DOI: http://doi.org/10.32792/utq.jceps.11.01.09

Intuitionistic Fuzzy Soft HX Ideals

[1]Walaa Hasan Ashour, [2]Riyam Thamir, [3]Mohammed Jassim Tuamah and [4]Hasan Ali Naser

[1,2and 3] The Ministry of Education, Thi-Qar, Iraq.

[4] University of Thi-Qar, College of Education for pure sciences, Department of Mathematics, Thi-Qar, Iraq.

Received 22/12/2020 Accepted 06/04/2021 Published 10/06/2021

This work is licensed under a Creative Commons Attribution 4.0 International License.

Abstract:
In this paper we defined the concept of intuitionistic fuzzy soft HX ideal of HX ring and prove some results about them, after that we study some types of intuitionistic fuzzy soft HX ideal of HX ring and prove some results about them. Also, we study the image and pre image of intuitionistic fuzzy soft HX ideal of HX ring and prove some results about them.

Keywords: fuzzy soft HX set, intuitionistic fuzzy soft HX set, fuzzy soft HX ideal, intuitionistic fuzzy soft HX ideal, homomorphism and anti homomorphism of an intuitionistic soft fuzzy HX ideal, image and pre-image of an intuitionistic soft fuzzy set.

1- introduction:

The concept of fuzzy sets was first introduced by L. A. Zadeh [7] in 1965 as an extension of the classical notion of a set and studied their properties. Atanassov [1] in 1986 an intuitionistic fuzzy set and studied their properties. Molodtsov [3] in 1999 defined the notion of the soft set and proved some results. In 2001 P. K. Maji, R. Biswas and A. R. Roy [4] are introduced the concept of fuzzy soft set. P. K. Maji, R. Biswas and A. R. Roy [5] defined the notation of intuitionistic fuzzy soft set. Jayanta Ghosh, Bivas Dinda and T.K. Samanta [6] in 2011 defined soft ideal and fuzzy soft ideal and discuss a few of its properties. In 2016, Shuker Mahmood, Zinab Al-Batat [2] studied of intuitionistic fuzzy soft ideals. In 2018, Walaa Hasan Ashour [8] defined the notion of intuitionistic fuzzy soft HX subring of a HX ring and some of their its related properties. In this paper we defined the concept of intuitionistic fuzzy soft HX ideal of HX ring and prove some results about them, after that we study some types of intuitionistic fuzzy soft HX ideal of HX ring and prove some results about them. Also we study the image and pre image of intuitionistic fuzzy soft HX ideal of HX ring and prove some results about them.

2- Preliminaries:

2.1. Definition [3]: Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if F is a function from E into the set of all subsets of the X , i.e. $F: E \to P(X)$, Where $P(X)$ is the power set of X . the set of all soft set over X is denoted by $SS(X, E)$.

2.2. Definition [4]:

Let X be an initial universe set , E a set of parameters and I^X is the set of all fuzzy sets of X .Apair (F, E) is called a fuzzy soft set over X ,Where $F: E \to I^X$ is mapping from E in to I^X.

2.3. Definition: [8]

Let R be a ring . Let (F, E) be a fuzzy soft set defined on R. let $K \subset 2^R\_\{\emptyset\}$ be HX ring on R .we defined a fuzzy soft HX set (G^F, E): $G^F: E \to I^K$ on K as follows :

For all $e \in E$, $G_e^F(A) = \max\{F_e(x) \ / \text{ for all } x \in A \subseteq R\}$.

2.4. Definition: [8]

Let R be a ring . Let $(I\tilde{F}, E)$ be intuitionistic fuzzy soft set defined on R. let $K \subset 2^R\_\{\emptyset\}$ be HX ring on R .we defined intuitionistic fuzzy soft HX set $(G^{\tilde{F}}, E)$ on K as follows :

Where For all $e \in E$, $\mu_{G_e^F}(A) = \max\{\mu_{F_e}(x) \ / \text{ for all } x \in A \subseteq R\}$.

$$v_{H_e^{\tilde{F}}}(A) = \min\{v_{\tilde{F}_e}(x)(x) \ / \text{ for all } x \in A \subseteq R\}.$$

2.5. Definition: [6]

Let $(R, +, .)$ be a ring and E be a parameter set and $A \subseteq E$. Let F be a mapping given by $F: E \to P(X)$. Then (F, E) is called a soft ideal over R if and only if for each $e \in E, F(e)$ is a ideal of R i.e.

(i) $x, y \in F(e) \Rightarrow x - y \in F(e)$.

(ii) $x \in F(e), r \in R \Rightarrow r.x \in F(e), x.r \in F(e)$.

2.6. Definition: [6]

Let $(R, +, .)$ be a ring and E be a parameter set and $A \subseteq E$. Let F be a mapping given by $F: E \to I^x$, where I^x denotes the collection of all fuzzy subsets of R. Then (F, E) is called fuzzy soft ideal over R if and only if for each $e \in E$, the corresponding fuzzy subset $F_e: E \to I$. is a fuzzy ideal of R i.e.

(i) $F_e(x - y) \geq F_e(x) * F_e(y)$.

(ii) $F_e(x.y) \geq F_e(x), \forall x, y \in R$.

