sqrt{a})^{n-k-1}}\lambda_k(x) = \mathrm{H}[g(x,t) - L(u(x,t)) - N(u(x,t))],$$
$$\mathrm{H}[u(x,t)] = \sum_{k=0}^{n-1} \frac{a(\sqrt{a})^n}{(\sqrt{a})^{n-k-1}}\lambda_k(x) + (\sqrt{a})^n \mathrm{H}[g(x,t) - L(u(x,t)) - N(u(x,t))],$$
$$= \sum_{k=0}^{n-1} a(\sqrt{a})^{k+1}\lambda_k(x) + (\sqrt{a})^n \mathrm{H}[g(x,t) - L(u(x,t)) - N(u(x,t))]. \tag{3.1.3}$$

In the second stage, we apply the inverse Yasser Jassim transform to both sides of (3.1.3) to obtain,

$$u(x,t) = \mathrm{H}^{-1}\left\{\sum_{k=0}^{n-1} a(\sqrt{a})^{k+1}\lambda_k(x)\right\} + \mathrm{H}^{-1}\{(\sqrt{a})^n \mathrm{H}[g(x,t) - L(u(x,t)) - N(u(x,t))]\} . \tag{3.1.4}$$

The equation above can be rewritten as

$$u(x,t) = \sum_{k=0}^{n-1} \frac{t^k}{k!}\lambda_k(x) + \mathrm{H}^{-1}\{(\sqrt{a})^n \mathrm{H}[g(x,t) - L(u(x,t)) - N(u(x,t))]\}. \tag{3.1.5}$$

The Daftardar Jafari method (DJM), an iterative approach proposed by Daftardar and Jafari, is applied in the third and final stage. The solution to (3.1.1) is expressed as an infinite series,

$$u(\mathrm{x},\mathrm{t}) = \sum_{n=0}^{\infty} u_n \,. \tag{3.1.6}$$

by Substituting (3.1.6) in Eq. (3.1.5) gives

$$\sum_{n=0}^{\infty} u_n = \sum_{k=0}^{n-1} \frac{\mathrm{t}^k}{k!} \lambda_k(\mathrm{x}) + \mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}[g(\mathrm{x},\mathrm{t}) - L(\sum_{n=0}^{\infty} u_n) - N(\sum_{n=0}^{\infty} u_n)]\} \,. \tag{3.1.7}$$

In (3.1.7), the nonlinear term goes decomposed as

$$N(\sum_{n=0}^{\infty} u_n) = N(u_0) + \sum_{n=1}^{\infty} \left[N(\sum_{k=0}^{\mathrm{n}} u_k) - N\left(\sum_{k=0}^{\mathrm{n}-1} u_k\right)\right] \;,$$

Substituting into (3.1.7) gives

$$\sum_{n=0}^{\infty} u_n = \sum_{k=0}^{n-1} \frac{\mathrm{t}^k}{k!} \lambda_k(\mathrm{x}) + \mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}\big[g(\mathrm{x},\mathrm{t}) - L(\sum_{n=0}^{\infty} u_n) - \big(N(u_0) + \sum_{n=1}^{\infty} \left[N(\sum_{k=0}^{\mathrm{n}} u_k) - N\left(\sum_{k=0}^{\mathrm{n}-1} u_k\right)\right]\big)\big]\} \,,$$

The following iteration is then deduced:

$$u_0 = \sum_{k=0}^{n-1} \frac{\mathrm{t}^k}{k!} \lambda_k(\mathrm{x}) + \mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}[g(\mathrm{x},\mathrm{t})]\},$$

$$u_1 = -\mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}\big[L(u_0) + \big(N(u_0)\big)\big]\},$$

$$u_2 = -\mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}\big[L(u_1) + \big(N(u_1 + u_0) - N(u_0)\big)\big]\},$$

$$\vdots$$

$$u_n = -\mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}\big[L(u_{n-1}) + \big(N(u_0 + \cdots + u_{n-1}) - N(u_0 + \cdots + u_{n-2})\big)\big]\}, \qquad n = 3,4,\ldots$$

As a result, the approximate solution is

$$u(\mathrm{x},\mathrm{t}) = u_0 + u_1 + u_2 + u_3 + \cdots = \sum_{n=0}^{\infty} u_n, \tag{3.1.8}$$

3.2- Yasser Jassim Transform Hussein Jassim Method (YJHJM)

Examine the general nonlinear differential equation that follows:

$$u^n(\mathrm{x},\mathrm{t}) + L\big(u(\mathrm{x},\mathrm{t})\big) + N\big(u(\mathrm{x},\mathrm{t})\big) = g(x,t), \qquad n \in N \tag{3.2.1}$$

using the initial condition $u_{\mathrm{t}}^{(i)}(\mathrm{x},0) = \lambda_i(\mathrm{x}),\; i = 0,1,2\ldots,n.$
where $u(x,t)$denotes the unknown function. $u^{(n)} = \frac{\partial^n u}{\partial \mathrm{t}^n}$, L is a linear operator, $g(x,t)$ is a known function, while N is a nonlinear operator.
By applying the Yasser Jassim transform to equation (3.2.1), we arrive at

$$\mathrm{H}[u^n(\mathrm{x},\mathrm{t})] = \mathrm{H}\big[g(x,t) - L\big(u(\mathrm{x},\mathrm{t})\big) - N\big(u(\mathrm{x},\mathrm{t})\big)\big]. \tag{3.2.2}$$

