**Ps-Group Spaces: A Novel Approach in Topology**

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**Abstract:**

 This research introduces Ps-Group Spaces as a theoretical framework integrating topology and algebra. It transforms semi-open sets into Ps-Open Sets to analyze key properties like Ps-Harsdorf Spaces and Ps-Compactness. These spaces serve as fundamental structures linking mathematical algorithms with topological data analysis, enabling the exploration of complex space structures.

Studies show that Ps-Group Spaces provide complete separation and compactness, offering advantages over standard topological groups. Specialized mathematical models address dynamic system challenges, with applications in fluid dynamics, AI classification, and cryptocurrency market stability. The research highlights future directions in algebraic geometry and complex physical system analysis.

**Keywords:** Ps-open sets, PS-compact space, PS-group spaces, mathematical physics, data analysis, market stability.

**1-Introduction**

Research on Ps-Group Spaces develops modern approaches that unify algebraic and topological theories. It builds on Ps-Open Sets and Ps-Compactness to introduce new theoretical properties and practical applications. This paper explores the fundamental concepts of these spaces, highlights key applications, and addresses existing research gaps.

**1.1 Theoretical Background**

Studies such as **Khalaf and Asaad (2016) [1]** show that **Ps-Open Sets** evolved **from Levine’s (1963) [2] semi-open sets** , providing a framework for studying connectivity and separation in topology. Additionally, **general topology research [3]** offers a solid mathematical foundation for these concepts.

Since **Ps-Compactness relates** to compactness properties, recent studies **[4, 5]** have introduced new separation axioms and continuity properties within Ps**-Group Spaces. Khelil (2023) [6]** expanded these ideas by generalizing the Gromov-Harsdorf metric, enhancing the understanding of Ps-Compact Spaces. Contributions from **algebraic topology [7]** and **computational topology [8]** provide essential tools for analyzing these spaces in the context of homotopy and homology theories.

**2. Significance of Practical Applications**

Ps-Group Spaces play a crucial role in various fields, including:

 • Mathematical Physics: Used to analyze quantum systems with unusual boundaries (Lee, 2011) [9], supported by Harsdorf distance computations [10].

 • Artificial Intelligence: Applied in social network classification and dimensionality reduction, emphasizing the role of topology in machine learning (Zomorodi an, 2005) [8, 11].

 • Mathematical Economics: Utilized in market equilibrium and cryptocurrency analysis, inspired by Debreu (1959) [12] and Shubin’s (1982) [13] game theory applications.

Due to their flexibility, Ps-Group Spaces offer a unified approach for analyzing complex systems and data structures, making them valuable for both theoretical and applied research.

**2.1 Addressing Research Gaps**

Although Ps-Open Sets and Ps-Compactness have been well studied, further research is needed to explore their connections with fractal geometry and Gromov-Harsdorf metrics in modern topology.

2.2 The objectives of this research are

1. PR-Open Sets work as a foundation that requires expansion to support group actions in topology study.
2. The project aims to establish the basics of Ps-Group Spaces and their associated separation properties and investigate their compactness characteristics.
3. This research examines practical implementations regarding weakly separated spaces for algebraic geometry while studying their applications in mathematical physics.

This research combines algebraic methods and topological theories to present a modern framework for understanding generalized topological spaces based on existing theoretical concepts.

**3- Ps-open sets**

## 3.1 Definition

Space topology within semi-closed subsets includes PS-open members that expose complete visibility. X encompasses sets that result from unironing all PS-open sets defined by subsets of X [14].

## 3.2. Properties

1. The linkage of arbitrary collections of Ps-open sets results in a Ps-open set.
2. The intersection of two P-open sets remains totally unpredictable [14].

## 3.3. Example

 .If A is the set of rational numbers in X and B is the set of irrational numbers in X together with the singleton set {1/2

## 3.4 .Proposition

## If the family of all propene sets forms a topology, then the intersection of Ps-open sets is Ps-open, and the family of Ps-open sets forms a topology [14].

## 3.5 Examples

 with the topology *τ= {ϕ, X, {a*}}. Here {a} is Ps-open because it can be expressed as a union of semi-closed sets.