2.7. Definition:

Let R be a ring . Let (F, E) be a fuzzy soft ideal defined on R. let $K \subset 2^R\_\{\emptyset\}$ be HX ring . A fuzzy soft subset $(G^F, E) \equiv \{G_e^F, e \in E\}$ of K is called fuzzy soft HX ideal on K or fuzzy soft ideal induced by F if the following conditions are satisfied for all A , B $\in$ K and $e \in E$

i- $G_e^F(A - B) \geq \min\{G_e^F(A), G_e^F(B)\}$

ii- $G_e^F(AB) \geq \max\{G_e^F(A), G_e^F(B)\}$

Where $G_e^F(A) = \max\{F_e(x) \ / \text{ for all } x \in A \subseteq R\}$.

2.8. Definition:

Let R be a ring . Let (F, E) be intuitionistic fuzzy soft ideal on R and a non-empty set $K \subset 2^R\_\{\emptyset\}$ is a HX ring. An intuitionistic fuzzy soft subset $M = \langle A, G_e^F(A), H_e^{\tilde{F}}(A)\rangle$ of a HX ring K is said to be an intuitionistic fuzzy soft HX (IFSHX) ideal of K if the following conditions are satisfied for all $A, B \in K$.

i- $G_e^F(A - B) \geq \min\{G_e^F(A), G_e^F(B)\}$

ii- $G_e^F(AB) \geq \max\{G_e^F(A), G_e^F(B)\}$

iii- $H_e^{\tilde{F}}(A - B) \leq \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

iv- $H_e^{\tilde{F}}(AB) \leq \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

Where $G_e^F(A) = \max\{F_e(x) \,/\, \text{for all } x \in A \subseteq R\}$

$H_e^{\tilde{F}}(A) = \min\{\tilde{F}_e(x) \,/\, \text{for all } x \in A \subseteq R\}$.

2.9. Definition:

Let R be a ring . Let (F, E) be intuitionistic fuzzy soft ideal on R and a nonempty set $K \subset 2^R\_\{\emptyset\}$ is a HX ring . An intuitionistic fuzzy soft subset $M = \langle A, G_e^F(A), H_e^{\tilde{F}}(A)\rangle$ of a HX ring K is said to be an intuitionistic anti fuzzy soft HX (IFSHX) ideal of K if the following conditions are satisfied for all $A, B \in K$.

i- $G_e^F(A - B) \leq \max\{G_e^F(A), G_e^F(B)\}$

ii- $G_e^F(AB) \leq \min\{G_e^F(A), G_e^F(B)\}$

iii- $H_e^{\tilde{F}}(A - B) \geq \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

iv- $H_e^{\tilde{F}}(AB) \geq \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

Where $G_e^F(A) = \min\{F_e(x) \,/\, \text{for all } x \in A \subseteq R\}$

$H_e^{\tilde{F}}(A) = \max\{\tilde{F}_e(x) \,/\, \text{for all } x \in A \subseteq R\}$.

2.10. Theorem:

If M_1 and M_2 be two intuitionistic fuzzy soft HX ideal K then $M_1 \cap M_2$ is also intuitionistic fuzzy soft HX ideal of HX ring K .

Proof:

Let $M_1 = \{\langle A, G_e^F(A), H_e^{\tilde{F}}(A)/A \in K\rangle\}$ and

$M_2 = \{\langle B, W_e^F(B), W_e^{\tilde{F}}(B)/B \in K\rangle\}$ be two intuitionistic fuzzy soft aHX ideal of HX ideal K .

i- $(G_e^F \cap W_e^F)(A - B) = \min\{G_e^F(A - B), W_e^F(A - B)\}$

$\geq \min\{\min\{G_e^F(A), G_e^F(B)\}, \min\{W_e^F(A), W_e^F(B)\}\}$

$= \min\{\min\{G_e^F(A), W_e^F(A)\}, \min\{G_e^F(B), W_e^F(B)\}\}$

$= \min\{(G_e^F \cap W_e^F)(A), (G_e^F \cap W_e^F)(B)\}$

Hence, $(G_e^F \cap W_e^F)(A - B) \geq \min\{(G_e^F \cap W_e^F)(A), (G_e^F \cap W_e^F)(B)\}$

ii- $(G_e^F \cap W_e^F)(AB) = \max\{G_e^F(AB), W_e^F(AB)\}$

$\geq \max\{\max\{G_e^F(A), G_e^F(B)\}, \max\{W_e^F(A), W_e^F(B)\}\}$

$= \max\{\max\{G_e^F(A), W_e^F(A)\}, \max\{G_e^F(B), W_e^F(B)\}\}$

$= \max\{(G_e^F \cap W_e^F)(A), (G_e^F \cap W_e^F)(B)\}$

Hence, $(G_e^F \cap W_e^F)(AB) \geq \max\{(G_e^F \cap W_e^F)(A), (G_e^F \cap W_e^F)(B)\}$

iii- $(H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(AB) = \max\{H_e^{\tilde{F}}(AB), W_e^{\tilde{F}}(AB)\}$ $\leq$
$\max\{\max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}, \max\{W_e^{\tilde{F}}(A), W_e^{\tilde{F}}(B)\}\}$ $=$
$\max\{\max\{H_e^{\tilde{F}}(A), W_e^{\tilde{F}}(A)\}, \max\{H_e^{\tilde{F}}(B), W_e^{\tilde{F}}(B)\}\}$

$= \max\{(H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(A), (H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(B)\}$

Hence, $(H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(AB) \leq \max\{(H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(A), (H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(B)\}$

iv- $(H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(AB) = \min\{H_e^{\tilde{F}}(AB), W_e^{\tilde{F}}(AB)\}$

$\leq \min\{\min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}, \min\{W_e^{\tilde{F}}(A), W_e^{\tilde{F}}(B)\}\}$

$= \min\{\min\{H_e^{\tilde{F}}(A), W_e^{\tilde{F}}(A)\}, \min\{H_e^{\tilde{F}}(B), W_e^{\tilde{F}}(B)\}\}$

$= \min\{(H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(A), (H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(B)\}$

Hence, $(H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(AB) \leq \min\{(H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(A), (H_e^{\tilde{F}} \cap W_e^{\tilde{F}})(B)\}$

Therefore the intersection of any two(IFSHX) ideal is also an (IFSHX) ideal HX ring K.