By applying the Yasser Jassim transform differentiation property, we have

$$\frac{1}{(\sqrt{a})^{\mathrm{n}}}\mathrm{H}[u(\mathrm{x},\mathrm{t})] - \sum_{k=0}^{n-1} \frac{a}{(\sqrt{a})^{\mathrm{n-k-1}}} \lambda_k(\mathrm{x}) = \mathrm{H}\big[g(x,t) - L\big(u(\mathrm{x},\mathrm{t})\big) - N\big(u(\mathrm{x},\mathrm{t})\big)\big] \tag{3.2.3}$$

$$\mathrm{H}[u(\mathrm{x},\mathrm{t})] = \sum_{k=0}^{n-1} \frac{a(\sqrt{a})^{\mathrm{n}}}{(\sqrt{a})^{\mathrm{n-k-1}}} \lambda_k(\mathrm{x}) + (\sqrt{a})^{\mathrm{n}}\mathrm{H}\big[g(x,t) - L\big(u(\mathrm{x},\mathrm{t})\big) - N\big(u(\mathrm{x},\mathrm{t})\big)\big]$$

$$= \sum_{k=0}^{n-1} a\big(\sqrt{a}\big)^{k+1} \lambda_k(\mathrm{x}) + (\sqrt{a})^{\mathrm{n}}\mathrm{H}\big[g(x,t) - L\big(u(\mathrm{x},\mathrm{t})\big) - N\big(u(\mathrm{x},\mathrm{t})\big)\big]. \tag{3.2.4}$$

Operating with Yasser Jassim inverse on both sides of (3.2.4), we obtain

$$u(x,t) = \mathrm{H}^{-1}\left\{\sum_{k=0}^{n-1} a\big(\sqrt{a}\big)^{k+1} \lambda_k(\mathrm{x})\right\} + \mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}\big[g(x,t) - L\big(u(\mathrm{x},\mathrm{t})\big) - N\big(u(\mathrm{x},\mathrm{t})\big)\big]\},$$

$$= \sum_{k=0}^{n-1} \frac{t^k}{k!} \lambda_k(\mathrm{x}) + \mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}[g(x,t) - L(u(\mathrm{x},\mathrm{t})) - N(u(\mathrm{x},\mathrm{t}))]\}. \quad (3.2.5)$$

We now use Maclaurin's expansion with respect to t to rewrite Eq. (3.2.5).

We formulate Eq. (3.2.5) using Maclaurin's extension with respect to t.

$$u(x,t) = \sum_{i=0}^{\infty} \frac{t^i}{i!} \mathrm{D}_t^i \left(\sum_{k=0}^{n-1} \frac{t^k}{k!} \lambda_k(\mathrm{x}) + \mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}[g(x,t) - L(u(\mathrm{x},\mathrm{t})) - N(u(\mathrm{x},\mathrm{t}))]\}\right)_{t=0} \quad (3.2.6)$$

Where $\mathrm{D}_\mathrm{t}^i = \frac{d^i}{dt^i}$. Assume that

$$u(x,t) = \sum_{i=0}^{\infty} u_i,$$

By substituting in Eq. (3.2.6), we have the outcome that

$$\sum_{i=0}^{\infty} u_i = \sum_{i=0}^{\infty} \frac{t^i}{i!} \mathrm{D}_t^i \left(\sum_{k=0}^{n-1} \frac{t^k}{k!} \lambda_k(\mathrm{x}) + \mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}[g(\mathrm{x},\mathrm{t}) - L(\sum_{i=0}^{\infty} u_i) - N(\sum_{i=0}^{\infty} u_i)]\}\right)_{t=0}$$

(3.2.7)

Put i=i+1, Eq. (3.2.7) become

$$u_0 + \sum_{i=0}^{\infty} u_{i+1} = \sum_{i=0}^{\infty} \frac{t^i}{i!} \mathrm{D}_t^i \left(\sum_{k=0}^{n-1} \frac{t^k}{k!} \lambda_k(\mathrm{x}) + \mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}[g(x,t) - L(\sum_{i=0}^{\infty} u_i) - N(\sum_{i=0}^{\infty} u_i)]\}\right)_{t=0}$$

(3.2.8)

By contrasting Eq. (3.2.8)'s two sides

$$\begin{cases} u_0 = u(x,0) = \lambda_0(\mathrm{x}) \\ u_1 = t\lambda_1(\mathrm{x}) + t\,\mathrm{D}_t^1(\mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}[g(x,t) - L(u_0) - N(u_0)]\})_{t=0} \\ u_2 = \frac{t^2}{2!}\lambda_2(\mathrm{x}) + \frac{t^2}{2!}\mathrm{D}_t^2(\mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}[g(x,t) - L(u_0+u_1) - N(u_0+u_1)]\})_{t=0} \\ u_3 = \frac{t^3}{3!}\lambda_3(\mathrm{x}) + \frac{t^3}{3!}\mathrm{D}_t^3(\mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}[g(x,t) - L(u_0+u_1+u_3) - N(u_0+u_1+u_3)]\})_{t=0} \\ \vdots \\ u_{i+1} = \frac{t^{i+1}}{(i+1)!}\mathrm{D}_t^{i+1}\left(\sum_{k=0}^{n-1} \frac{t^k}{k!} \lambda_k(\mathrm{x}) + \mathrm{H}^{-1}\{(\sqrt{a})^{\mathrm{n}}\mathrm{H}[g(x,t) - L(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i} u_j)]\}\right)_{t=0} \end{cases}$$

Consequently, the approximate solution is

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots = \sum_{i=0}^{\infty} u_i, \quad (3.2.9)$$

4. illustrative examples:

4.1 YJDJM:

Example 4.1.1: Examine PDE

$$u_{tt}(x,t) + u_{\mathrm{xx}}(x,t) = 0,$$

with $u(\mathrm{x},0) = -e^x$ and $u_x(\mathrm{x},0) = 0$

$$\mathrm{H}[u_{tt}(x,t)] + \mathrm{H}[u_{\mathrm{xx}}(x,t)] = 0,$$

$$\frac{1}{a}\mathrm{H}[u(x,t)] - \sqrt{a}\,u(x,0) - au_x(\mathrm{x},0) = -\mathrm{H}[u_{\mathrm{xx}}(x,t)],$$

$$\mathrm{H}[u(x,t)] = -a\sqrt{a}e^x - a\mathrm{H}[u_{\mathrm{xx}}(x,t)],$$

$$u(x,t) = -e^x - \mathrm{H}^{-1}\{a\mathrm{H}[u_{\mathrm{xx}}(x,t)]\},$$

$Let \quad u(x,t) = \sum_{i=0}^{\infty} u_i,$

$\sum_{i=0}^{\infty} u_i = -e^x - \mathrm{H}^{-1}\{a\mathrm{H}[\sum_{i=0}^{\infty} u_{ixx}]\},$