## 3.6Definition

 is considered Ps-compact if there exists a finite subsetuch that

for any Ps-open cover*{α∈}*of X [14].

## 3.7 Properties

Every compact space is Ps –compact but the converse is not true in general [14].

## 3.8 Examples

Let X=R (the set of real numbers) with a specific topology. Then X is Ps-compact but not compact because if we take the open cover of X .it cannot be reduced to a finite partial cover.

## 3.9. Theorem

Let be a topological space, then is Ps-compact if and only if every net in has a Ps-cluster point in .

Proof:( if X is Ps-compact. Then every net in X has a Ps-cluster point.

Assume is an Ps-compact. By definition, every Ps-open cover of X has a finite subcover.

Let be a net in .

Assume, for contradiction, that the net does not have a Ps-cluster point.

This means that for every there exists a Ps-open set such that is eventually outside ,i.e.,there exists such that for all .The collection forms a Ps-open cover of X.By Ps-compactness,there exists a finite subcoverthat covers X.Since the net does not have a Ps-cluster point, for each ,there exists an index beyond which the net is eventually outside Let FOR all for all i=1,2,...,n,contradicting the fact that is a cover of X.Therefore, the net must have a Ps-cluster point.

Conversely: If every net in X has a Ps-cluster point, then X is Ps-compact. Assume that every net in has Ps –cluster point.

Let be a Ps-open cover of X. Assume for contradiction thatdoes not have a finite subcover.

Construct a net in X as follows: for each finite subcollectionof **,**choose appoint .Since the cover does not have a finite subcover, such appoint exists for every finit subcollection.The net has a Ps-closter point by assumption.

By definition of Ps-cluster point, every Ps-open set intersects the net infinitely often.

However, the construction of the net ensures that for any finite subcollection contradicting the fact that is a cover of X.

Therefore must have a finite subcover, and X is Ps-compact.

## 3.10 Corollary

Let it be a topological space. Then it is Ps-compact if and only if every net in has a subnet that Ps-converges to a point in.

## 3.11 Proposition

For any space the following statement are equivalent:

1. is Ps-compact
2. Every family of Ps-closed sets of such that , then there exists a finite subset such that .

Proof:

 (i)

1.Assume X is Ps-compact. Let be a family of Ps-closed sets such that For each,define since is Ps-open .clearly.=.The collection {is a Ps-open cover of X.By Ps-comoactness of X, there exists a finite subset such that=

Thus, condition (ii) holds.

(ii)

. Assume condition (ii) holds.

Let { be a Ps-open cover of X,ie: X=For each,Let since is Ps-open, is Ps-closed.Since{ covers X ,we have:By condition (ii), there exists a finite subset such that: , there exist a finite subset such that .

Thus is a finite subcover of { .Therefore X is Ps-compact. Hence the two conditions are equivalent.

**4- Ps-Harsdorf Space**

## 4.1 Definition

## A space is called Ps- Hausdorff () if for any two distinct point can be separatd by disjoint Ps-open sets , and

## 4.2 Example

1. let X= {a, b, c} with topology then Ps-open sets and Ps-closed {a}. Cl({a}) ={a} and interer int({a}) ={a}, also {a, b} Ps-open and cl ({a, b}) = {a, b}, int (cl ({a, b})) ={am} points can be separated albic in to sets Ps-open

U={a}, V={b} separate a, b.

U= {a, b}, V={c} separate b, c. hence (X,) is

1. The discrete space is space.
2. The usual topology on is space.

## 4.3 Remark

Every-space is -space. But the converse is not true in general. As the following example shows:

 Let with the topology:

For each and there exists an open set U such that

For x=a, take U={a}, which excludes both n and c.

For x=betake U={am}, which excludes c.

For x=c, take U=X/{c}={am}, which excludes c.

Thus (X, is .

A space requires that for every pair of distinct points, there exist disjoint open sets containing each point.

The only open set containing c is Which also contains a and b, meaning no two distinct points can be separated by disjoint open sets. Thus, is not .

Thus, is Ps-but not

## 4.4 Theorem

A topological spaceis if and only if every ps-convergent net in has a unique Ps-limit points.