2.11. Theorem:

If M be an intuitionistic fuzzy soft HX ideal of HX ring K if and only if M^c is an antuitionistic anti fuzzy soft HX ideal of HX ring K.

Proof:

Let $M = \{\langle A, G_e^F(A), H_e^{\tilde{F}}(A)/A \in K\rangle\}$ be antuitionistic fuzzy soft HX ideal of HX ring K.

i- $(G_e^F)^c(A-B) = 1 - G_e^F(A-B) \leq 1 - \min\{G_e^F(A), G_e^F(B)\}$

$= 1 - \min\{1 - (G_e^F)^c(A), 1 - (G_e^F)^c(B)\} = \max\{(G_e^F)^c(A), (G_e^F)^c(B)\}$

Hence, $(G_e^F)^c(A-B) \leq \max\{(G_e^F)^c(A), (G_e^F)^c(B)\}$

ii- $(G_e^F)^c(AB) = 1 - G_e^F(AB)$

$\leq 1 - \max\{G_e^F(A), G_e^F(B)\} = 1 - \max\{1 - (G_e^F)^c(A), 1 - (G_e^F)^c(B)\} = \min\{(G_e^F)^c(A), (G_e^F)^c(B)\}$

Hence , $(G_e^F)^c(AB) \leq \min\{(G_e^F)^c(A), (G_e^F)^c(B)\}$

iii- $(H_e^{\tilde{F}})^c(A-B) = 1 - H_e^{\tilde{F}}(A-B)$

$\geq 1 - \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\} = 1 - \max\left\{1 - (H_e^{\tilde{F}})^c(A), 1 - (H_e^{\tilde{F}})^c(B)\right\}$

$= \min\left\{(H_e^{\tilde{F}})^c(A), (H_e^{\tilde{F}})^c(B)\right\}$

Hence, $(H_e^{\tilde{F}})^c(A-B) \geq \min\left\{(H_e^{\tilde{F}})^c(A), (H_e^{\tilde{F}})^c(B)\right\}$

iv- $(H_e^{\tilde{F}})^c(AB) = 1 - H_e^{\tilde{F}}(AB)$

$\geq 1 - \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\} = 1 - \min\left\{1 - (H_e^{\tilde{F}})^c(A), 1 - (H_e^{\tilde{F}})^c(B)\right\}$

$= \max\left\{(H_e^{\tilde{F}})^c(A), (H_e^{\tilde{F}})^c(B)\right\}$

Hence, $(H_e^{\tilde{F}})^c(AB) \geq \max\left\{(H_e^{\tilde{F}})^c(A), (H_e^{\tilde{F}})^c(B)\right\}$

Hence $M^c = \{\langle A, (G_e^F)^c(A), (H_e^{\tilde{F}})^c(A)/A \in K\rangle\}$ be intuitionistic fuzzy soft aHX ideal of HX ring K.

Conversely, let M^c be intuitionistic fuzzy soft aHX ideal HX ring K

i- $G_e^F(A-B) = 1 - (G_e^F)^c(A-B)$

$\geq 1 - \max\{(G_e^F)^c(A), (G_e^F)^c(B)\}$

$= 1 - \max\{1 - G_e^F(A), 1 - G_e^F(B)\} = \min\{G_e^F(A), G_e^F(B)\}$

Hence, $G_e^F(A-B) \geq \min\{G_e^F(A), G_e^F(B)\}$

ii- $G_e^F(AB) = 1 - (G_e^F)^c(AB) \leq 1 - \min\{(G_e^F)^c(A), (G_e^F)^c(B)\}$

$= 1 - \min\{1 - G_e^F(A), 1 - G_e^F(B)\} = \max\{G_e^F(A), G_e^F(B)\}$

Hence, $G_e^F(AB) \leq \max\{G_e^F(A), G_e^F(B)\}$

iii- $H_e^{\tilde{F}}(A-B) = 1-\left(H_e^{\tilde{F}}\right)^c(A-B)$
$\leq 1-\min\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\}$
$= 1-\min\{1-H_e^{\tilde{F}}(A), 1-H_e^{\tilde{F}}(B)\} = \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$
Hence, $H_e^{\tilde{F}}(A-B) \leq \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

iv- $H_e^{\tilde{F}}(AB) = 1-\left(H_e^{\tilde{F}}\right)^c(AB) \leq 1-\max\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\}$
$= 1-\max\{1-H_e^{\tilde{F}}(A), 1-H_e^{\tilde{F}}(B)\} = \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$
Hence, $H_e^{\tilde{F}}(AB) \leq \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$
$M = \{\langle A, G_e^F(A), H_e^{\tilde{F}}(A)/A \in K\rangle\}$ be an intuitionistic fuzzy soft HX ideal of HX ring K.