$u_0 = -e^x,$

$u_1 = -\mathrm{H}^{-1}\{a\mathrm{H}[u_{0xx}]\} = \mathrm{H}^{-1}\{a\mathrm{H}[e^x]\},$

$= \mathrm{H}^{-1}\{a(a\sqrt{a}e^x)\} = \mathrm{H}^{-1}\left\{a(\sqrt{a})^3 e^x\right\} = \frac{t^2}{2!}e^x,$

$u_2 = -\mathrm{H}^{-1}\{a\mathrm{H}[u_{1xx}]\} = -\mathrm{H}^{-1}\left\{a\mathrm{H}\left[\frac{t^2}{2!}e^x\right]\right\}$
$= -\mathrm{H}^{-1}\left\{a\left(2!\,a(\sqrt{a})^3\frac{e^x}{2!}\right)\right\} = -\mathrm{H}^{-1}\left\{a(\sqrt{a})^5 e^x\right\} = -\frac{t^4}{4!}e^x,$

$\vdots$

$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots$
$u(x,t) = -e^x + \frac{t^2}{2!}e^x - \frac{t^4}{4!}e^x \cdots$

$$u(x,t) = -e^x \cos(x).$$

Example. 4.1.2 Examine PDE

$u_t(x,t) + u(x,t)u_x(x,t) = 0,$
$with\; u(x,0) = x$

$H[u_{t(x,t)}] = -H[u(x,t)u_x(x,t)],$

$\frac{1}{\sqrt{a}}H[u(x,t)] - au(x,0) = -H[u(x,t)u_x(x,t)],$

$H^{-1}[u(x,t)] = H^{-1}\{a\sqrt{a}x\} - H^{-1}\{\sqrt{a}H[u(x,t)u_x(x,t)]\},$

$u(x,t) = x - H^{-1}\{\sqrt{a}H[u(x,t)u_x(x,t)]\},$

$Let \quad u(x,t) = \sum_{i=0}^{\infty} u_i, \qquad u(x,t)u_x(x,t) = \sum_{i=0}^{\infty} \omega_i,$

$\sum_{i=0}^{\infty} u_i = x - H^{-1}\{\sqrt{a}H[\sum_{i=0}^{\infty} \omega_i]\},$

$u_0 = x,$

$u_1 = x - H^{-1}\{\sqrt{a}H[\omega_0]\},$

$= -H^{-1}\{\sqrt{a}H[u_0]\},$

$u_1 = -H^{-1}\{\sqrt{a}H[x]\},$

$= -H^{-1}\left\{xa(\sqrt{a})^2\right\} \quad = -xt$

$u_2 = -H^{-1}\{\sqrt{a}H[\omega_1]\},$

$= -H^{-1}\{\sqrt{a}H[(u_0+u_1)(u_0+u_1)_x - (u_0)(u_0)_x]\},$

$$= -H^{-1}\left\{-2xa\left(\sqrt{a}\right)^3 + xa\left(\sqrt{a}\right)^4\right\} = xt^2 - x\frac{t^3}{3!}$$

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots$$

$$= x - xt + xt^2 - xt^3 + \cdots$$

$$u(x,t) = \frac{x}{1+t}$$

Example. 4.1.3 Examine PDE

$$u_t(x,t) + u_x(x,t) + 2v(x,t) = 0,\ with\ u(x,0) = cos(x),$$

$$v_t(x,t) + v_x(x,t) - 2u(x,t) = 0,\ with\ v(x,0) = sin(x),$$

$$H[u_t(x,t)] = -H[2v(x,t)] - H[u_x(x,t)],$$

$$H[v_t(x,t)] = H[2u(x,t)] - H[v_x(x,t)],$$

$$\frac{1}{\sqrt{a}}H[u(x,t)] - au(x,0) = -H[2v(x,t)] - H[u_x(x,t)],$$

$$\frac{1}{\sqrt{a}}H[v(x,t)] - av(x,0) = H[2u(x,t)] - H[v_x(x,t)],$$

$$H[u(x,t)] = a\sqrt{a}cos(x) - \sqrt{a}H[2v(x,t)] - \sqrt{a}H[u_x(x,t)],$$

$$H[v(x,t)] = a\sqrt{a}sin(x) + \sqrt{a}H[2u(x,t)] - \sqrt{a}H[v_x(x,t)],$$

$$u(x,t) = cos(x) - H^{-1}\{\sqrt{a}H[2v(x,t)]\} - H^{-1}\{\sqrt{a}H[u_x(x,t)]\},$$

$$v(x,t) = sin(x) + H^{-1}\{\sqrt{a}H[2u(x,t)]\} - H^{-1}\{\sqrt{a}H[v_x(x,t)]\},$$