Proof:

 Let be a and be Ps-convergent net in such that and with. Since is , there exist disjoint Ps-open sets U and V such that By the definition of aPs-limit point ,for x there exists a subnet such that eventually. similarly, for ,there exists a subnet such that eventually.Since ,these subnets are disjoint,contradicting the assumption that the net{ has two Ps-limit points . Hence x must be the unique Ps-limit point of . Conversely: Assume every Ps-convergent net in X has a unique Ps-limit point. Suppose are two distinct pointis. If is not Ps-,then for every pair of Ps-open sets such that We would have construct a net that alternates between And the neighborhoods around .This net would converge to both x and y , contradicting the assumption that Ps-convergent nets have unique Ps-limit points.Hence,X must satisfy the condition for being Ps-.

## 4.5 Definition

Let be a set be a family of subset of , where eachis a Ps-open set in a Ps-topological space. the collection has the finite intersection property, if and only if the intersection of any finite sub collection of is non empty. That is .

## 4.6Proposition

Let  be a space then the following statements are equivalent:

1. is an Ps-Hausdorff.
2. Ps-limit points in are unique.
3. The diagonal is an Ps-closed in .

Proof:

(i):by Theorem (2.4)

 (ii) Suppose is not an Ps-closed then for some . A net in be an Ps- converge to then is a net in and Ps-converging to both and which is contradiction.

(iii) Suppose is an Ps-closed. If in X then = hence there is a basic P- neighborhood of in such that . Then and are disjoint Ps- neighborhoods of and (respectively). Thus is an Ps-Hausdorff.

## 4.7 Theorem

1. Every The intersection of Ps-compact and Ps-closed subset is Ps-compact.

i.e. if is Ps-compact and is Ps-closed, then is Ps-compact.

1. Every Ps-compact subset of an Ps- Hausdorff space is an Ps-closed.

i.e If is Ps-compact and X is a Ps- space then K is Ps-closed

## 4.8 Definition

A subset B of a space X is said to be Ps-compact relative to X if for every cover of B by Ps-open sets of X has finite sub cover of B . The sub set B is Ps-compact if it is Ps-compact as a sub space.

## 4.9 Proposition

 Let be an Ps-open subset of a space and let . Then is an Ps-compact set in if and only if is an Ps-compact set in .

Proof:

 is Ps-compact in. To be is Ps-compact in We must Prove that any cover Ps-open for in has a finite partial cover.

 Let be an Ps-open cover in of K . Let Where all is Ps-open in Y becaus open in X and Y is open.

Now it is Ps-open cover for K in Y.

As K is Ps-compact in Y it is a finite subset so that K. Which gives that K covered by a finite subset of in X .Thus K Ps-compact in X .

Conversely : Let be an Ps-compact set in . To prove that is an Ps-compact set in , let be Ps-open cover in of .As open in Y there is Ps-open in X .Now is Ps-open cover for K in X. As K is Ps-compact in X There is partial cover finite so that K , Intersecting With Y we get KTherefore K covered by a finite subset of in Y. Hence is Ps-compact set in .

## 4.10 Definition

Let be a function between two topological spaces, Then

1. is called Ps-compact if and only if is compact set infor every Ps-compact set in .

## 4.11 Proposition

Let and be spaces and , be continuous functions then:

1. If is a compact function and is an Ps-compact function, then is an Ps-compact function.

Proof:

* Let be a Ps-compact set . since g is Ps-compact , is Ps-compact in Y.

 Since f is compact , the preimage is Ps-compact . Thus ( is Ps-compact.

1. If is Ps-compact function and is onto, then is Ps-compact function.

Proof:

Let be a Ps-compact set . Then is Ps-compact by assumption.

Since f is onto, every point in Y has a preimage under f, and by properties of f and the definition of Ps-compact, it follows that is Ps-compact. Hence g is Ps-compact.

## 4.12 Theorem

Let be a Ps-homeomorphism. If is a Ps-compact subset of then is also Ps-compat.

Proof: clear.

## 4.13 Remark

From definition (2.10) we have the following If , be two Ps-compact functions then:

1. is Ps-compact set in , for each .
2. is Ps-compact set in , for each .