2.12. Definition:

$M = \{\langle A, G_e^F(A), H_e^{\tilde{F}}(A)/A \in K\rangle\}$ be intuitionistic fuzzy soft subset of HX ideal of K. We define the following " necessity " and "possibility " operations

$$\square M = \{\langle A, G_e^F(A), 1- G_e^F(A)\rangle/A \in K\}$$
$$\diamond M = \left\{\langle A, H_e^{\tilde{F}}(A), 1- H_e^{\tilde{F}}(A)\rangle/A \in K\right\}$$

2.13. Theorem:

If M is an be intuitionistic fuzzy soft HX ideal of a HX ring K then □M is an intuitionistic fuzzy soft HX ideal of HX ring deal K.

Proof:

Let $\square M = \{\langle A, G_e^F(A), (G_e^F)^c(A)\rangle/A \in K\}$ and
$M = \left\{\langle A, G_e^F(A), (H_e^{\tilde{F}})(A)\rangle/A \in K\right\}$ be intuitionistic fuzzy soft HX ideal of K

Now

i- $(G_e^F)^c(A-B) = 1-G_e^F(A-B) \leq 1-\min\{G_e^F(A), G_e^F(B)\}$
$= 1-\min\{1-(G_e^F)^c(A), 1-(G_e^F)^c(B)\} = \max\{(G_e^F)^c(A), (G_e^F)^c(B)\}$

ii- $(G_e^F)^c(AB) = 1-G_e^F(AB) \leq 1-\max\{G_e^F(A), G_e^F(B)\}$
$= 1-\max\{1-(G_e^F)^c(A), 1-(G_e^F)^c(B)\} = \min\{(G_e^F)^c(A), (G_e^F)^c(B)\}$
Hence , $G_e^F((A-B) \geq \min\{G_e^F(A), G_e^F(B)\}$,
$G_e^F((AB) \geq \max\{G_e^F(A), G_e^F(B)\}$
$(G_e^F)^c(A-B) \leq \max\{(G_e^F)^c(A), (G_e^F)^c(B)\}$
$(G_e^F)^c(AB) \leq \min\{(G_e^F)^c(A), (G_e^F)^c(B)\}$
Therefore □M is an intuitionistic fuzzy soft HX ideal of HX ring K .

2.14. Theorem:

If M is an be intuitionistic fuzzy soft HX ideal of a HX ring K then ⋄ M is an intuitionistic fuzzy soft HX ideal of HX ring K .

Proof:

Let $\diamond M = \left\{\langle A, \left(H_e^{\tilde{F}}\right)^c(A), H_e^{\tilde{F}}(A)\rangle / A \in K\right\}$ and

$M = \{\langle A, G_e^F(A), H_e^{\tilde{F}}(A)\rangle / A \in K\}$ be intuitionistic fuzzy soft HX ideal of K

Now

i- $\left(H_e^{\tilde{F}}\right)^c(A-B) = 1 - H_e^{\tilde{F}}(A-B) \geq 1 - \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

$= 1 - \max\left\{1 - \left(H_e^{\tilde{F}}\right)^c(A), 1 - \left(H_e^{\tilde{F}}\right)^c(B)\right\} = \min\left\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\right\}$

ii- $\left(H_e^{\tilde{F}}\right)^c(AB) = 1 - H_e^{\tilde{F}}(AB) \geq 1 - \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

$= 1 - \min\left\{1 - \left(H_e^{\tilde{F}}\right)^c(A), 1 - \left(H_e^{\tilde{F}}\right)^c(B)\right\} = \max\left\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\right\}$

Hence, $H_e^{\tilde{F}}(A-B) \leq \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$, $H_e^{\tilde{F}}(AB) \leq \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$,

$\left(H_e^{\tilde{F}}\right)^c(A-B) \geq \min\left\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\right\}$,

$\left(H_e^{\tilde{F}}\right)^c(AB) \geq \max\left\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\right\}$

Hence $\diamond M = \left\{\langle A, \left(H_e^{\tilde{F}}\right)^c(A), 1 - H_e^{\tilde{F}}(A)\rangle / A \in K\right\}$ be intuitionistic fuzzy soft aHX ideal of HX ring K.

2.15. Theorem:

An IFSS $M = \{\langle A, G_e^F(A), H_e^{\tilde{F}}(A) / A \in K\rangle\}$ be intuitionistic fuzzy soft HX ideal of HX ring K if and only if the fuzzy soft subsets $G_e^F(A)$, $\left(H_e^{\tilde{F}}\right)^c$ are fuzzy soft HX ideal of HX ring k.

Proof:

Let $M = \{\langle A, G_e^F(A), H_e^{\tilde{F}}(A) / A \in K\rangle\}$ be intuitionistic fuzzy soft HX ideal of HX ring K

i- $\left(H_e^{\tilde{F}}\right)^c(A-B) = 1 - H_e^{\tilde{F}}(A-B) \geq 1 - \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

$= 1 - \max\left\{1 - \left(H_e^{\tilde{F}}\right)^c(A), 1 - \left(H_e^{\tilde{F}}\right)^c(B)\right\}$

$= \min\left\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\right\}$

Hence, $\left(H_e^{\tilde{F}}\right)^c(A-B) \geq \min\left\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\right\}$

ii- $\left(H_e^{\tilde{F}}\right)^c(AB) = 1 - H_e^{\tilde{F}}(AB) \geq 1 - \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

$= 1 - \min\left\{1 - \left(H_e^{\tilde{F}}\right)^c(A), 1 - \left(H_e^{\tilde{F}}\right)^c(B)\right\}$

$= \max\left\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\right\}$

Hence, $\left(H_e^{\tilde{F}}\right)^c(AB) \geq \max\left\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\right\}$

Thus $\left(H_e^{\tilde{F}}\right)^c$ is a fuzzy soft HX ideal of K.