Let $u(x,t) = \sum_{n=0}^{\infty} u_n,$

$$v(x,t) = \sum_{n=0}^{\infty} v_n,$$

$$\sum_{n=0}^{\infty} u_n = cos(x) - H^{-1}\left\{\sqrt{a}H\left[2\sum_{n=0}^{\infty} v_n\right]\right\} - H^{-1}\left\{\sqrt{a}H\left[\sum_{n=0}^{\infty} u_{nx}\right]\right\},$$

$$\sum_{n=0}^{\infty} v_n = sin(x) + H^{-1}\left\{\sqrt{a}H\left[2\sum_{n=0}^{\infty} u_n\right]\right\} - H^{-1}\left\{\sqrt{a}H\sum_{n=0}^{\infty} v_{nx}\right\},$$

$$u_0 = cos(x),$$

$$v_0 = sin(x),$$

$$u_1 = -H^{-1}\{\sqrt{a}H[2v_0] + \sqrt{a}H[u_{0x}]\},$$

$$= -\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2sin(\mathrm{x})] - \sqrt{a}\mathrm{H}[sin(\mathrm{x})]\} = -tsin(\mathrm{x})$$

$$\mathrm{v}_1 = \mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2u_0] - \sqrt{a}\mathrm{H}[\mathrm{v}_{0x}]\},$$

$$= \mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2cos(\mathrm{x})] - \sqrt{a}\mathrm{H}[cos(\mathrm{x})]\} = tcos(\mathrm{x})$$

$$u_2 = -\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2\mathrm{v}_1] + \sqrt{a}\mathrm{H}[u_{1x}]\},$$

$$= -\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2tcos(\mathrm{x})] - \sqrt{a}\mathrm{H}[tcos(\mathrm{x})]\} = -\frac{t^2}{2!}cos(\mathrm{x})$$

$$\mathrm{v}_2 = \mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2u_1] - \sqrt{a}\mathrm{H}[\mathrm{v}_{1x}]\},$$

$$= \mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[-2tsin(\mathrm{x})] + \sqrt{a}\mathrm{H}[tsin(\mathrm{x})]\} = -\frac{t^2}{2!}sin(\mathrm{x})$$

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots$$

$$= cos(\mathrm{x}) - tsin(\mathrm{x}) - \frac{t^2}{2!}cos(\mathrm{x}) + \frac{t^3}{3!}sin(\mathrm{x}) + \cdots$$

$$= \left(cos(\mathrm{x}) - \frac{t^2}{2!}cos(\mathrm{x}) + \cdots\right) + \left(-tsin(\mathrm{x}) + \frac{t^3}{3!}sin(\mathrm{x}) + \cdots\right)$$

$$u(x,t) = cos(\mathrm{x})cos(t) - sin(\mathrm{x})sin(t)$$

$$\omega(x,t) = \omega_0 + \omega_1 + \omega_2 + \omega_3 + \cdots$$

$$= sin(\mathrm{x}) + tcos(\mathrm{x}) - \frac{t^2}{2!}sin(\mathrm{x}) - \frac{t^3}{3!}cos(\mathrm{x}) + \cdots$$

$$= \left(sin(\mathrm{x}) - \frac{t^2}{2!}sin(\mathrm{x}) + \cdots\right) + \left(tcos(\mathrm{x}) - \frac{t^3}{3!}cos(\mathrm{x}) + \cdots\right)$$

$$\omega(x,t) = sin(\mathrm{x})cos(t) - cos(\mathrm{x})sin(t)$$

4.2 YJHJM:

Example. 4.2.1 Examine PDE

$$u_{tt}(x,t) + u_{\mathrm{xx}}(x,t) = 0,$$

with $u(\mathrm{x},0) = -e^x$ and $u_x(\mathrm{x},0) = 0$

$$\mathrm{H}[u_{tt}(x,t)] + \mathrm{H}[u_{\mathrm{xx}}(x,t)] = 0,$$

$$\frac{1}{a}\mathrm{H}[u(x,t)] - \sqrt{a}\,u(x,0) - au_x(\mathrm{x},0) = -\mathrm{H}[u_{\mathrm{xx}}(x,t)],$$

$$\mathrm{H}[u(x,t)] = -a\sqrt{a}e^x - a\mathrm{H}[u_{\mathrm{xx}}(x,t)],$$

$$u(x,t) = -e^x - \mathrm{H}^{-1}\{a\mathrm{H}[u_{\mathrm{xx}}(x,t)]\},$$

$$u(x,t) = \sum_{i=0}^{\infty}\frac{t^i}{i!}\,\mathrm{D}_t^i\left(-e^x - \mathrm{H}^{-1}\{a\mathrm{H}[u_{\mathrm{xx}}(x,t)]\}\right)_{t=0}$$

$$let\ u(x,t)=\sum_{i=0}^{\infty} u_i,$$

$$\sum_{i=0}^{\infty} u_i=\sum_{i=0}^{\infty}\frac{t^i}{i!}\,\mathrm{D}_t^i\left(-e^x-\mathrm{H}^{-1}\{a\mathrm{H}[\sum_{i=0}^{\infty} u_{ixx}]\}\right)_{t=0},$$

$$u_0=u(x,0)=-e^x$$

$$u_1=t\,\mathrm{D}_t^1(-e^x-\mathrm{H}^{-1}\{a\mathrm{H}[u_{0xx}]\})_{t=0},$$

$$=t\,\mathrm{D}_t^1(-e^x+\mathrm{H}^{-1}\{a\mathrm{H}[e^x]\})_{t=0}=t\,\mathrm{D}_t^1\left(-e^x+\mathrm{H}^{-1}\{a(a\sqrt{a})e^x\}\right)_{t=0},$$

$$=t\,\mathrm{D}_t^1\left(-e^x+\frac{t^2}{2!}e^x\right)_{t=0}=t\,(0+te^x)_{t=0}=0$$

$$u_2=\frac{t^2}{2!}\,\mathrm{D}_t^2(-e^x-\mathrm{H}^{-1}\{a\mathrm{H}[u_{0xx}+u_{1xx}]\})_{t=0},$$

$$=\frac{t^2}{2!}\,\mathrm{D}_t^2(-e^x+\mathrm{H}^{-1}\{a\mathrm{H}[e^x+0]\})_{t=0}=\frac{t^2}{2!}\,\mathrm{D}_t^2\left(-e^x+\mathrm{H}^{-1}\{a(a\sqrt{a})e^x\}\right)_{t=0},$$