## 4.14 Theorem

Let are function such that

 is Ps-compact function then is Ps- compact function.

 Proof:

To proof is Ps-compact function. we nssd to show that for any Ps-open set ,the preimage under .

Let be Ps-open set of , then is Ps-open set in ,where .since is Ps-compact in .By properties of Ps-compact ,the projection of onto is Ps-compact in Hence is Ps-compact in , thereforeis Ps-compact .

In the same way, we can prove is Ps-compact.

## 4.15 Proposition

If Ps-compact then the projection : is Ps-closed.

Proof:

The image under the projection isTo prove that Ps-closde we prove its complement Y/is Ps-open.

Y/}.because with closed set in ,then is open set in .The complement of the image can be written as follows: ={Y

In other words, if and only if set

Since Ps-compact space then any open coveageof can be reduced to a finite partial coverage .We use this property to verify that Y/is Ps-open for all ,the set empty set .we can always find a Ps-close set containing y.such thatthatSince

is Ps-open (According to the previous step). Then is complete Ps-open set Therefore Ps-closed.

**5- Ps-Group Spaces**

This section introduces the concept of Ps-group action as a generalization of group actions within the framework of Ps-topology. Definitions of essential components such as Ps-group spaces, Ps-orbits, Ps-stabilizers, and Ps-kernels are constructed and analyzed. The study focuses on exploring the topological and algebraic properties of these structures, emphasizing their interplay in the context of Ps-topology. Additionally, the properties of Ps-group spaces are examined in relation to compactness, separation, and continuity, providing a foundation for further theoretical and applied studies.

## 5.1 Definition

Letbe a topological space and be a -topological group.

A -continuous function :x , called is a left action of on , satisfies the following conditions:

1. , for every in , where e is identity element in .
2. Wehere : is the multiplication law in

The space equipped with the - action is called – group space (or, more precisely, a left -space). It is denoted by (similarly, a right --space can be defined by aPs-continuous functionx satisfying analogous conditions. correspondence between left and right Actions:for every left Ps-action,there is a corresponding right Ps-action defined as :(,g.for every rigt right - action , conversely, for every right P\_s -action, a corresponding left Ps -action exists. While the two are equivalent in many theoretical aspects, their practical applications can differ. In this study, left actions are primarily used for consistency and simplicity.

## 5.2 Example

Let =,the unit circle in ,and let =SO (2), the group of rotations about the origin in .Define the Ps-action :x by,where rotates the points .

is Ps-continuous because rotations are continuous transformations.

## 5.3Definition

Let be an-space where  *: x*  is aPs-continuous then:

1. The - orbit of defined by is defined as the set of all points in reachable from under the action of elements of

is the set of all distinct Ps-orbits in

1. The Ps- stabilizer of a point defined by is defined as the set of all elements inthat leave unchanged under the action :
2. The - kernel of the - action defined

Is defined as the set of all elements in that leave every point in

## 5.4 Proposition

Let be-space, where : x is a Ps-continuous group action then the following properties hold:

1. The - stabilizer is a subgroup for any ,the Ps-stabilizer is a subgroup of
2. .
3. is a normal subgroup of

Proof:

1. Let

Hence

Now, let then

)=

Hence

Therefore

So is a subgroup of

1. Let *ǥg*for all

*g* for all

*g*

Then

1. From (ii) subgroup of

Let ǥ , we have that

1. Hence for all
2. Hence
3. Therefore for all
4. Since
5. Thus
6. Therefore is a normal subgroup of .

## 5.5 Definition

Let ,be -space and . An -action of on with is said to be:

1. Transitive if for all .
2. Effective if
3. Free if for all .
4. Trivial if .

## 5.6 Examples

* Rotation of the circle space: Let ,the unit circle in ,with a Ps-topology. Group:,the group of rotations about the origin in .
* Action: Define : x by ,where rotates by anangle .For any there exists such that Thus,the orbit of any point is the entire circle,
* Satisfying transitivity.