Conversely, let $G_e^F(A)$, $\left(H_e^{\tilde{F}}\right)^c$ are fuzzy soft HX ideal of HX ring K

Now

i- $H_e^{\tilde{F}}(A-B) = 1 - \left(H_e^{\tilde{F}}\right)^c(A-B \quad \leq 1 - \min\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\}$

$= 1 - \min\{1 - H_e^{\tilde{F}}(A), 1 - H_e^{\tilde{F}}(B)\} = \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

Hence, $H_e^{\tilde{F}}(A - B) \leq \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$

ii- $H_e^{\tilde{F}}(AB) = 1 - \left(H_e^{\tilde{F}}\right)^c(AB) \leq 1 - \max\{\left(H_e^{\tilde{F}}\right)^c(A), \left(H_e^{\tilde{F}}\right)^c(B)\}$
$= 1 - \max\{1 - H_e^{\tilde{F}}(A), 1 - H_e^{\tilde{F}}(B)\} = \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$
Hence, $H_e^{\tilde{F}}(AB) \leq \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$
Already we have
$G_e^F(A - B) \geq \min\{G_e^F(A), G_e^F(B)\}$, $G_e^F(AB) \geq \max\{G_e^F(A), G_e^F(B)\}$
Hence, $M = \{\langle A, G_e^F(A), H_e^{\tilde{F}}(A)/\in K\rangle\}$ be intuitionistic fuzzy soft HX ideal of HX ring K.

2.16. Theorem:

An IFSS $M = \{\langle A, G_e^F(A), H_e^{\tilde{F}}(A)/A \in K\rangle\}$ be intuitionistic fuzzy soft HX ideal of HX ring K if and only if the fuzzy soft subsets $(G_e^F)^c, H_e^{\tilde{F}}$ are anti fuzzy soft HX ideal of HX ring K.

Proof:

Let $M = \{\langle A, G_e^F(A), H_e^{\tilde{F}}(A)/A \in K\rangle\}$ be intuitionistic fuzzy soft HX ideal of HX ring K

i- $(G_e^F)^c(A - B) = 1 - G_e^F(A - B)$
$\leq 1 - \min\{G_e^F(A), G_e^F(B)\} = 1 - \min\{1 - (G_e^F)^c(A), 1 - (G_e^F)^c(B)\}$
$= \max\{(G_e^F)^c(A), (G_e^F)^c(B)\}$
Hence, $(G_e^F)^c(A - B) \leq \max\{(G_e^F)^c(A), (G_e^F)^c(B)\}$

ii- $(G_e^F)^c(AB) = 1 - G_e^F(AB) \leq 1 - \max\{G_e^F(A), G_e^F(B)\}$
$= 1 - \max\{1 - (G_e^F)^c(A), 1 - (G_e^F)^c(B)\}$
$= \min\{(G_e^F)^c(A), (G_e^F)^c(B)\}$
Hence, $(G_e^F)^c(AB) \leq \min\{(G_e^F)^c(A), (G_e^F)^c(B)\}$
Hence, $(G_e^F)^c$ and $H_e^{\tilde{F}}$ are anti – fuzzy soft HX ideal of HX ring K.
Conversely, $(G_e^F)^c$ and $H_e^{\tilde{F}}$ are anti – fuzzy soft HX ideal of HX ring K.

i- $G_e^F(A - B) = 1 - (G_e^F)^c(A - B \geq 1 - \max\{(G_e^F)^c(A), (G_e^F)^c(B)\}$
$= 1 - \max\{1 - G_e^F(A), 1 - G_e^F(B)\} = \min\{G_e^F(A), G_e^F(B)\}$
Hence $G_e^F(A - B) \geq \min\{G_e^F(A), G_e^F(B)\}$

ii- $G_e^F(AB) = 1 - (G_e^F)^c(AB) \geq 1 - \min\{(G_e^F)^c(A), (G_e^F)^c(B)\}$
$= 1 - \min\{1 - G_e^F(A), 1 - G_e^F(B)\} = \max\{G_e^F(A), G_e^F(B)\}$
Hence, $G_e^F(AB) \geq \max\{G_e^F(A), G_e^F(B)$
Thus
$H_e^{\tilde{F}}(A - B) \leq \max\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$, $H_e^{\tilde{F}}(AB) \leq \min\{H_e^{\tilde{F}}(A), H_e^{\tilde{F}}(B)\}$
Hence Let $M = \left\{\langle A, G_e^F(A), \frac{H_e^{\tilde{F}}(A)}{A} \in K\rangle\right\}$ be intuitionistic fuzzy soft HX ideal of HX ring K.