$$=\frac{t^2}{2!}\,\mathrm{D}_t^2\left(-e^x+\frac{t^2}{2!}e^x\right)_{t=0}=\frac{t^2}{2!}\,\mathrm{D}_t^1(0+te^x)_{t=0}=\frac{t^2}{2!}\,(0+e^x)_{t=0}=\frac{t^2}{2!}\,e^x$$

$$u(x,t)=u_0+u_1+u_2+u_3+\cdots$$

$$u(x,t)=-e^x+0+\frac{t^2}{2!}\,e^x+0-\frac{t^4}{4!}e^x\cdots$$

$$u(x,t)=-e^x\cos(x).$$

Example. 4.2.2 Examine PDE

$$u_{t(x,t)}+u(x,t)u_x(x,t)=0,$$

$$with\ u(x,0)=x$$

$$H[u_t(x,t)]=-H[u(x,t)u_x(x,t)],$$

$$\frac{1}{\sqrt{a}}H[u(x,t)]-au(x,0)=-H[u(x,t)u_x(x,t)],$$

$$H^{-1}[u(x,t)]=H^{-1}\{a\sqrt{a}x\}-H^{-1}\{\sqrt{a}H[u(x,t)u_x(x,t)]\},$$

$$u(x,t)=x-H^{-1}\{\sqrt{a}H[u(x,t)u_x(x,t)]\},$$

$$u(x,t)=\sum_{i=0}^{\infty}\frac{t^i}{i!}\,\mathrm{D}_t^i\left(x-H^{-1}\{\sqrt{a}H[u(x,t)u_x(x,t)]\}\right)_{t=0},$$

Let

$$u(x,t)=\sum_{i=0}^{\infty} u_i, \qquad uu_x=\sum_{i=0}^{\infty}\omega_i,$$

$$\sum_{i=0}^{\infty} u_i=\sum_{i=0}^{\infty}\frac{t^i}{i!}\,\mathrm{D}_t^i\left(x-H^{-1}\{\sqrt{a}H[\sum_{i=0}^{\infty}\omega_i]\}\right)_{t=0},$$

$$u_0=u(x,0)=x$$

$$u_1=t\,\mathrm{D}_t^1\left(x-H^{-1}\{\sqrt{a}H[\omega_0]\}\right)_{t=0},$$

$$=t\,\mathrm{D}_t^1\left(x-H^{-1}\{\sqrt{a}H[u_0u_{0x}]\}\right)_{t=0},$$

$= t\ D_t^1(x - xt)_{t=0}$

$= t\ (0 - x)_{t=0} = -tx$

$u_2 = \frac{t^2}{2!}\ D_t^2\left(x - H^{-1}\{\sqrt{a}H[\omega_1]\}\right)_{t=0}$,

$= \frac{t^2}{2!}\ D_t^1(2xt)_{t=0}$,

$= \frac{t^2}{2!}\ (2x)\ = xt^2$

$u_3 = \frac{t^3}{3!}\ D_t^3\left(x - H^{-1}\{\sqrt{a}H[\omega_2]\}\right)_{t=0}$,

$= \frac{t^3}{3!}\ D_t^3\left(x - H^{-1}\{\sqrt{a}H[3xt^2]\}\right)_{t=0}$,

$= -\frac{t^3}{3!}\ D_t^1(6xt^1)_{t=0} = -xt^3$

$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots$

$= -x + xt^2 - xt^3 + \cdots$

$$u(x,t) = \frac{x}{1+t}$$

Example. 4.2.3 Examine PDE

$u_t(x,t) + u_x(x,t) + 2v(x,t) = 0,\ with\ u(x,0) = cos(x),$

$v_t(x,t) + v_x(x,t) - 2u(x,t) = 0,\ with\ v(x,0) = sin(x),$

$H[u_t(x,t)] = -H[2v(x,t)] - H[u_x(x,t)],$

$H[v_t(x,t)] = H[2u(x,t)] - H[v_x(x,t)],$

$\frac{1}{\sqrt{a}}H[u(x,t)] - au(x,0) = -H[2v(x,t)] - H[u_x(x,t)],$

$\frac{1}{\sqrt{a}}H[v(x,t)] - av(x,0) = H[2u(x,t)] - H[v_x(x,t)],$

$H[u(x,t)] = a\sqrt{a}cos(x) - \sqrt{a}H[2v(x,t)] - \sqrt{a}H[u_x(x,t)],$

$H[v(x,t)] = a\sqrt{a}sin(x) + \sqrt{a}H[2u(x,t)] - \sqrt{a}H[v_x(x,t)],$

$u(x,t) = cos(x) - H^{-1}\{\sqrt{a}H[2v(x,t)] + \sqrt{a}H[u_x(x,t)]\},$

$v(x,t) = sin(x) + H^{-1}\{\sqrt{a}H[2u(x,t)] - \sqrt{a}H[v_x(x,t)]\},$

$u(x,t) = \sum_{i=0}^{\infty}\frac{t^i}{i!}\ D_t^i\left(cos(x) - H^{-1}\{\sqrt{a}H[2v(x,t)] + \sqrt{a}H[u_x(x,t)]\}\right)_{t=0}$,

$$\mathrm{v}(x,t)=\sum_{i=0}^{\infty}\frac{t^i}{i!}\,\mathrm{D}_t^i\left(\sin(\mathrm{x})+\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2u(x,t)]-\sqrt{a}\mathrm{H}[\mathrm{v_x}(x,t)]\}\right)_{t=0},$$