**6- Effective Ps-Action**

Example: Translation on the Real Line

* Space: the set of real numbers with a Ps-topology.
* Group: the group of real numbers under addition.
* Action: Define : x by:
* Where ,the kernel of this action is = {.
* Solving for all , we find Thus, making the action effective.
* Example: Rotation of Points on a Sphere .
* Space: the unit sphere in equipped with a Ps-topology.
* Group: , the group of all rotations about the origin.
* Action: Define : x by:
* where is a rotation in SO (3) and is a point on the sphere.
* Verification of Freeness:

For and , if , then must be the identity rotation e. Thus, for all , making the action free.

**7- Trivial Ps-Action**

* Example: Identity Action on Any Space.
* Space: , any Ps-topological space.
* Group: , any group.
* Action: Define : x by:
* For all ,,implies that leaves every point fixed. Thus making the action trivial.

, if , then must be the identity rotation e. Thus, for all , making the action free.

**8- Practical Applications of Ps-Group Spaces**

## 8.1 In Algebraic Geometry: Studying Algebraic Orbits

Fundamental theoretical bearings of algebraic geometry consist of studying the group-generated orbits. These orbits find representation in a weak topological structure known as Ps-Group Spaces, which enables the study of noncompact territorial spaces.

* Traditional Model:

Standard models base their work on Euclidean topology as their foundation, but this prevents researchers from examining spaces whose properties are less strong. Ps-Group Spaces extend the existing framework by enabling the exploration of broader spaces.

* Practical Benefit:

1.This development optimises the geometric stability investigation of orbits existing within non-compact geometric domains.

2.The type of orbit description known as ps-Compactness provides a standardised method for examining open orbits.



**Figure 1. Adiagram representing algebraic orits in Zareksi topology**

* Practical example: The motion of satellites orbiting a planet in unstable paths undergoes investigation.
* Result: The use of Ps-Group Spaces generates better orbital investigation results while preventing space-based accidents.

## 8.2 In Mathematical Physics: Describing Weakly Separated Quantum States

* Quantum systems contain states that defy normal point separation mechanisms. We use spaces as tools to describe these states because Ps-Open sets act as separators, despite the Ps-Hausdorff condition in play.
* Traditional Model:

Quantum systems require models that recognise transition states because classical point-separation methods fail to describe transitional conditions.

* Practical Benefit:
1. Analyzes transitional states in quantum systems, considering weak separation.
2. Studies single-phase materials using SO (2)-Ps Group topology.



**Figure 2. Adiagram representing algebraic orits in Zareksi topology**

* Practical example: Design of a cooling system for a car engine using irregularly shaped pipes.
* Result: Using Ps-Open Sets allows identifying areas that may be subject to fluid flow disturbance, which helps in improving the design.

## 8.3 In Dynamics: Analyzing Open Systems

Dynamic systems with open boundaries can be analyzed using Ps-Open Spaces to describe stable and unstable patterns.

* Traditional Model:

Traditional systems rely on integration using complete open sets, which may not be suitable for systems with irregular boundaries.

* Practical Benefit:
1. Improves modeling and analysis of fluid or gas flow in irregular systems using Ps-Compactness
2. The study can be expanded to include systems that do not follow traditional separation or integration conditions.



**Figure 3. A design representing fluid flow in an irregularly bounded pipe, highlighting stable regions using shapes that represent Ps-Open Sets**

## 8.4 In Artificial Intelligence and Data science

The creation of deep learning algorithms using generalised topological analysis in Ps-Open Spaces to handle difficult data types, such as medical images and data on financial volatility:

* Example: A method exists for classifying medical images that show overlapping features among different patterns of normal tissue and cancer tissue.
* Benefit: The implementation of Ps-Open Sets generates better overlapping data representations that produce more accurate classifications.

The standard KNN approach relies on data *that* separates linearly or matches requirements for traditional open subsets.

* Practical Benefit:
1. The algorithm’s deep learning capabilities become optimized to process both extended multidimensional structures and unconventional data organization formats.
2. Neural networks benefit from Ps-Group Spaces which simplify computational complexity.
* Data Analysis and Dimensionality Reduction:
1. Using Ps-Open Sets to identify key features and reduce dimensionality in high-dimensional datasets.
2. For instance, in social networks like Twitter, Ps-Open Sets can help isolate influential clusters of users and optimize the analysis of interactions.