2.17. Definition:

Let R_1 and R_2 be any two ring let $K_1 \subset 2^{R_1} - \{\emptyset\}$ and $K_2 \subset 2^{R_2} - \{\emptyset\}$ be two HX ring .

i- let $A = \{\langle x, \mu_{F(x)}(x), \upsilon_{F(x)}(x)\rangle / x \in R_1\}$ be any intuitionistic fuzzy soft sets on R_1 . let $C = \{\langle (U, G_e^F(U), H_e^{\tilde{F}}(U))\rangle / U \in K_1\}$ be intuitionistic fuzzy soft sets on K_1 and Let f be a function from K_1 in to K_2 then the image of C on K_1 under f is defined as :

$$f(G_e^F(U)) = \begin{cases} \max\{G_e^F(U) : U \in f^{-1}(V)\}, f^{-1}(V) \neq \emptyset\}, \\ 0 \qquad \text{otherwise} \end{cases}$$

$$f(H_e^{\tilde{F}}(U)) = \begin{cases} \min\{H_e^{\tilde{F}}(U) : \{U \in f^{-1}(V)\}, f^{-1}(V) \neq \emptyset\} \\ 0 \qquad \text{otherwise} \end{cases}$$

ii- $B = \{\langle y, \alpha_{F(B)}(y), \beta_{F(B)}(y)\rangle / y \in R_2\}$ be intuitionistic fuzzy soft sets on R_2.

$D = \{\langle V, M_e^F(V), N_e^{\tilde{F}}(V)\rangle \ / \ V \in K_2\}$ be intuitionistic fuzzy soft sets on K_2.then pre – image of D on K_2 under f is defined $f^{-1}\left(M_e^F(V)\right)(U) = M_e^F(f(U))$ Also $f^{-1}\left((N_e^{\tilde{F}}(V))\right) = N_e^{\tilde{F}}(f(V))$

2.18. Theorem:

Let R_1 and R_2 be any two rings. $A = \{\langle x, \mu_{F(x)}(x), \upsilon_{F(x)}(x)/\rangle x \in R_1\}$ and $B = \{\langle y, \alpha_{F(B)}(y), \beta_{F(B)}(y)\rangle / y \in R_2\}$ any two intuitionistic fuzzy soft sets on R_1 and R_2 respectively and $C = \{\langle U, G_e^F(U), H_e^{\tilde{F}}(U)\rangle / U \in K_1\}$ be intuitionistic fuzzy soft sets on K_1 .

Let f be a on to homomorphism from K_1 to K_2. If C intuitionistic fuzzy soft HX ideal of K_1 then f(C) is a intuitionistic fuzzy soft HX ideal of K_2.

Proof:

Let C intuitionistic fuzzy soft HX ideal of K_1 and let $V = f(U)$, $W = f(T) \in K_2$ Where $U, T \in K_1$

i- $M_e^F[f(U) - f(T)] = M_e^F[f(U - T)]$

$= G_e^F[U - T]$

$\geq \min\{G_e^F(U), G_e^F(T)\} = \min\{M_e^F(f(U), M_e^F(f(T)\}$

Hence , $M_e^F[f(U) - f(T)] \geq \min\{M_e^F(f(U), M_e^F(f(T)\}$

ii- $M_e^F[f(U)f(T)] = M_e^F[f(UT)]$

$= G_e^F[UT]$

$\geq \max\{G_e^F(U), G_e^F(T)\} = \max\{M_e^F(f(U), M_e^F(f(T)\}$

Hence, $M_e^F[f(U)f(T)] \geq \max\{M_e^F(f(U), M_e^F(f(T)\}$

iii- $N_e^{\tilde{F}}[f(U) - f(T)] = N_e^{\tilde{F}}[f(U - T)]$ (f is hom)

$= H_e^{\tilde{F}}[U - T] \leq \max\{H_e^{\tilde{F}}(U), G_e^F(T)\}$

$= \max\{N_e^{\tilde{F}}(f(U), N_e^{\tilde{F}}(f(T)\}$

Hence, $N_e^{\tilde{F}}[f(U) - f(T)] \leq \max\{N_e^{\tilde{F}}(f(U), N_e^{\tilde{F}}(f(T)\}$

iv- $N_e^{\tilde{F}}[f(U)f(T)] = N_e^{\tilde{F}}[f(UT)]$ (f is hom)

$= H_e^{\tilde{F}}[UT] \leq \min\{H_e^{\tilde{F}}(U), G_e^F(T)\}$

$= \min\{N_e^{\tilde{F}}(f(U), N_e^{\tilde{F}}(f(T)\}$

Hence, $N_e^{\tilde{F}}[f(U) - f(T)] \leq \min\{N_e^{\tilde{F}}(f(U), N_e^{\tilde{F}}(f(T)\}$

Thus f(C) is a intuitionistic fuzzy soft HX ideal of K_2.

2.19. Theorem:

Let R_1 and R_2 be any two rings. $A = \{\langle x, \mu_{F(x)}(x), \upsilon_{F(x)}(x)\rangle / x \in R_1\}$ and $B = \{\langle y, \alpha_{F(B)}(y), \beta_{F(B)}(y)\rangle / y \in R_2\}$ any two intuitionistic fuzzy soft sets on R_1 and R_2 respectively and $D = \{\langle V, M_e^F(V), N_e^{\tilde{F}}(V)\rangle / V \in K_2\}$ be intuitionistic fuzzy soft sets on K_2.

Let f be a on to homomorphism from K_1 to K_2. If D intuitionistic fuzzy soft HX ideal of K_2 then $f^{-1}(D)$ is a intuitionistic fuzzy soft HX ideal of K_1.