Let $u(x,t)=\sum_{n=0}^{\infty}u_n$

$$\mathrm{v}(x,t)=\sum_{n=0}^{\infty}\mathrm{v}_n,$$

$$\sum_{n=0}^{\infty}u_n=\sum_{i=0}^{\infty}\frac{t^i}{i!}\,\mathrm{D}_t^i\left(\cos(\mathrm{x})-\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2\sum_{n=0}^{\infty}\mathrm{v}_n]+\sqrt{a}\mathrm{H}[\sum_{n=0}^{\infty}u_{nx}]\}\right)_{t=0},$$
$$\sum_{n=0}^{\infty}\mathrm{v}_n=\sum_{i=0}^{\infty}\frac{t^i}{i!}\,\mathrm{D}_t^i\left(\sin(\mathrm{x})+\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2\sum_{n=0}^{\infty}u_n]-\sqrt{a}\mathrm{H}[\sum_{n=0}^{\infty}\mathrm{v}_{nx}]\}\right)_{t=0},$$

$$u_0=\cos(\mathrm{x}),$$
$$\mathrm{v}_0=\sin(\mathrm{x}),$$

$$u_1=t\,\mathrm{D}_t^1\left(\cos(\mathrm{x})-\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2\mathrm{v}_0]+\sqrt{a}\mathrm{H}[u_{0x}]\}\right)_{t=0},$$
$$=t\,\mathrm{D}_t^1\left(\cos(\mathrm{x})-\mathrm{H}^{-1}\left\{2a\left(\sqrt{a}\right)^2\sin(\mathrm{x})-a\left(\sqrt{a}\right)^2\sin(\mathrm{x})\right\}\right)_{t=0},$$
$$=-t\sin(\mathrm{x})$$

$$\mathrm{v}_1=t\,\mathrm{D}_t^1\left(\sin(\mathrm{x})+\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2u_0]-\sqrt{a}\mathrm{H}[\mathrm{v}_{0x}]\}\right)_{t=0},$$
$$=t\,\mathrm{D}_t^1\left(\sin(\mathrm{x})+\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2\cos(\mathrm{x})]-\sqrt{a}\mathrm{H}[\cos(\mathrm{x})]\}\right)_{t=0},$$
$$=t\cos(\mathrm{x})$$

$$u_2=\frac{t^2}{2!}\,\mathrm{D}_t^2\left(\cos(\mathrm{x})-\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2\mathrm{v}_1]+\sqrt{a}\mathrm{H}[u_{1x}]\}\right)_{t=0},$$
$$=\frac{t^2}{2!}\,\mathrm{D}_t^2\left(\cos(\mathrm{x})-\mathrm{H}^{-1}\left\{a\left(\sqrt{a}\right)^3\cos(\mathrm{x})\right\}\right)_{t=0},$$
$$=-\frac{t^2}{2!}\cos(\mathrm{x})$$

$$\mathrm{v}_2=\frac{t^2}{2!}\,\mathrm{D}_t^2\left(\sin(\mathrm{x})+\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[2u_1]-\sqrt{a}\mathrm{H}[\mathrm{v}_{1x}]\}\right)_{t=0},$$
$$=\frac{t^2}{2!}\,\mathrm{D}_t^2\left(\sin(\mathrm{x})+\mathrm{H}^{-1}\{\sqrt{a}\mathrm{H}[-2t\sin(\mathrm{x})]+\sqrt{a}\mathrm{H}[t\sin(\mathrm{x})]\}\right)_{t=0},$$
$$=-\frac{t^2}{2!}\sin(\mathrm{x})$$

$$u(x,t)=u_0+u_1+u_2+u_3+\cdots$$

$$=\cos(\mathrm{x})-t\sin(\mathrm{x})-\frac{t^2}{2!}\cos(\mathrm{x})+\frac{t^3}{3!}\sin(\mathrm{x})+\cdots$$

$$=\left(\cos(\mathrm{x})-\frac{t^2}{2!}\cos(\mathrm{x})+\cdots\right)+\left(-t\sin(\mathrm{x})+\frac{t^3}{3!}\sin(\mathrm{x})+\cdots\right)$$

$$u(x,t)=\cos(\mathrm{x})\cos(t)-\sin(\mathrm{x})\sin(t)$$

$$\omega(x,t)=\omega_0+\omega_1+\omega_2+\omega_3+\cdots$$

$$=\sin(\mathrm{x})+t\cos(\mathrm{x})-\frac{t^2}{2!}\sin(\mathrm{x})-\frac{t^3}{3!}\cos(\mathrm{x})+\cdots$$

$$=\left(\sin(\mathrm{x})-\frac{t^2}{2!}\sin(\mathrm{x})+\cdots\right)+\left(t\cos(\mathrm{x})-\frac{t^3}{3!}\cos(\mathrm{x})+\cdots\right)$$

$$\omega(x,t)=\sin(\mathrm{x})\cos(t)-\cos(\mathrm{x})\sin(t)$$

5-Discussion and Comparison

In this section, the numerical and graphical results are presented for **Example 4.1.2 and 4.2.2**, in order to demonstrate the accuracy and efficiency of the proposed methods.

x	y	u_{ADM}	u_{YJDJM}	u_{YJHJM}	u_{exact}	Abs_{Error}
0.00100	0.00300	0.00300	0.00300	0.00300	0.00300	0.00000
0.01411	0.01600	0.01578	0.01578	0.01578	0.01578	0.00000
0.02721	0.02900	0.02823	0.02823	0.02823	0.02823	0.00000
0.04032	0.04200	0.04037	0.04037	0.04037	0.04037	0.00000
0.05342	0.05500	0.05221	0.05221	0.05221	0.05221	0.00000
0.06653	0.06800	0.06376	0.06376	0.06376	0.06376	0.00000
0.07963	0.08100	0.07503	0.07503	0.07503	0.07503	0.00000
0.09274	0.09400	0.08602	0.08602	0.08602	0.08602	0.00000
0.10584	0.10700	0.09676	0.09676	0.09676	0.09676	0.00000
0.11895	0.12000	0.10724	0.10724	0.10724	0.10724	0.00000
0.13205	0.13300	0.11749	0.11749	0.11749	0.11749	0.00000
0.14516	0.14600	0.12749	0.12749	0.12749	0.12749	0.00000
0.15826	0.15900	0.13727	0.13727	0.13727	0.13727	0.00000
0.17137	0.17200	0.14683	0.14683	0.14683	0.14684	0.00000
0.18447	0.18500	0.15618	0.15618	0.15618	0.15619	0.00001
0.19758	0.19800	0.16532	0.16532	0.16532	0.16533	0.00001
0.21068	0.21100	0.17427	0.17427	0.17427	0.17428	0.00002
0.22379	0.22400	0.18302	0.18302	0.18302	0.18304	0.00002
0.23689	0.23700	0.19158	0.19158	0.19158	0.19161	0.00003
0.25000	0.25000	0.19995	0.19995	0.19995	0.20000	0.00005