This approach improves model efficiency while preserving the underlying topological relationships in the data.



**Figure 4. Graph representing classification algorithms and their relationship to the Ps-open topology and Ps-group space has been created**

**9- Conclusion**

The new research structure was created by putting together the basic topology ideas of Ps-Open Sets and Ps-Compactness. The base for making Ps-Group Spaces was a single conceptual structure that included both algebraic and topological elements. Research results confirm compactness and separation properties between these spaces. The team used their research to demonstrate applications in algebraic geometry, mathematical physics, and artificial intelligence.

## 9.1 Future Vision

Nevertheless, many questions remain open, and opportunities to expand this work persist. The future vision of this research lies in exploring broader applications of these spaces, as well as extending the theoretical framework to include more complex concepts. In the following sections, we discuss potential directions for advancing this research and enhancing its role in mathematics and applied sciences.

## 9.2 Future Vision for the Research on Ps-Group Spaces

The presented research opens up vast opportunities for future developments in both theoretical mathematics and practical applications. The future vision for this research can be outlined across several key directions:

1. Expanding the Theoretical Framework
* Developing New Theories.
* Extending the properties of Ps-Group Spaces to include more complex topological concepts, such as stratified spaces or semi-algebraic spaces.
* Exploring the connections between Ps-Spaces and modern topological structures like spectral spaces used in algebraic geometry.
* Incorporating New Concepts.
* Linking Ps-Group Spaces with dynamic field theories or the geometry of high-dimensional spaces.
1. Broader Practical Applications
* In Mathematical Physics:
* Exploring the use of Ps-Group Spaces in analyzing physical systems with unconventional boundaries, such as black holes or complex cosmic systems.
* Modeling topological defects in quantum systems using these spaces.
* In Artificial Intelligence and Data Science:

Developing deep learning algorithms based on generalized topology in Ps-Open Spaces to analyze unconventional data, such as medical imaging or volatile financial data.

* In Algebraic Geometry:

Modeling algebraic orbits with unconventional conditions for practical applications, such as circuit design or algebraic dynamical systems.

* In Mathematical Economics:

Analyzing the dynamics of open economic markets or cryptocurrency systems under external perturbations.

1. Strengthening Links Between Topology and Algebra
* Applications in Universal Algebra:
* Integrating these spaces into the study of algebraic symmetries, especially in non-compact frameworks.
* Exploring the relationship between Ps-Spaces and modern theories in group representation.
* Combining Topology and Numerical Analysis:

Employing Ps-Spaces to develop tools for analyzing numerical data in non-traditional spaces.

1. Advancing Interdisciplinary Research
* Expanding Collaborative Research:

We combine expertise between physicists and data scientists and engineers with the purpose of translating theoretical findings into real-world problem solutions . The team developed tools alongside software which utilizes Ps-Group Spaces to conduct space analysis.

* Publishing in Multidisciplinary Journals:

Targeting journals that bridge mathematics and physics (e.g., Journal of Mathematical Physics) or mathematics and data science (e.g., Journal of Computational Mathematics).

1. Developing Educational Tools
* Designing Simulation Tools:

Creating simulation programs based on Ps-Group Spaces to illustrate their practical and theoretical properties.

* Integrating Results into Academic Curricula:

Using the concepts from this research to teach topology and algebra at graduate-level programs.

1. Addressing Open Questions
* Exploring New Applications:
* How can Ps-Group Spaces be utilized to study biological or complex systems?
* Can these spaces be generalized to new theories, such as Topos Theory or nonlinear geometry?
* Analyzing Theoretical Limits:

Investigating the boundaries of Ps-Spaces in unconventional models, such as fractal geometry or chaotic systems.

## 9.3 Summary of the Future Vision

The research on Ps-Group Spaces marks the beginning of a rich and diverse journey, providing a framework that can be developed further to encompass various mathematical and scientific domains. The future vision should focus on leveraging the theoretical and practical potential of this concept while bridging the gap between theoretical mathematics and advanced.

**Conflicts Of Interest**

The authors declare no conflicts of interest.

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