Proof:

let D intuitionistic fuzzy soft HX ideal of K_2 .

let $V = f(U)$, $W = f(T) \in K_2$ Where $U, T \in K_1$

i- $[f^{-1}(M_e^F)](U - T) = M_e^F[(f(U - T)]$

$= M_e^F[f(U) - f(T)]$

$\geq \min\{M_e^F(f(U), f(T)\} = \min\{f^{-1}(M_e^F(U), f^{-1}(M_e^F(T)\}$

Hence, $[f^{-1}(M_e^F)](U - T) \geq \min\{f^{-1}(M_e^F(U), f^{-1}(M_e^F(T)\}$

ii- $[f^{-1}(M_e^F)](UT) = M_e^F[(f(UT)]$

$= M_e^F[f(U)f(T)]$

$\geq \max\{M_e^F(f(U), f(T)\} = \max\{f^{-1}(M_e^F(U), f^{-1}(M_e^F(T)\}$

Hence, $[f^{-1}(M_e^F)](UT) \geq \max\{f^{-1}(M_e^F(U), f^{-1}(M_e^F(T)\}$

iii- $f^{-1}(N_e^{\tilde{F}})(U - T) = N_e^{\tilde{F}}[f(U - T)]$

$= N_e^{\tilde{F}}[f(U - T)] \leq \max\{N_e^{\tilde{F}}(f(U)), N_e^{\tilde{F}}(f(T))\}$

$= \max\{[f^{-1}(N_e^{\tilde{F}})](U), [f^{-1}(N_e^{\tilde{F}})](T)\}$

Hence, $f^{-1}(N_e^{\tilde{F}})(U - T) \leq \max\{[f^{-1}(N_e^{\tilde{F}})](U), [f^{-1}(N_e^{\tilde{F}})](T)\}$

iv- $f^{-1}(N_e^{\tilde{F}})(UT) = N_e^{\tilde{F}}[f(UT)] = N_e^{\tilde{F}}[f(UT)]$

$\leq \min\{N_e^{\tilde{F}}(f(U)), N_e^{\tilde{F}}(f(T))\} = \min\{[f^{-1}(N_e^{\tilde{F}})](U), [f^{-1}(N_e^{\tilde{F}})](T)\}$

Hence, $f^{-1}(N_e^{\tilde{F}})(U - T) \leq \min\{[f^{-1}(N_e^{\tilde{F}})](U), [f^{-1}(N_e^{\tilde{F}})](T)\}$

Therefore $f^{-1}(D)$ is a intuitionistic fuzzy soft HX ideal of K_1.

2.20. Theorem:

Let R_1 and R_2 be any two rings. $A = \{\langle x, \mu_{F(x)}(x), \upsilon_{F(x)}(x)\rangle / x \in R_1\}$ and $B = \{\langle y, \alpha_{F(B)}(y), \beta_{F(B)}(y)\rangle / y \in R_2\}$ any two intuitionistic fuzzy soft sets on R_1 and R_2 respectively and $C = \{\langle U, G_e^F(U), H_e^{\tilde{F}}(U)\rangle / U \in K_1\}$ and $D = \{\langle V, M_e^F(V), N_e^{\tilde{F}}(V)\rangle / V \in K_2\}$ any two intuitionistic fuzzy soft sets on K_1 and K_2 respectively . Let f be an on to anti-homomorphism from K_1 to K_2. If C intuitionistic fuzzy soft HX ideal of K_1 then f(C) is a intuitionistic fuzzy soft HX ideal of K_2.

Proof:

If C intuitionistic fuzzy soft HX ideal of K_1. Let

$V = f(U)$, $W = f(T) \in K_2$ Where $U, T \in K_1$

i- $M_e^F[f(U) - f(T)] = M_e^F[f(T - U)]$

$= G_e^F[T - U]$

$\geq \min\{G_e^F(T), G_e^F(U)\} \geq \min\{G_e^F(U), G_e^F(T)\} = \min\{M_e^F(f(U), M_e^F(f(T)\}$

Hence, $M_e^F[f(U) - f(T)] \geq \min\{ M_e^F(f(U), M_e^F(f(T)\}$

ii- $M_e^F[f(U)f(T)] = M_e^F[f(TU)]$

$= G_e^F[TU]$

$\geq \max\{G_e^F(T), G_e^F(U)\} \geq \max\{G_e^F(U), G_e^F(T)\}$

$= \max\{ M_e^F(f(U), M_e^F(f(T)\}$

Hence, $M_e^F[f(U)f(T)] \geq \max\{ M_e^F(f(U), M_e^F(f(T)\}$

iii- $N_e^{\tilde{F}}[f(U) - f(T)] = N_e^{\tilde{F}}[f(T - U)]$

$= H_e^{\tilde{F}}[T - U] \leq \max\{H_e^{\tilde{F}}(T), H_e^{\tilde{F}}(U)\} \leq \max\{H_e^{\tilde{F}}(U), H_e^{\tilde{F}}(T)\} =$ $\max\{ N_e^{\tilde{F}}(f(U), N_e^{\tilde{F}}(f(T)\}$

Hence, $N_e^{\tilde{F}}[f(U) - f(T)] \leq \max\{ N_e^{\tilde{F}}(f(U), N_e^{\tilde{F}}(f(T)\}$

iv- $N_e^{\tilde{F}}[f(U)f(T)] = N_e^{\tilde{F}}[f(UT)]$

$= H_e^{\tilde{F}}[UT] \leq \min\{H_e^{\tilde{F}}(U), H_e^{\tilde{F}}(T)\} \leq \min\{H_e^{\tilde{F}}(T), H_e^{\tilde{F}}(U)\}$

$= \min\{ N_e^{\tilde{F}}(f(U), N_e^{\tilde{F}}(f(T)\}$

Hence, $N_e^{\tilde{F}}[f(U) - f(T)] \leq \min\{ N_e^{\tilde{F}}(f(U), N_e^{\tilde{F}}(f(T)\}$

Thus $f(C) = D$ is a intuitionistic fuzzy soft HX ideal of K_2.