Table 1. Numerical results for **Example 4.1.2 and 4.2.2**, comparing ADM, YJDJM, and YJHJM solutions with the exact solution, including the absolute error..

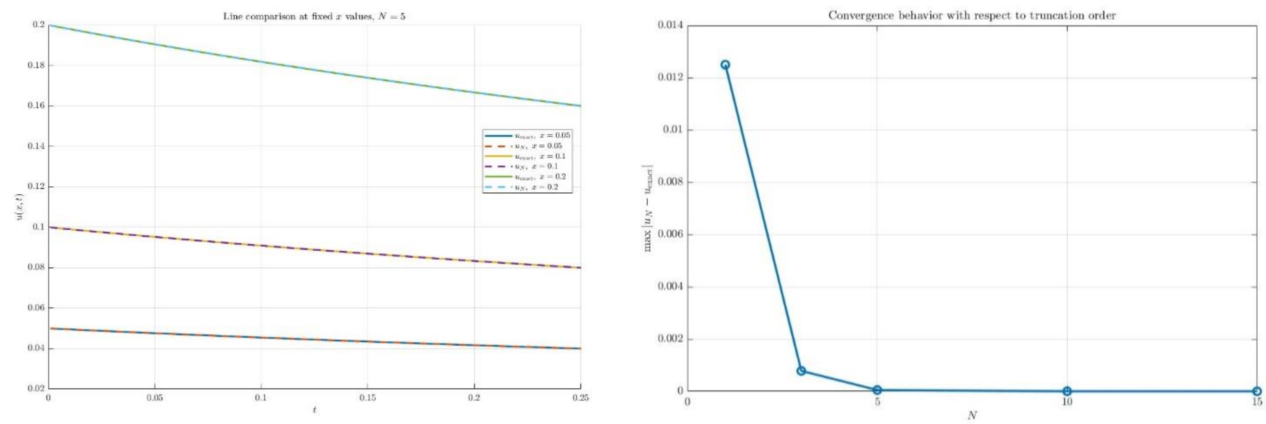

Figure 2. Line comparison at fixed $x$values and convergence behavior of the approximate solution.

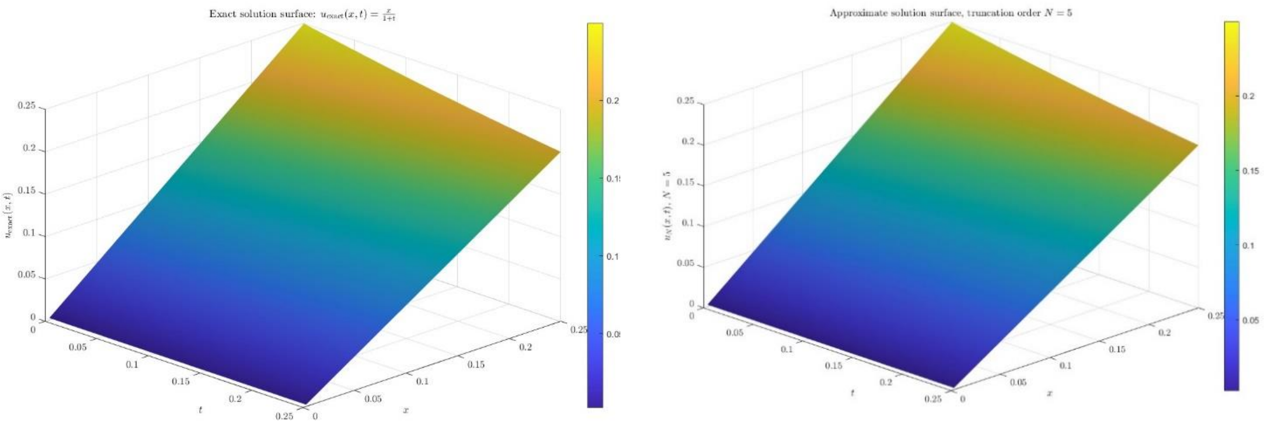

Figure 2. Exact and approximate solution surfaces illustrating the accuracy of the proposed method for $N = 5$.

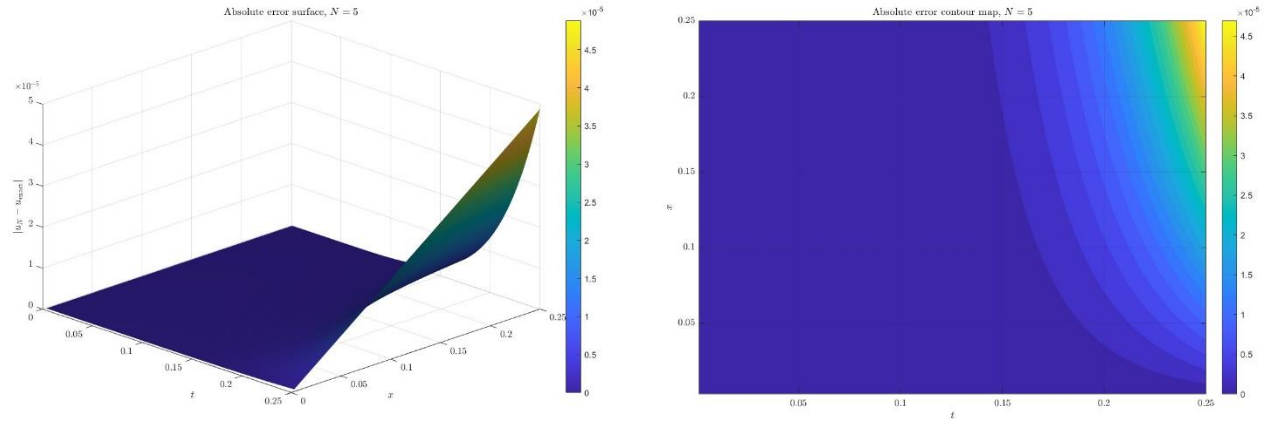

Table 3. Absolute error surface and contour plot for $N = 5$.

6-Conclusion

This research effectively combines the Yasser Jassim Transform with the Daftardar–Jafari Method and the Hussein Jassim Method to solve linear and nonlinear partial differential equations. The suggested methods significantly simplify the solution procedure by avoiding direct integrations and reducing computational complexity.

The numerical and graphical results, including solution surfaces, error analysis, and convergence behaviour, demonstrate that the approximate solutions obtained by YJDJM and YJHJM closely correspond with the exact solution. Moreover, the comparative study reveals that the proposed approaches yield numerical results commensurate with those obtained via direct integration, with negligible absolute errors even at low truncation orders.

The results indicate that the proposed approaches are accurate, efficient, and easy to implement, making them reliable options for solving a wide range of partial differential equations.

Conflicts of Interest.

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] **A. K. Mutashar, M. A. Hussein, and H. K. Jassim**, "An Innovative Computational Method for Solving Partial Differential Equations," Journal of Information Systems Engineering and Management, vol. 10, no. 37s, pp. 1038–1056, 2025. DOI: https://doi.org/10.52783/jisem.v10i37s.6757

[2] **V. Daftardar-Gejji and H. Jafari**, "An iterative method for solving nonlinear functional equations," Journal of Mathematical Analysis and Applications, vol. 316, pp. 753–763, 2006. DOI: https://doi.org/10.1016/j.jmaa.2005.05.009

[3] **M. T. Yasser and H. K. Jassim**, "A new integral transform for solving integral and ordinary differential equations," Mathematics and Computational Sciences, vol. 6, no. 2, pp. 32–42, 2025. DOI: https://doi.org/10.30511/mcs.2025.2045547.1254

[4] **M. M. Abdelrahim Mahgoub**, "The new integral transform (Mohand transform)," Advances in Theoretical and Applied Mathematics, vol. 12, no. 2, pp. 113–120, 2017.

[5] **M. M. Abdelrahim Mahgoub**, "The new integral transform (Sawi transform)," Advances in Theoretical and Applied Mathematics, vol. 14, no. 1, pp. 81–87, 2019.

[6] **K. S. Aboodh**, "The new integral transform (Aboodh transform)," Global Journal of Pure and Applied Mathematics, vol. 9, no. 1, pp. 35–43, 2013.

[7] **G. Adomian**, Solving Frontier Problems of Physics: The Decomposition Method, Boston, MA: Kluwer Academic Publishers, 1994. DOI: https://doi.org/10.1007/978-94-015-8289-6

[8] **M. Siddiqui, M. A. Hameed, B. M. Siddiqui, and Q. K. Ghori**, "Use of Adomian decomposition method in the study of parallel plate flow of a third-grade fluid," Communications in Nonlinear Science and Numerical Simulation, vol. 15, pp. 2388–2399, 2010. DOI: https://doi.org/10.1016/j.cnsns.2009.05.073

[9] **J. H. He**, "Variational iteration method — a kind of nonlinear analytical technique: some examples," International Journal of Non-Linear Mechanics, vol. 34, pp. 699–708, 1999. DOI: https://doi.org/10.1016/S0020-7462(98)00048-1

[10] **S. J. Liao**, Beyond Perturbation: Introduction to Homotopy Analysis Method, Boca Raton, FL: Chapman & Hall/CRC Press, 2004. DOI: https://doi.org/10.1201/9780203491164

[11] **J. H. He**, "Homotopy perturbation technique," Computer Methods in Applied Mechanics and Engineering, vol. 178, pp. 257–262, 1999. DOI: https://doi.org/10.1016/S0045-7825(99)00018-3

[12] **J. H. He**, "A coupling method of a homotopy technique and a perturbation technique for nonlinear problems," International Journal of Non-Linear Mechanics, vol. 35, pp. 37–43, 2000. DOI: https://doi.org/10.1016/S0020-7462(98)00085-7

[13] **J. K. Zhou**, Differential Transformation and Its Applications for Electrical Circuits, Wuhan, China: Huaxhong University Press, 1986 (in Chinese).

[14] **N. Bildik, A. Konuralp, F. Bek, and S. Kucukarslan**, "Solution of different types of partial differential equations by differential transform method and Adomian's decomposition method," Applied Mathematics and Computation, vol. 172, pp. 551–567, 2006. DOI: https://doi.org/10.1016/j.amc.2005.02.037

[15] **S. A. Khuri and A. Sayfy**, "A Laplace variational iteration strategy for the solution of differential equations," Applied Mathematics Letters, vol. 25, pp. 2298–2305, 2012. DOI: https://doi.org/10.1016/j.aml.2012.06.020

[16] **A. F. Abdulhassan and T. M. Elzaki**, "*On Transform-Enhanced Iterative Methods for Nonlinear Fractional Partial Differential Equations*," Journal of Education for Pure Science, vol. 16, no. 1, 2026. doi.org/10.32792/jeps.v16i1.829.