2.21. Theorem*:*

Let R_1 and R_2 be any two rings. $A = \{\langle x, \mu_{F(x)}(x), \upsilon_{F(x)}(x)\rangle / x \in R_1\}$ and $B = \{\langle y, \alpha_{F(B)}(y), \beta_{F(B)}(y)\rangle / y \in R_2\}$ any two intuitionistic fuzzy soft sets on R_1 and R_2 respectively and $C = \{\langle U, G_e^F(U), H_e^{\tilde{F}}(U)\rangle\rangle / U \in K_1\}$ and $D = \{\langle V, M_e^F(V), N_e^{\tilde{F}}(V)\rangle / V \in K_2\}$ be any two intuitionistic fuzzy soft sets on K_1 and K_2 respectively .

Let f be an on to anti- homomorphism from K_1 to K_2. If D intuitionistic fuzzy soft HX ideal of K_2 then $f^{-1}(D)$ is a intuitionistic fuzzy soft HX ideal of K_1.

Proof:

let D intuitionistic fuzzy soft HX ideal of K_2 .

let $V = f(U)$, $W = f(T) \in K_2$ Where $U, T \in K_1$

i- $[f^{-1}(M_e^F)](U - T) = M_e^F[(f(U - T)]$

$= M_e^F[f(T) - f(U)]$

$\geq \min\{M_e^F(f(T), f(U)\} \geq \min\{M_e^F(f(U), f(T)\}$

$= \min\{[f^{-1}(M_e^F)](U), [f^{-1}(M_e^F)](T)\}$

Hence , $[f^{-1}(M_e^F)](U - T) \geq \min\{[f^{-1}(M_e^F)](U), [f^{-1}(M_e^F)(T)]\}$

ii- $[f^{-1}(M_e^F)](UT) = M_e^F[(f(UT)]$

$= M_e^F[f(U)f(T)]$

$\geq \max\{M_e^F(f(T), f(U)\} \geq \max\{M_e^F(f(U), f(T)\}$

$= \max\{[f^{-1}(M_e^F)](U), [f^{-1}(M_e^F)](T)\}$

Hence, $[f^{-1}(M_e^F)](UT) \geq \max\{[f^{-1}(M_e^F)](U), [f^{-1}(M_e^F)](T)\}$

iii- $[f^{-1}(N_e^{\tilde{F}})](U - T) = N_e^{\tilde{F}}[f(U - T)]$

$= N_e^{\tilde{F}}[f(T - U)]$

$\leq \max\{N_e^{\tilde{F}}(f(T)), N_e^{\tilde{F}}(f(U))\} \leq \max\{N_e^{\tilde{F}}(f(U)), N_e^{\tilde{F}}(f(T))\}$

$= max\ \{[f^{-1}(N_e^{\tilde{F}})](U), [f^{-1}(N_e^{\tilde{F}})](T)\}$

Hence, $[f^{-1}(N_e^{\tilde{F}})](U-T) \leq \max\ \{[f^{-1}(N_e^{\tilde{F}})](U), [f^{-1}(N_e^{\tilde{F}})](T)\}$

iv- $f^{-1}(N_e^{\tilde{F}})(UT) = N_e^{\tilde{F}}[f(UT)]$

$= N_e^{\tilde{F}}[f(T)f(U)]$

$\leq \min\{N_e^{\tilde{F}}(f(T)), N_e^{\tilde{F}}(f(U))\} \leq \min\{N_e^{\tilde{F}}(f(U)), N_e^{\tilde{F}}(f(T))\}$

$= \min\ \{[f^{-1}(N_e^{\tilde{F}})](U), [f^{-1}(N_e^{\tilde{F}})](T)\}$

Hence, $f^{-1}(N_e^{\tilde{F}})(U-T) \leq \min\ \{[f^{-1}(N_e^{\tilde{F}})](U), [f^{-1}(N_e^{\tilde{F}})](T)\}$

Therefore $f^{-1}(D)$ is a intuitionistic fuzzy soft HX ideal of K_1.

Reference:

[1] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems 20, 87 – 96, 1986.

[2] S. Mahmood, Zinab Al-Batat, ntuitionistic Fuzzy Soft LA- Semigroups and Intuitionistic Fuzzy Soft Ideals, International Journal of Applications of Fuzzy Sets and Artificial Intelligence, Vol. 6 (2016), 119 – 132.

[3] D. Molodtsov, "Soft set theory-first results," Computers & Mathematics with Applications, vol. 37, no.4-5, pp. 19–31, 1999.

[4] P. K. Maji, A. R. Roy and R. Biswas, Fuzzy soft sets, Journal of Fuzzy Mathematics. 9 (3) 589-602 (2001).

[5] P. K. Maji, R. Biswas, and A. R. Roy, "Intuitionistic fuzzy soft sets," The Journal of Fuzzy Mathematics, vol. 9, no. 3, pp. 677-692, 2001.

[6] Jayanta Ghosh, Bivas Dinda and T.K. Samanta, Fuzzy Soft Rings and Fuzzy Soft Ideals, Int. J. Pure Appl. Sci. Technol., 2(2) (2011), pp. 66-74.

[7] L.A. Zadeh, Fuzzy sets, Information and Control, 338-353, 8 (1965).

[8] W.H. Ashour, On Fuzzy Soft HX Ring and Intuitionistic Fuzzy Soft HX Ring.